Enlargement of the EU and Weighted Voting in its Council of Ministers

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Preface

Of all the decision-making bodies of the European Union (EU), the Council of Ministers (CM) is by far the most important, and is likely to remain so for the foreseeable future, although the European Parliament—at first virtually impotent—is gradually winning considerable authority.

The CM has several different rules for adopting decisions, depending on the kind of issue involved. The greatest, and growing, number of issues (except those concerned with the constitution of the EU itself) are decided by a weighted voting rule, known in EU parlance as *qualified majority voting* (QMV): each member state is assigned a number of bloc-votes, or *weight*, and a proposed resolution is carried if the total weight of those voting for it equals or exceeds a certain *quota*. The weights and quota must of course be fixed afresh each time the EU is enlarged (see Table 2).

The weights correspond very roughly to population sizes: more populous states have greater weights. But there is no strict proportionality: the ratio of weight to population is highest for the small member states and lowest for the most populous ones (see Table 3). This has created a widespread feeling that the present weighting system makes QMV very inequitably biased in favour of the smaller members and against the larger ones, and that it should therefore be readjusted so as to eliminate this apparent bias. The pressure for such a rejigging has become very powerful in view of the planned enlargement of the EU, which will in any case require new weights and quota to be put in place. Several plans to this effect have been proposed, and are at present being discussed by officials of the EU and ministers of the member states.

The proper evaluation of a decision rule of an assembly, committee or council raises some theoretical questions that are considerably more complex than they seem at first glance. Among the most intricate are those concerning two-tier systems, in which the direct voters in a council, such as the CM, are themselves representatives of constituencies—which may be of unequal size—such as the EU states, whose electors therefore exercise *indirect* influence over the council's decisions. Some of what the scientific theory of voting power, founded in 1946 by Lionel Penrose [38, 39], has to say about these matters is rather surprising, as it goes against the expectation of naïve common sense.

As authors of several papers on voting power [14]–[18], [20]–[25], [35] and of the only monograph wholly devoted to this subject [19], we are acutely aware of the fact that the present discussions of QMV in the context of the EU's prospective enlargement, like earlier opinions on weighted voting pronounced in US courts, have been bedevilled by insufficient knowledge, let alone understanding, of the relevant theory. People commonly make assumptions that appear to be commonsensical, even obvious, but that on closer examination turn out to be fallacious. This applies not only to members of the general public but also to journalists, officials, politicians and jurists.

For example, since the inception of the EU and through its four successive enlargements to date, the quota in the CM has been held steady at about 71% of the total weight. This was evidently done with the intention of keeping things as they are. But the actual and clearly unintended result was to make it, with each successive enlargement of the EU, progressively more difficult to adopt a resolution (and thus change the status quo) under QMV.

Our aim in writing this booklet is to help bridge this information gap. It is not addressed to academic experts, so we omit mathematical proofs and similar technicalities, for which we refer the more mathematically erudite reader to our book [19] and other sources. Rather, we aim to present the conclusions of the theoretical analysis and outline the reasoning leading to them in as non-technical a way as possible without dumbing them down.

After a brief outline of the basic concepts of the theory of voting power, we discuss several criteria for evaluating any proposed decision rule for the CM or any similar council of representatives.

First and foremost is the basic democratic desideratum of *equitability*—'one person, one vote' (OPOV)—according to which the (indirect) influence of electors in the various constituencies ought to be as nearly equal as possible, irrespective of the different sizes of these constituencies. A citizen of Germany ought, in principle, to have just as much influence over a decision of the CM as a citizen of Greece or of Luxembourg. The problem as to which decision rules of a council of representatives are equitable in this sense was solved by Penrose [38]; and the solution was rediscovered twenty years later by John Banzhaf [2, 3].

Second, there is the desideratum of *majoritarianism*: the rule used by the council should arguably come as close as possible to producing outcomes that conform to the wishes of the majority of the entire electorate (in the case of the CM: the entire citizenry of the EU). Majoritarianism has often been confused with equitability; but in fact the two are quite distinct. The solution to the problem of satisfying majoritarianism was conjectured by Peter Morriss [36] and proved by us [19, 20].

A third desideratum is that of *sensitivity* or *responsiveness*. To define this concept precisely we must first provide some preliminary explanations; we shall get to it in Subsection 1.2.4. Roughly speaking, the sensitivity of a council's decision rule is the degree to which the members of the council collectively are empowered as decision-makers. The concept of sensitivity was defined by Pradeep Dubey and Lloyd Shapley [13] and an index of *relative* sensitivity was proposed by us in [16].

Our fourth criterion for evaluating a decision rule of a council is its *resistance*: the degree to which the rule tends to favour the defeat of any proposed bill—the preservation of the status quo—as opposed to its approval. Arguably, a decision rule that has a very low degree of resistance may be undesirable, as it tends to make approving new bills *too* easy. On the other hand, a decision rule whose degree of resistance is very high tends to engender immobilism. An index of resistance was proposed by us [19]; it is closely related to 'the power of a collectivity to act' as defined by James Coleman [6].

After presenting these theoretical concepts and results, we apply them to evaluating nine different decision rules for an enlarged CM of 28 members. Five of these rules have been proposed and discussed at recent meetings of the EU's Intergovernmental Conference (IGC) on Institutional Reform [8, 9, 10]; the remaining four are rules that we believe are worth considering, if only for the sake of comparison.

We hope that this booklet will contribute to a better-informed public discussion of the various proposals regarding the decision rule of the enlarged CM.

Chapter 1

Introduction: Measuring Voting Power

In this introductory chapter and in the next chapter we outline some of the concepts and results of the theory of voting power. In the main text we avoid as far as we can the use of technical jargon and mathematical formulas: these are mostly relegated to the footnotes, which also contain references to the literature for additional technical matters such as rigorous statements and proofs of theorems. A reader uninterested in these technicalities may skip the footnotes.

1.1 Decision rules

Before we address the problem of measuring voting power, we must first explain what we mean here by *decision rule*. Such a rule applies to some *assembly* of *voters*.[1] When the assembly is called upon to make a decision on a bill, each of its members votes either 'yes' or 'no'.[2] This creates a *division* of the assembly into two camps: the 'yes' voters and the 'no' voters.[3] For an assembly consisting of two voters, there are four possible divisions (both voters say 'yes', both say 'no', voter a says 'yes' and b says 'no', or a says 'no' and b 'yes'); for an assembly of three voters, the number of possible divisions

[1]A voter may be an individual person or a bloc of several persons who invariably act as one.

[2]The rules that apply in the CM do not treat abstention or absence as a distinct option: under QMV, abstention and absence are tantamount to 'no', whereas in matters that require unanimity abstention counts as 'yes' and absence as 'no'. Cf. [19, p. 148, fn. 10 and p. 156, fn. 26].

[3]In [19] such a division is referred to as a *bipartition*.

is eight; an assembly of four gives rise to 16 divisions; and so on.[4]

Any division must have one of two possible outcomes: *positive*, in which case the bill is passed; or *negative*, in which case the bill is blocked. The function of a decision rule for a given assembly is to specify the outcome—positive or negative—of each possible division of the assembly.

1.1.1 Weighted rules The most common kind of decision rule is exemplified by the five rules shown in Table 2: each voter is assigned a non-negative number of votes, or *weight*; and a positive number, which is not greater than the sum of all weights, is specified as *quota*. What determines the outcome of a division is the total weight of the 'yes' voters in it: the outcome is positive just in case the total weight of the 'yes' camp equals or exceeds the quota.

Note that in principle a voter can be assigned weight 0 (which is of course a non-negative number); but in practice this would be silly, because it would make such a voter a *dummy*, whose vote—whether 'yes' or 'no'—would not make the slightest difference to the outcome. However, a voter having positive weight can also be a dummy. In fact, this was the case with Luxembourg under the QMV in the period 1958–72, when its weight was 1: as the reader can easily check by consulting the column headed '1958' in Table 2, Luxembourg's vote could never affect the outcome. (This didn't matter all that much, because the Treaty of Rome stipulated that QMV would not be used until 1966; and even in 1966–72 it was only used on rare occasions. Still, it seems a bit of a blunder; and as we shall see in Subsection 1.2.1 it was not the last one in the history of QMV in the CM.)

1.1.2 Decision rules in general[5] More generally, and put rather abstractly, a decision rule for a given assembly is *any* classification of all its possible divisions into those with positive or negative outcome, subject only to the following three conditions:

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[4]Putting it generally, an assembly with n members has 2^n possible divisions.

[5]For reasons that will be mentioned in Subsection 1.3.1, the terminology used in the literature on voting power is largely borrowed from the jargon of cooperative game theory. Thus, a decision rule (in the sense to be explained here) is referred to as a *simple game* or *simple voting game* (SVG); a weighted rule is called a *weighted voting game* (WVG); and voters are often called *players*. Any set of voters is referred to as a *coalition*—a particularly confusing usage, which we shall avoid here, because it falsely suggests the connotation of 'coalition' in normal political discourse, as an alliance of persons, parties or states that agree to act together on a more or less regular basis. The set of 'yes' voters in a division whose outcome is positive is referred to as a *winning* coalition; while any other set of voters is called a *losing* coalition—which is another unfortunate usage, because the set of 'no' voters in a division with negative outcome are also winners in the ordinary sense of this word.

(1) The division in which all the voters unanimously vote 'yes' must have positive outcome.

(2) The division in which all the voters unanimously vote 'no' must have negative outcome.

(3) If a division has positive outcome, and a voter is transferred from its 'no' camp to the 'yes' camp, then the resulting division must likewise have positive outcome.

These conditions exclude perverse would-be rules. The meaning of (1) and (2) is obvious. Condition (3) can be paraphrased as stipulating that increased support for a bill cannot hurt it.[6]

All the rules we shall consider in this booklet satisfy the following additional condition:

(4) If two divisions are each other's mirror image (so that the 'yes' voters in one are 'no' voters in the other and vice versa) then they cannot both have positive outcome.

A rule satisfying (4) is said to be *proper*. An improper rule may be acceptable where the issues to be decided are of a very restricted kind,[7] but ordinarily such a rule will not do, as it may give rise to an impasse. For, suppose an assembly divides one day on a bill and the outcome is positive, so that the bill is passed. Next day the defeated opponents of the bill table a second bill that says the exact opposite of the first. Presumably, the division on this second bill will be the mirror image of the first division. If this bill is now passed, the supporters of the first bill will want to re-table it. And so an endless pingpong could ensue. (If both divisions that mirror each other have *negative* outcome, this may lead to two mutually opposite bills being blocked. However, this does not lead to an impasse but to the preservation of the status quo.)

In the case of a weighted rule, condition (4) is satisfied if the quota is set higher than half the total weight of all voters.[8]

1.2 Penrose's measure of voting power

By the *voting power* of a voter under a given decision rule we mean, roughly speaking, the amount of influence over the outcome of a division, which the

[6] This is referred to as the *monotonicity* condition.

[7] See [19, pp. 12–13.].

[8] Note, by the way, that not all decision rules are—or even can be—presented as weighted rules. See [19, pp. 31–32].

voter possesses by virtue of the rule. Our aim in this chapter is to make this somewhat vague notion more precise by quantifying the *amount of influence*.

At first sight it may seem that, at least in the case of a weighted rule, there is no difficulty: surely, the voting powers of the voters are proportional to their weights. But on closer examination this widely held common-sense view is easily seen to be fallacious.

1.2.1 Luxembourg gets lucky In 1958–72, QMV assigned Luxembourg weight 1, while Belgium was assigned weight 2 and France 4 (see Table 2). But as we noted in Subsection 1.1.1, Luxembourg was then a dummy, with no influence at all over decisions under QMV; so it could not possibly have half as much influence as Belgium, or a quarter as much as France.

In the third period of the EU,[9] Luxembourg was assigned weight 2, while Ireland and Denmark got 3 each (see column headed '1981' in Table 2). Surely, the intention was to give Ireland and Denmark—with populations about ten times and over fourteen times as large as Luxembourg's—more voting power than to the minuscule grand duchy. But a close examination of the 1981 weights and quota shows that if Luxembourg were to exchange weights with Ireland or Denmark, that would not affect the outcome of any possible division. (For example, if the three Benelux members and Greece voted 'no' while the other six members voted 'yes', the 'yes' camp had total weight 46, exceeding the quota by 1, so the outcome was positive; if Luxembourg were to swap weights with Denmark, the 'yes' camp would have total weight 45, exactly equal to the quota, so the outcome would still have been positive.) Therefore the 1981 QMV in fact gave Luxembourg *exactly the same* amount of influence as Ireland or Denmark.

This shows conclusively that, contrary to naïve common sense, voting powers are not in general strictly proportional to voting weights. A scientific analysis is indispensable.

1.2.2 Penrose's measure of voting power The scientific study of voting power was initiated in 1946 by Lionel Penrose [38]. His first key idea was simple: *the more powerful a voter is, the more often will the outcome go the way s/he votes.* In other words, a more powerful voter is a more successful one, one that is more often on the successful side of a division.

So consider a given decision rule (say the 1958 QMV) and a given voter (say Belgium). Let us denote by r the proportion of all divisions of the assembly in which the given voter is on the successful side: s/he votes 'yes'

[9] Strictly speaking, the name 'European Union' applies only following the Maastricht Treaty, which took effect in November 1993; but we shall use this name also for the earlier stages.

and the outcome is positive, or votes 'no' and the outcome is negative. This can also be expressed in terms of probabilities: assuming that all the voters vote independently and at random (for example, each voter flips a true coin and votes 'yes' if it shows heads and 'no' if it shows tails)[10] then r is the probability that our voter will be in the successful camp.

In our example, the 1958 QMV, there are 64 possible divisions, and Belgium is in the successful camp in 38 of them; so in this case $r = \frac{19}{32}$.

Actually, r itself is not a very convenient measure of voting power, because it runs together luck and genuine influence.[11] Indeed, even for a dummy $r = \frac{1}{2}$, because even a dummy finds himself, by sheer luck, in the successful camp in half of all divisions. So Penrose proposed

$$\psi = 2r - 1$$

(ψ is the Greek letter *psi*) as measure of voting power.[12] This takes the element of luck out of it: for a dummy $\psi = 0$, whereas for a 'dictator' (a voter whose weight equals or exceeds the quota, while the other voters are dummies) $\psi = 1$. For Belgium under the 1958 QMV we get $\psi = \frac{3}{16}$. The values of ψ for all member states under QMV in the first five periods of the EU are shown in Table 4. Note that the sum of the powers of the voters (shown at the bottom of the table) is not constant; we shall return to this important point later on.

Penrose also observed that ψ can be interpreted directly in the following striking way: *it is the probability that the given voter can be decisive*; in other words, the probability of getting a division in which our voter can reverse the outcome by reversing his or her vote.[13]

For example, the 64 possible divisions of the 1958 CM can be arranged in 32 pairs, so that the two divisions in each pair differ from each other in one respect only: the vote of Belgium. In 26 of these pairs, the outcome under

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[10] Regarding the significance of this assumption, see Subsection 1.3.2.

[11] This way of putting it is due to Barry [4].

[12] Penrose [39]. In [38] he had used $\psi/2$ rather than ψ itself as measure of voting power. The difference between the two is inessential and does not affect any of the arguments presented by him or by us.

Note that ψ is in fact a *function*, whose value depends on the decision rule and the voter under consideration. When we need to emphasize this, we denote by $\psi_a[\mathcal{W}]$ the value of ψ for voter a under rule $\mathcal{W}$.

In using the symbol ψ here we follow Owen [37]. In [19], following other authors, we denote Penrose's measure by β'; this notation is due to historical reasons that will be mentioned in a moment (see p. 6 footnote 15).

[13] Another term used instead of 'decisive' is *critical*. For a proof of the equivalence of the two ways of characterizing ψ, see [19, pp. 45–46].

QMV is the same for both divisions of the pair; so in these divisions Belgium's vote makes no difference. But in the remaining 6 pairs Belgium's vote makes all the difference: in one division of the pair, in which Belgium votes 'yes', the outcome is positive; while in the other division, in which Belgium votes 'no', the outcome is negative. So Belgium is decisive in 6 out of 32 pairs of divisions (or in 12 divisions out of 64). Therefore the probability of Belgium being decisive is $\frac{3}{16}$, which is exactly the value of ψ in this case.

1.2.3 The wheel reinvented Penrose's pioneering work lay for many years unnoticed or forgotten by mainstream writers on social choice (the science of collective decision-making). But his ideas on measuring voting power are so natural, so compelling, that they forced themselves on several other scholars who tackled the problem of measuring voting power: without knowing of Penrose's—or of one another's—work, they re-invented some of his ideas.

The first among them (as far as we know) was the American jurist John F Banzhaf, who began to address this problem in 1965 [1] (see also [2, 3]).[14] Banzhaf approached the problem in much the same way as Penrose. But since he was interested in voting power not as an absolute magnitude, but only in the *ratio* of one voter's power to another's, the *Banzhaf index of voting power* named after him and denoted by β (the Greek letter *beta*), gives only the *relative* power of each voter. The value of β for any voter can be obtained very simply from ψ by dividing the value of ψ for that voter by the sum of the values of ψ of all the voters in the assembly.[15] So, unlike ψ,

[14] Others who reinvented some of Penrose's ideas include Rae [40], who in 1969 reinvented the quantity r; Coleman [6], who in 1971 invented two variants of Penrose's measure ψ; and Barry [4], who in 1980 reinvented both r and ψ. None of these knew about Penrose's work. It also appears that Rae did not know about Banzhaf's work and that Coleman knew about neither Banzhaf's nor Rae's. Barry knew about the Banzhaf index, but misunderstood the reasoning behind it. On the unwitting generalization of the Penrose measure by Steunenberg et al. in 1999 [46], see below, p. 12, footnote 28.

[15] In symbols:

$$\beta_a[\mathcal{W}] = \frac{\psi_a[\mathcal{W}]}{\sum_{x \in N} \psi_x[\mathcal{W}]},$$

where a is any voter, N is the assembly and $\mathcal{W}$ is the decision rule.

In 1979, Dubey and Shapley [13], who were familiar with both Banzhaf's and Rae's work, but not with Penrose's, reinvented ψ, which they regarded as 'in many ways more natural' than β, and denoted it by β'. They also commented that the connection between this measure and r 'was not noticed for several years' after 1969, not realizing that Penrose had explicitly noted it back in 1946. Thereafter ψ, alias β', came to be known as the *absolute Banzhaf index* (as opposed to β, the *relative* Banzhaf index), or as the *Banzhaf measure* or *Banzhaf value.*

the values of β for all the voters in an assembly always add up to 1. The values of β for all member states under QMV in the first five periods of the EU are shown in Table 5. Note that this table can be obtained from Table 4 by dividing the figures in each column of the latter table by the total shown at the bottom of the column.

The reader must be warned that the Banzhaf index β can only be used for comparing the voting powers of several voters *under the same decision rule*; it is not a reliable yardstick for comparing the voting powers of different voters, or even of the same voter, under two different decision rules. *For the latter purpose the Penrose measure ψ must be used.*

For example, from Table 5 we can reliably infer that under the 1981 QMV Greece had twice as much voting power as Ireland. But the fact that the value of β for Ireland was greater in 1986 than in 1981, does not mean that Ireland's voting power increased when Spain and Portugal were admitted to the EU. Turning to Table 4, we see that the admission of Spain and Portugal caused *all* ten old members to lose some power. However, Ireland and Denmark happened to lose much less than all the others, so their *relative* positions improved *compared to the others'*. It is of course hardly surprising if an old member of the EU loses some power when new members are admitted. In fact, the only member that actually gained power on such occasions was Luxembourg, to which it happened three times: in 1973, 1981 and 1995; but that was mostly a result of blunders in allocating weights.

1.2.4 Sensitivity Another pitfall against which we must warn the reader is that of treating ψ or β as an *additive* quantity like money, for example. If Betty and Norman have €0.1132m each and Lucy has €0.0195m, then it makes perfect sense to say that the three jointly have €0.2459m, because $0.1132 + 0.1132 + 0.0195 = 0.2459$. But it would be a mistake to infer from Table 4 that under the 1973 QMV the joint voting power of the Benelux countries was 0.2459. This is because ψ is computed under the assumption that voters act independently of each other. If the Benelux countries were to conclude a pact binding them to vote always together, as a bloc, then instead of the decision rule shown in the '1973' column of Table 2 we would have a rather different rule: the three individual Benelux voters with weights 5, 5 and 2 would be replaced by a single Benelux bloc voter with weight 12; and a simple calculation shows that under this rule the voting power ψ of the Benelux bloc would be 0.2031 rather than 0.2459.[16] Nevertheless, the sum-total of the values of ψ for all members of the assembly (shown, for the first

[16]For further discussion of the non-additivity of voting power, see [19, § 7.2]. For an analysis of the conditions under which forming a bloc is advantageous to its members, see [24].

five periods of the EU, in the *total* row of Table 4) is of great significance. In the words of Dubey and Shapley,

> [It] reflects the 'volatility' or 'degree of suspense' in the decision rule. It gives an indication of the likelihood of a close decision, i.e., one so close that a single voter could tip the scales. ... [I]t is also a kind of democratic participation index, measuring the decision rule's sensitivity to the desire of the 'average voter' or to the 'public will'.[17]

But the 'raw' numerical value of the sensitivity of a decision rule—the sum of the values of ψ for all the voters—is not easy to interpret at a glance. For example, according to Table 4, the sensitivity of the 1986 QMV was 1.0850; but it is not obvious whether this means that the sensitivity was low or high. It is more informative to compare the sensitivity of a given decision rule with that of the least sensitive and most sensitive decision rules for the same number of voters. This is what the *relative sensitivity* index S does.

For a given number of voters, the least possible sensitivity is attained by the *unanimity* rule, under which a bill is passed only if all the voters are unanimously for it; and the greatest possible sensitivity is attained by the *[simple] majority* rule, under which a bill is passed provided more than half of the voters support it. Now, the values of the relative sensitivity index S for these two extreme cases are 0 and 1 respectively.[18] So, for example, Table 4 shows that the relative sensitivity of the 1986 QMV—compared, on a scale from 0 to 1, to other decision rules for 12 voters—was 0.8510, which is fairly high. Note however that it is not meaningful to compare the values of S of two decision rules that apply to assemblies of different sizes. So it

[17] See [13, p. 106]. In fact, Dubey and Shapley are referring here not to the sum of the values of ψ for all the voters under a decision rule $\mathcal{W}$, which we shall denote by $\Sigma[\mathcal{W}]$, but to the quantity $\mathrm{H}[\mathcal{W}] = 2^{n-1}\Sigma[\mathcal{W}]$, where n is the number of voters in the assembly. However, what they say applies equally to $\Sigma[\mathcal{W}]$, because it differs from $\mathrm{H}[\mathcal{W}]$ by a factor that depends only on n.

[18] For a given number n of voters, the least possible value of $\mathrm{H}[\mathcal{W}]$ (see footnote 17), attained by the unanimity rule, is n (see [19, Thm. 3.3.11]). The maximal value of $\mathrm{H}[\mathcal{W}]$ for n voters, attained by the majority rule, is $\mathrm{H}_n = m\binom{n}{m}$, where m is the least integer greater than $n/2$ (see [19, Thm. 3.3.14]). Since the the growth of H_n/n—the ratio between these two extreme values of $\mathrm{H}[\mathcal{W}]$—is approximately exponential in n, we use a logarithmic scale to define the relative sensitivity $\mathrm{S}[\mathcal{W}]$ of any decision rule for n voters:

$$\mathrm{S}[\mathcal{W}] := \frac{\log(\mathrm{H}[\mathcal{W}]/n)}{\log(\mathrm{H}_n/n)}$$

(see [16] and [19, p. 61]).

would be wrong to conclude from Table 4 that the relative sensitivity of the 1986 QMV was greater than that of the 1973 QMV.

1.2.5 Resistance A no less important criterion for assessing a decision rule is its *resistance* to changes in the status quo: the degree to which it is biased in favour of a negative outcome—one in which the proposed bill is blocked, and the status quo maintained—rather than a positive one. This is measured by the *resistance coefficient* R,[19] whose possible values for proper decision rules (see Subsection 1.1.2) range from 0 to 1. For the majority rule with an odd number of voters, R is exactly 0; with an even number of voters R is positive, but becomes negligibly small when the the number of voters is large. For the unanimity rule, R is exactly 1.

From Table 4 it is clear that with each enlargement of the EU, the resistance of the QMV rule has crept up. The politicians and officials who decided on the weights and quota may have assumed that by pegging the quota at about 71% of the total weight, they would be keeping the resistance of the rule more or less constant: with each enlargement of the EU, the same proportion of the total weight—about 29%—would be needed to block a resolution. But this naïve assumption is fallacious: pegging the quota at a constant percentage of the total weight tends to *increase* the resistance of a proper weighted rule as the number of voters increases.[20]

1.3 Other approaches to voting power

In this section we discuss briefly alternative approaches to the problem of measuring voting power, and explain why they are not appropriate for the present assessment of the various proposals for allocating QMV weights and quota in the CM of a future enlarged EU.

1.3.1 The Shapley–Shubik index In 1954, Lloyd S Shapley and Martin Shubik [45] proposed a new approach to the problem of measuring voting

[19]The resistance coefficient $\mathrm{R}[\mathcal{W}]$ of a decision rule $\mathcal{W}$ with n voters is defined by

$$\mathrm{R}[\mathcal{W}] := \frac{2^{n-1} - \omega[\mathcal{W}]}{2^{n-1} - 1},$$

where $\omega[\mathcal{W}]$ is the number of divisions with positive outcome under $\mathcal{W}$ (see [19, p. 62]).

[20]This is easy to demonstrate with a rule that assigns equal weights to all voters. With n voters and a quota of 75% of the total weight, the probability of a bill passing is equal to the probability of getting at least 75% 'heads' in n tosses of a fair coin. For $n = 4$ this probability is 5/16. For $n = 16$ the probability is less than 1/25; for $n = 100$ you can forget about it.

power. This approach was based on a branch of mathematics known as *game theory*; more specifically, on the mathematical theory of *cooperative games [with transferable utility]*. A year earlier, Shapley [44] had proposed what was to become known as the *Shapley value* in connection with such games. Following each play of a cooperative game, every player receives some monetary or money-like *payoff*, which can be positive, negative, or 0. But before the game is played the result is in general uncertain; and the quantity known as *the Shapley value of the game to a given player* is accepted by many game-theorists as a prior estimate of the payoff that the player can expect, on the average.

The Shapley–Shubik index is just the Shapley value applied to decision rules, which can be dressed up as cooperative games of a special kind: so-called *simple* games. The idea is that the voters in an assembly can be regarded as 'players' in a game. A play of the game consists in bringing about a division of the assembly. If the outcome of a division is positive, the camp of 'yes' voters is awarded a fixed prize. (If the outcome is negative, no prize is awarded.) The fixed prize—the spoils of victory—is a monetary or money-like quantity which the victors share among themselves according to a prior binding agreement, arrived at through bargaining, concluded in advance of the division.[21] The Shapley–Shubik index of a voter ('player') under a given decision rule ('simple game') is then presumed to be a prior estimate of that voter's expected payoff. For convenience, the value of the fixed prize to be shared out is set as 1 unit; so the sum of the values of this index for all the voters is always 1.

The Shapley–Shubik index is widely used—alongside the Penrose measure or its 'relativized' form, the Banzhaf index—as a measure of voting power. In many cases the Shapley–Shubik and Banzhaf indices have fairly similar values; but in general they behave quite differently.[22]

As Coleman [6] pointed out in 1971, the the notion of voting power quantified by the Shapley-Shubik index is not the power to affect the outcome of a division in the the usual sense, that is, whether a bill is passed or blocked (see Section 1.1). Rather, it is the power to appropriate a share in the spoils of victory, available solely to a victorious 'yes' camp. This notion of power may be appropriate in some contexts, in which such a division of spoils does occur; for example, a convention for nominating a candidate for the US presidency, ' ... for there are spo[i]ls to be distributed among those delegations that

[21] For this reason, any set of voters that constitutes the 'yes' camp in some division with positive outcome is called a 'winning coalition'. Any other set of voters is said to be a 'losing coalition'.

[22] See [19, pp. 277–278] for a brief summary of the main differences.

support the winner, and particularly those delegations that cast the deciding ballots in favor of the winner. But this is an unusual case, in which there is a winning nominee, who does have spoils to distribute.' But the Shapley–Shubik index does not measure voting power in the sense appropriate to the more usual context,

> ... for the usual problem is not one in which there is a division of the spoils among the winners, but rather the problem of controlling the action of the collectivity. The action is ordinarily one that carries its own consequences or distribution of utilities, and these cannot be varied at will, i.e. cannot be split up among those who constitute the winning coalition. Instead, the typical question is ... the passage of a bill, a resolution, or a measure committing the collectivity to an action.[23]

For the more usual context, Coleman proposed what was in effect an approach to power-as-influence, similar to that taken earlier by Penrose, Banzhaf and Rae, of whose work he was not aware (see Subsection 1.2.3).[24]

In this booklet we use the Penrose measure ψ and its relativized form, the Banzhaf index β; but not the Shapley–Shubik index. This is because—irrespective of whether the latter index is right for dealing with other issues of voting power—it is inappropriate for analysing voting power in the CM. A resolution passed by the CM determines policy, which is a public good, whose fruits affect, for better or worse, all member states and their citizens. It does not involve a fixed prize to be distributed exclusively among those who have voted for it.[25]

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[23] [6, pp. 299, 272].

[24] For an amplification of Coleman's distinction between the two notions of voting power, which we call 'I-power' (power as influence) and 'P-power' (power as a prize), see [25] and especially [19, *passim*].

In 1954, when [45] was published, and for a long time thereafter, Penrose's work was unknown to mainstream social-choice theorists, who therefore credited Shapley and Shubik with the inauguration of the scientific study of voting power. And because the notion of voting power posited by these two authors was inextricably enmeshed in cooperative game theory, it was widely assumed that the *whole* study of voting power must be part of that theory. So when Banzhaf and others came up with their quite different approach—unwittingly reinventing Penrose's ideas—it was misinterpreted as belonging to cooperative game theory. Thus the Banzhaf index was seen by many authors, quite incorrectly, as measuring a voter's share in some fixed total payoff. Conversely, the Shapley–Shubik index was often misinterpreted as measuring a voter's influence over the outcome (in the sense of Section 1.1). For a detailed critique of this confusion, see [19, *passim*]; and for an additional discussion, particularly relevant to the distribution of power in the CM, see [23, 24].

[25] In the technical literature, other indices of voting power have been proposed: by

1.3.2 A priori v. actual voting power The voting power that the Penrose measure—and for that matter also the Shapley–Shubik index—is intended to quantify is the power that a voter has *solely by virtue of the decision rule itself*.[26] Voting power in this restricted sense is said to be *a priori*, in contradistinction to *a posteriori* or *actual* voting power. The latter may take into account—apart from the decision rule—additional real-life (and transient) factors such as voters' actual interests, preferences and temperaments; their persuasive skills; and their mutual affinities and disaffinities. For example, a middle-of-the-road voter may well tend to be on the 'wrong' side of a division less often—other things being equal—than a voter whose temperament or interests are far removed from those of most other voters. This will tend to increase the former's and reduce the latter's actual voting powers as compared to their a priori voting powers. Similarly, a voter who is able, by fair means or foul, to win other voters' support will thereby gain added actual power.

For this reason some social-choice theorists have argued that indices of a priori voting power are quite useless for analysing the real power distribution in existing (or past) bodies, such as the current CM.[27] Against this, Shapley and Shubik [45, p. 791] and many others have argued, in our view rightly, that a priori voting power is an important analytic tool even for studying actual voting power. The latter is after all a resultant of the power derived from the decision rule itself and the power gained or lost due to additional, political and social factors such as voters' actual preferences etc. A priori voting power can therefore serve as a useful benchmark: if actual voting power can be reliably computed—a big *if*, by the way—then the disparity between it and a priori voting power can be used to assess the importance of those additional factors.[28]

Deegan and Packel [11, 12]; Johnston [31]; and Holler [28]. But in practical applications they are used much more rarely than the Banzhaf and Shapley–Shubik indices. We avoid using them here not only because they are not based on an appropriate notion of what voting power is all about, but also because they display undesirable or 'pathological' mathematical behaviour, which makes them unreliable. For a detailed critique, see [19, §§ 6.4 and 7.9].

[26] The same also applies to the other indices, mentioned in footnote 25.

[27] For a fervent presentation of this argument, see Garrett and Tsebelis [26, 27].

[28] In this connection it is interesting to note that the elaborate measure of actual voting power proposed by Steunenberg et al. in 1999 [46], which is designed to take into account an enormous variety of information on voters' preferences, has turned out to be a generalization of the Penrose measure. In [23] we show that if that information is reduced to an absolute minimum, then the measure proposed in [46] becomes essentially the Penrose measure. The authors have assured us (in personal communication) that when proposing their measure they neither suspected nor intended any connection between it and the

However, when it comes to designing the constitution of a future decision-making body, there is a near-consensus that the a priori approach is the only right one, and indeed often the only possible one. As Roth puts it, the a priori approach

> ... abstracts from the particular personalities and political interests present in particular voting environments, but this abstraction is what makes the analysis focus on the rules themselves rather than on the other aspects of the political environment. This kind of analysis seems to be just what is needed to analyze the voting rules in a new constitution, for example, long before the specific issues to be voted on arise or the specific factions and personalities that will be involved can be identified.[29]

When designing a new constitution we must take an aprioristic stance for two reasons.

First, because it would be unfair to tailor the decision rule to the specific interests, preferences and affinities of the voters. Even if such information is available, we must go 'behind a veil of ignorance' (to use an apt expression due to Rawls [41]) and act impartially, considering only the voting power that the voters will derive from the decision rule itself.

Second, in general there is very little reliable information about the *future* interests, preferences and affinities of the voters. This is particularly true where the voters—as in the CM—are states, whose political colours and orientations are fluid and subject to change every few years.

Because we are taking an aprioristic stance in this booklet, when we speak in probabilistic terms we always assume, as we have done in Subsection 1.2.2, that the voters act independently and vote at random 'yes' or 'no' with equal probability. Of course, this is not how voters *actually* behave: they don't decide how to vote by tossing a coin. Rather, our random-voting assumption is the most neutral one we can make a priori, 'behind a veil of ignorance'.

1.3.3 Procedural factors A specific caution against using a priori measures of voting power, even when designing a decision rule for a future enlarged CM, has been voiced by some scholars[30] on the grounds that the CM is only one component of a larger decision-making structure, whose components interact according to quite complex procedural rules. Thus, the European

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Penrose measure.

[29]See [43, p. 9]. For similar views with specific reference to the EU see Holler and Widgrén [30] and Lane and Berg [33].

[30]For example, Garret and Tsebelis [26].

Commission acts as *agenda setter* under QMV, because in order for a resolution to pass in the CM under QMV, it must first be approved by the European Commission, who then submits it to the CM. The European Parliament too has a significant role—which is likely to grow in importance—on issues to which the 'co-decisionmaking' procedure applies.

However, these facts do not affect the relative distribution of voting power within the CM *as measured by the Penrose measure and the Banzhaf index*. The interaction between the Commission and the CM under QMV is, in effect, equivalent to adding the Commission as an 'additional blocker', whose approval is needed for any resolution. But the internal relative distribution of voting power, as measured by Penrose and Banzhaf, is not affected by such an additional blocker.[31] For this reason, our assessment in this booklet of various proposals for QMV in a future enlarged CM is unaffected by the agenda-setting role of the Commission. The same applies also to the role of the European Parliament in 'co-decisionmaking'.

[31] See [19, p. 270]. In [19, § 7.9] we show that the same *does not apply* to the Shapley–Shubik, Deegan–Packel and Johnston indices, mentioned in footnote 25. But it does apply to the Holler index mentioned in the same footnote.

Chapter 2

Representative Councils

The voters in the CM are ministers who act as representatives of constituencies—member states—of different sizes. (By 'size' in this context we always mean the size of the population, or, more precisely, of the electorate of each constituency.) Choosing the 'right' decision rule for such a council of representatives is fraught with subtle problems, to which naïve common sense is a misleading guide. Often, what may superficially appear self-evident turns out to be untrue. In this chapter, while avoiding technicalities as far as we can, we outline the scientific approach to these problems.

2.1 A council as part of a two-tier system

2.1.1 The two-tier model In the theory of voting power in a council of representatives, such as the CM, we must make two assumptions.

First, we assume that in each division of the council every representative votes in accordance with the majority opinion in his or her constituency.[1] This no doubt involves a certain degree of 'democratic idealization'; but without it very little can be said about what decision rule the council itself ought to adopt. No scientific theory about real-world phenomena is possible without *some* idealization; the question is only whether the idealization is at all reasonable. The assumption we are making here is certainly much more reasonable than the one that, as we shall soon see (Section 2.1.2), is implicit in the naïve view. Besides, if it transpires, for example, that the Greek representative frequently votes in the CM in defiance of majority opinion in Greece, then it is a matter for the electors of that country themselves to

[1]This leaves open the question as to how a representative votes in case his or her constituency is exactly evenly split—which can happen if the size of the constituency is an even number. See however footnote 3 below.

rectify this 'democratic deficit'; the decision rule of the CM cannot be rigged so as to do it for them.

Second, the main results of the theory involve mathematical approximation.[2] In order to ensure that the error of the approximation is negligibly small, we must assume that the constituencies, though they may be of different sizes, are all quite large—say at least $100,000$ persons. This assumption certainly holds true for the EU: the population of its present smallest member, Luxembourg, is about $429,000$; and that of the smallest prospective member, Malta, is about $379,000$ (see Table 8).[3]

Thanks to our democratic idealization, we can view the council in two ways. First, we can regard it, as before, as a decision-making body in its own right. But we can also view it as the upper tier in a composite, two-tier voting system that operates as follows. When a bill is proposed, we can suppose that the entire citizenry—the total electorate of all the constituencies, which are the ground tier of the system—divides on it. The representatives then act as tellers or rapporteurs, each voting in the council according to the majority within their respective constituencies.

In this two-tier model, the ultimate—albeit *indirect*—decision makers are the electors at the lower tier. Moreover, any given decision rule for the council acts indirectly as a decision rule for the 'grand assembly' of the entire electorate: any possible division of this grand assembly will result, via the votes of the representatives at the council, in a positive or negative outcome.

The great importance of this dual view of the council is that, for any given decision rule for the council, we can calculate not only the direct voting power of each representative, but also *the resultant indirect voting power of each citizen, as ultimate indirect decision maker*. This enables us correctly to raise and answer questions that are vital to the democratic functioning of the entire composite system.

Before doing so in detail, we must clarify two points on which the naïve view of these matters is mistaken or confused.

2.1.2 False analogy with proportional representation Most people—including politicians and otherwise well-informed journalists—automatically tend to assume that the 'fairest' or 'most democratic' decision rule for a council of representatives such as the CM would assign to its members weights that are strictly proportional to the respective size of their constituencies. According to this view, the present distribution of weights in the CM ('1995'

[2] The required approximations are based on Stirling's approximation formula for $n!$, which can be found in books on the calculus or on probability theory.

[3] In view of this assumption we need not worry about the quandary raised in footnote 1: the probability of an exactly even split is negligibly small.

column in Table 2) is grossly biased against the five biggest member states and in favour of all the other ten. Indeed, from the '1995' column in Table 3 we can see that Germany has only 52.3% of the weight it 'deserves' on grounds of strict weight–size proportionality, and even Spain (the smallest of the big five) has only 87.2% of what it 'deserves'; whereas Spain's former dominion, The Netherlands (the largest of the remaining ten), has 38.3% 'too much' weight and Luxembourg has more than 21 times the weight it 'should' have.[4] Accordingly, much of the discussion around enlargement of the EU is permeated with the feeling that this 'imbalance' ought to be rectified to some extent, and that any such remaining 'imbalance' would constitute a great concession on the part of the larger member states.

However, as we shall see in §§ 2.2 and 2.3, strict proportionality of weight to size does not in fact produce, in a council of the kind we are discussing, voting-power distribution that is in any reasonable sense fair or democratic (except of course where all the constituencies are of equal size).

The reasons why so many people nevertheless tend to hold an unsound view on this matter are no doubt quite complex, and we shall have more to say on this later on, in §§ 2.2 and 2.3. But the most basic reason seems to be that they tend, perhaps unconsciously, to draw an analogy between a council such as the CM, in which each member represents *a fixed geographical or political-administrative constituency* and an assembly or legislature of quite a different sort, in which each party bloc represents what may be called *an opinion constituency*: a section of political-ideological interest or opinion within the electorate.

In an assembly of the latter sort, if party discipline is maintained so that all representatives of each party vote in unison, then each party bloc may be regarded as a single bloc-voter whose voting weight equals the number of its seats in the assembly. The ideal of proportional representation—whereby the weight (number of seats) of each party bloc is strictly proportional to the number of electors who have voted for it—is justified by its advocates on the grounds that the assembly ought to be a microcosm of the electorate at large, so that a division of the former on any bill would reflect in true proportion a hypothetical division of the latter, as if a plebiscite were held on that bill.

The underlying key presumption here is that each party bloc votes in the assembly in the same way that *all* members of its opinion constituency would vote if given the chance. This presumption may not be absolutely correct but is nevertheless fairly realistic. (To some extent it is self-fulfilling, because

[4]Cf. Teasdale, in a generally well-informed article [47, p. 108]: 'The scale of current over-representation of small countries in the Council [of Ministers] is striking.'

where proportional representation is rigorously practised, the number and diversity of parties tend to be large, so that every elector can vote for a party that corresponds quite closely to his or her views.)

But for a council such as the CM a similar presumption is virtually certain to be not only false, but wide off the mark. Here we can assume *at best* that a representative votes according to the *majority* opinion in his or her constituency: this was our *democratic idealization*. In some cases the margin of this majority—the number by which it exceeds the minority—may be quite large; but in other cases it may be very small. Unanimity within a constituency can be virtually ruled out even on issues that are deemed to touch on vital common interests of the constituency, because opinions as to where common interests lie may and in fact do differ.

Politicians who represent the larger member states on the CM may perhaps be tempted to claim that they speak for the whole of their respective countries, and that their votes ought to be weighted accordingly. But we can now see that it is at least questionable whether such weighting will produce a fair or democratic decision rule. A closer, more rigorous questioning is necessary.

2.1.3 Equality, majority rule and citizens' power So far we have spoken of a 'fair' or 'democratic' decision rule for a council of representatives, without examining what these terms really mean. We must now be more precise.

In fact, three different desiderata or criteria can be invoked in this connection.

The first and most obvious desideratum is *equitability* or *equal suffrage*: the decision rule of the council ought to be such that the (indirect) voting powers of all citizens—the electors at the ground level of the two-tier system—in the various constituencies should be as nearly equal as possible, irrespective of the different sizes of these constituencies.

This surely is the primary meaning of 'fairness' of a decision rule, encapsulated in the slogan 'one person, one vote' (OPOV).

While OPOV is a fundamental democratic principle, it is not the only one that a democratic decision rule may be required to satisfy. A second such requirement is *majoritarianism* or *majority rule*: the rule used by the council should arguably come as close as possible to producing outcomes that conform to the wishes of the majority of the entire electorate.

Equitability and majoritarianism are often conflated with each other. This is the case not only in popular discourse: political and legal experts of the highest calibre have been known to argue for (or against) one of these principles, using arguments that logically apply only to the other.[5] But in fact these two criteria are quite distinct: a decision rule that satisfies one of them need not automatically satisfy the other.[6]

(This can be seen even in the case of simple one-tier systems. Imagine a country with 1m citizens, in which decisions are made directly by plebiscite. Suppose that 0.5m citizens, chosen completely at random, are given double votes, while the other 0.5m are given ordinary single votes. Decisions are made by ordinary majority of the votes: a bill is passed if it receives more than 750,000 votes. By the laws of probability, the outcomes of plebiscites in this patently inequitable system will with virtual certainty conform to the wishes of a majority of the citizens. On the other hand, if the decision rule is that of unanimity, then clearly all voters have equal power, but the outcomes of half of all plebiscites will not satisfy a majority of the citizens.)

We must therefore discuss the implications of each of these criteria separately. We shall do so in §§ 2.2 and 2.3.

But before that we would like to mention a third desideratum, which is perhaps less frequently invoked than the other two, but is arguably of great democratic importance. This is the desideratum of *citizens' empowerment*: the decision rule of the council ought to be such that the sum total of the (indirect) voting powers of all citizens should be as great as possible. Note that if this principle can be implemented at the same time as equitability, then it clearly follows that the equal voting power of *each citizen* is as great as it can possibly be.[7]

Fortunately, we need not analyse the implications of this third criterion separately, because, as we shall see in Subsection 2.3.2, it turns out that it is mathematically equivalent to majoritarianism: of any two decision rules for the council, the one that is better from a majoritarian viewpoint also grants greater total power to the citizens.

2.2 Equitability

We noted in Subsection 2.1.2 that the idea that voting weights in a council such as the CM ought to be proportional to the respective sizes of the

[5] For instances of such confusion in opinions pronounced by justices of the US Supreme Court, see [19, §§ 4.1 and 4.4].

[6] There are even some—admittedly rather artificial—examples of two-tier systems in which the two principles are mutually incompatible. See [19, Ex. 3.4.12].

[7] The importance of citizens' empowerment in combination with equitability is forcefully urged by Morriss [36, § 24.3].

constituencies is based on a false analogy with an assembly elected by proportional representation. But the same widespread idea arises, more specifically, in connection with explicit attempts to implement equitability (OPOV). It is derived, in this particular connection, from two conscious or subconscious premisses, which are in fact fallacious. We shall now state these two premisses, and then examine each of them in turn.

2.2.1 Premiss of power–weight proportionality: *In a council operating a weighted voting rule, the voting powers of the members are strictly proportional to their respective weights.*

2.2.2 Premiss of size–power proportionality: *The indirect voting powers (or amounts of influence) of the citizens in a two-tier system are equalized, if the voting powers of their representatives on the council are strictly proportional to the sizes of their respective constituencies.*

From these two premisses it would indeed follow that equitability would be achieved if voting weights on the council were assigned in strict proportion to the sizes of the respective constituencies.

2.2.3 Critique of power–weight proportionality It is quite easy to refute this premiss: we have in fact done so in Subsection 1.2.1, where we also noted that this very fallacy must have been responsible for past blunders in allocating voting weights on the CM.

However, this easily-refuted fallacy matters relatively little in the context of a projected decision rule for the CM of an enlarged EU. This is because of a remarkable theorem due to L Penrose, according to which, as the number of voters (members of the council) increases, their voting powers tend to become more and more nearly proportional to their weights.[8]

An excellent illustration of this is provided by Table 6. This table shows, for each of the first five periods of the EU and each member state, the ratio between the member's relative power (β) and relative weight (the member's weight divided by the total weight). If voting powers were strictly proportional to weights, then all the figures in the table would be 1. In fact, they are not; but they do, on the whole, approach 1. In 1958, Luxembourg had (as we know) 100% less relative power than its relative weight; and we can also see that Belgium and The Netherlands had 21.4% more relative power

[8] See [39, Appendix]. The theorem depends on various conditions, the main one being that the ratio between the greatest and smallest weights does not increase indefinitely but remains bounded. Penrose assumes that the quota in the council is half (or a small fraction over half) of the total weight, but the same holds for a larger quota, provided it is not too near the total weight.

than relative weight. In 1973 the most extreme deviations from 1 in either direction were −54.3% (Luxembourg) and +28.1% (Denmark and Ireland). In 1981 they were −13.9% (Denmark and Ireland) and +29.2% (Luxembourg). In 1986 they were −31.6% (Luxembourg) and +16.3% (Denmark and Ireland). Note that it is the smallest ('lightest') members that have so far shown these quite large deviations; in the case of the larger members the deviations were very much smaller. But in 1995, when the EU was enlarged to its present 15 members, even the smaller members fell into line: the extreme deviations were now only −2.9% (France, Germany, Britain and Italy) and +4.2% (Denmark, Ireland and Finland).

So in practice, when it comes to an enlarged EU of 28 members, if we want to design a decision rule for the CM with a particular distribution of relative voting powers, we may always start by assigning weights in the same relative proportions. Of course, to be on the safe side,[9] we can then check by calculating the values of ψ that we have got the right thing.

2.2.4 Critique of size–power proportionality The widespread belief in Premiss 2.2.2 is no doubt an instance of the fallacy against which we warned at the beginning of Subsection 1.2.4: that of regarding voting power as an additive money-like quantity, which can be simply added up or distributed.

It is extremely tempting to believe that the voting power of a member of a council is simply distributed, divided evenly, among all his or her constituents. For example, since under the 1995 QMV the voting power of Luxembourg is 0.0229 (see Table 4), it seems that each of the 400,000 Luxembourgeois (See Table 1) 'possesses' $0.0229/400,000 = 0.000,000,055,725$ as his or her 'share' in that power.[10]

But voting power, which is essentially a *probability*, just does not behave in this way. Unfortunately—unlike the less harmful fallacy of power–weight proportionality, which is conclusively refuted by means of very simple counter-examples—the present fallacy, which has much more serious implications, is also much more deeply entrenched, because it can only be refuted by means of a detailed mathematical argument.[11]

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[9]Penrose's theorem only guarantees a *tendency*, not complete identity between relative powers and relative weights.

[10]To be more pedantic, we ought to have divided 0.0229 by the size of Luxembourg's electorate rather than its population; but this does not affect the argument.

[11]See [19, pp. 99–100] for a lengthy quotation from the judgment of the New York State Court of Appeals in the case of *Ianucci v Board of Supervisors of Washington County, NY*, in which the court accepts Banzhaf's index and his argument against the fallacy of power–weight proportionality, but walks with its eyes wide shut deep into the pitfall of size–power proportionality.

We can nevertheless outline this argument here, without going into technical details.

2.2.5 Penrose's square-root rule The solution to the equitability problem is easily deduced from two mathematical facts.

First, by the laws of probability, a citizen's indirect voting power—the amount of his or her influence in the two-tier decision-making system—is equal to the product:

citizen's direct *voting power* × *representative's voting power in the council*.

By the 'citizen's direct voting power' we mean the voting power that the citizen has (or would have) under the simple majority rule in a plebiscite, a possible division of his or her entire constituency. The representative mentioned in the formula is of course the representative of the citizen's constituency.[12]

The second mathematical fact concerns the first factor in the above formula, the citizen's direct voting power within his or her constituency. Quite obviously, the larger the constituency, the smaller the citizen's direct voting power within it. But, somewhat surprisingly, the citizen's direct voting power is inversely proportional not to the constituency's size, but to the *square root* of that size.[13] So, for example, if we have three constituencies, A, B and C, whose respective sizes are 1m, 4m and 25m, then the direct voting power of a citizen in B is twice (rather than four times) as small as that of a citizen in A; and a citizen in C has one fifth (rather than one 25th) as much direct power as a citizen in A.

From these two mathematical facts it clearly follows that:

> *Equitability is achieved by making the voting power of each member in the council proportional to the square root of the size of the member's constituency.*

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[12] For a rigorous argument establishing this formula, see [19, p. 67] or [37, p. 283].

[13] More precisely, it equals

$$\frac{m}{2^{n-1}}\binom{n}{m},$$

where n is the size of a constituency and m is the least integer greater than $n/2$. For large n, Stirling's formula yields the value

$$\sqrt{\frac{2}{n\pi}},$$

as an excellent approximation. See [19, pp. 55–56].

This is *Penrose's square-root rule.*[14]

So, in our hypothetical example, the decision rule of the council ought to give the members representing B and C twice and five times as much voting power, respectively, as to the member representing A.

Let us examine the present distribution of voting power in the CM in the light of Penrose's square-root rule. We refer to the last column of Table 7.[15] If voting power were distributed equitably, all the figures in this column would have been 1. We see that the distribution of power is indeed biased against the six largest members and in favour of the remaining nine. The most extreme cases in either direction are post-unification Germany, which has 20.3% less power than equitability would demand; and Luxembourg, which has 130.9% too much power. Ireland, Portugal Belgium and Greece (in this order) also have considerably more power than equitability would justify. But in the case of the remaining nine members the deviations are much smaller. And, more importantly, the true imbalance in the distribution of power is far, far less extreme than according to the naïve view, which assumes that voting weight ought to be strictly proportional to size (cf. the corresponding column in Table 3!).

2.3 Majoritarianism

2.3.1 The majority deficit Let us look at the present QMV decision rule of the CM (last column of Table 2). Consider a (purely hypothetical) resolution that is supported by the members representing France, Germany, Italy, Britain, Belgium, The Netherlands, Spain and Sweden but opposed by the remaining seven members. The total weight of the eight members supporting the resolution is 62, so the resolution is passed. According to our democratic idealization, the resolution is presumably supported by a majority of the electorate in each of the eight countries whose representatives voted for it, and opposed by a majority in each of the other seven. But suppose now that the margin of the *pro* majority in each of the eight countries is extremely slim, and that of the *contra* majority in the seven is extremely large. In this case a majority in the 'grand assembly' of the entire citizenry of the EU (made up of the large *contra* minorities in the eight countries and of the overwhelming *contra* majorities in the seven) is actually opposed to

[14] See Penrose [38, p. 57]; for a detailed proof see [19, pp. 66–67].

[15] Here and in the sequel we are using population size as proxy for the size of the electorate. This does not introduce a significant error, if the ratio between the two is fairly constant across the EU.

the resolution; and the margin of this grand *contra* majority can be very large.

Similarly, if a resolution is opposed by very narrow margins in France, Germany, Denmark and Ireland but supported by very large margins in each of the remaining eleven countries, then it is blocked in the CM, but supported by a huge margin in the entire citizenry of the EU.

A moment's reflection will convince the reader that the possibility of such cases is not an accidental defect of the current QMV. In fact—even granted our democratic idealization—it is inevitable under any conceivable decision rule in a council of representatives such as the CM. It is *always possible* that a majority of the entire electorate will disagree with the outcome of a division of the council.[16]

A majoritarian prescription for a decision rule of the council cannot possibly eliminate this phenomenon altogether; but it can aim to minimize it as far as possible.

Here we need to be more precise. Let us consider a given two-tier system, with a council in which each member represents a constituency of a given size. (To fix ideas, think, for example, of the proposed enlarged EU with 28 members.) Next, consider a given decision rule of the council. We shall now define what we mean by the *majority deficit* of any particular decision of the council.

If the council divides on a bill, and a majority of the entire electorate disagrees with the outcome (produced under the given decision rule), then the margin by which this majority exceeds the minority is the *majority deficit* of this particular decision. (For example, if the size of the entire electorate is 400m persons and 225m of them disagree with the outcome, then the majority deficit of this particular decision is 225m − 175m = 50m persons.) On the other hand, if a majority of the electorate agrees with the outcome, then the *majority deficit* of the decision is 0.[17]

Of course, what counts in assessing a given decision rule is not the size of the majority deficit of this or that isolated decision, but the *statistical average* or *mean value* of the majority deficit.[18] From a majoritarian viewpoint, the mean majority deficit (MMD) is a measure of the badness of the given

[16]However, such cases are theoretically excluded in an assembly elected by strictly proportional representation.

[17]In the highly unlikely event that the electorate is evenly split, the majority deficit is also 0.

[18]In the technical language of probability theory, this is the *mathematical expectation* of the majority deficit. It is obtained by multiplying each possible value of the majority deficit by the probability of this value occurring, and summing all these products.

decision rule. It is always a positive number,[19] but the aim is to choose a decision rule that makes it as small as possible.[20] We shall now outline the solution to this majoritarian problem.

2.3.2 The second square-root rule The naïve view is that the decision rule that is best from a majoritarian viewpoint is that which assigns to each member of the council weight in strict proportion to the size of his or her constituency, and sets the quota at just over half the total weight. But this view, which is probably based on the misleading analogy with proportional representation (see Subsection 2.1.2), is mistaken.

The correct solution follows at once from two theorems. The first of these makes a connection between the MMD and the sum total of the indirect voting powers (ψ) of all the citizens. This sum total is what we have called the *sensitivity* of the decision rule. But note that here we are concerned with the *overall* sensitivity of the rule, in its capacity as (indirect) decision rule *for the entire two-tier system*—which is quite different from the sensitivity of the rule for the council considered as a decision-making body in its own right.[21]

Our first theorem says that of any two decision rules in the council, the one that has the greater overall sensitivity has the smaller MMD.[22]

Our second theorem determines, for any given two-tier system, the decision rule(s) for its council yielding the greatest possible overall sensitivity.[23]

An immediate conclusion from these two theorems is:

> *For any two-tier structure, the proper weighted decision rule for its council that minimizes the MMD assigns to each member of the council weight proportional to the square root of his or her constituency, and fixes the quota at just over half the total weight,*

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[19] This is because it is the mean value of a random variable that has some positive values but no negative ones.

[20] Instead of trying to minimize the MMD, we could try to *maximize* the mean majority *surplus*: the mean value of the margin by which a majority of the electorate that *agrees* with the outcome of a decision exceeds the minority that disagrees with it. But it is easy to prove that the sum of the MMD and this mean majority surplus is a constant, which depends only on the size of the entire electorate. Therefore minimizing the MMD *automatically* also maximizes the mean majority surplus. (The constant just mentioned is denoted by Σ_n in [19, p. 55], where its exact value is given.)

[21] The latter is just the sum of the voting powers ψ of the *representatives* in the council; see Subsection 1.2.4.

[22] More precisely, for any decision rule $2\Delta = \Sigma_n - \Sigma$, where Δ is the MMD, Σ is the overall sensitivity, and Σ_n is the constant mentioned in footnote 20. See [19, p. 60].

[23] See [19, p. 74].

> *so that a bill is passed just in case the joint weight of the members voting for it exceeds half the total weight.*

This is *the second square-root rule.*[24] Note that this second rule speaks about the council members' *weights* and the quota, whereas Penrose's square-root rule (Subsection 2.2.5), which is concerned with equitability, does not mention any weights, but rather the council members' *voting powers.*

However, in view of what we said in Subsection 2.2.3, if the number of constituencies is reasonably large we can use weights as near-proxies for powers. So in this case the second square-root rule provides, in addition to the solution to the problem of majoritarianism, also a good approximation to a solution to the problem of equitability. On the other hand, equitability does not require the quota to be set as laid down in the second rule.

[24] This result was conjectured, in somewhat imprecise form, by Morriss [36, pp. 187–189] and proved in [20], [19, § 3.4]. In some cases there are also other decision rules that minimize the MMD, but they are not proper weighted ones.

Chapter 3

Assessment of Nine Proposals

In this chapter we consider nine proposals for the decision rule of an enlarged 28-member CM. In § 3.1 we list the proposals and give their relevant data and characteristics. In § 3.2, we compare the proposals according to the various criteria explained in the previous two chapters.

3.1 The proposals

Nine proposals are listed below. Of these, Proposals A, B, C and I have been devised by us in order to illustrate various points. Proposals D–H are, at the time of writing, being discussed by the EU's Inter-Governmental Conference (IGC).

For each proposal, we provide a table, giving the following data.

In the first column, the 28 present and prospective member states are listed in order of population size (as given in Table 8).

The next column, headed 'w', gives the weight assigned to each member.

The third column gives the Penrose measure (ψ) of the voting power of each member under the proposed rule.

The fourth column, headed '100β', gives the relative voting power of each member as a percentage of the total. The entries in this column are obtained from the previous column by dividing each value of ψ by the sum of all these values (which we shall denote by Σ); the result (which is the value of the Banzhaf index β) is then multiplied by 100 to yield the required percentage.

The last column, headed 'Quotient', gives the result of dividing the number shown in the previous column (100β) by the number shown for the same member in the last column of Table 8. In other words, the figures in this last column have the same meaning as those given in Table 7 for the first five periods of QMV.

Under each table we state the values of the following two characteristics.

q The quota. Note however that two of the proposals—G and I—are not pure weighted rules but prescribe a *double majority*: for a resolution to pass, it is required not only that the *joint weight* of the members voting for it exceed q, but also that the *number* of these members exceed a certain threshold.

min # The least number of members whose 'yes' vote is required to pass a resolution. In the case of the two double majority rules, this least number is imposed as the threshold just referred to; and this is accordingly noted next to the value of min #. In the remaining proposals, min # is the least number of members whose joint weight equals or exceeds q; it is obtained by going down the 'w' column in the table and adding up the weights until the running total reaches or exceeds q.

In describing each proposal, we state its provenance, describe its structure and give the numerical values of eight characteristic parameters, which are used in assessing the proposals.

The first four of these parameters provide information regarding the equitability of the proposal. According to Penrose's square-root rule (Subsection 2.2.5), if a proposal were perfectly equitable, its '100β' column would be the same as the last column of Table 8, and therefore all the entries in its 'Quotient' column would be 1. These four parameters allow us to assess how close the '100β' column of the given proposal is to the ideal presented by the last column of Table 8.

ρ Pearson's product-moment coefficient of correlation between the '100β' column of the given proposal and the last column of Table 8.[1] The closer ρ is to 1, the better the fit between these two columns, and hence the more equitable is the given proposal.

χ^2 Chi-squared: another way of measuring the closeness of fit between the '100β' column of the given proposal and the last column of Table 8.[2] The *smaller* the value of χ^2, the closer the fit.

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[1]For the definition of ρ, see any textbook on statistics.

[2]If b_i is the i-th entry in the '100β' column, and s_i is the corresponding entry in the last column of Table 8, then

$$\chi^2 = \sum_{i=1}^{28} \frac{(b_i - s_i)^2}{s_i}.$$

While ρ and χ^2 measure the *overall* equitability of a proposal, the following two parameters focus on the most extreme *individual* deviations from equitability, which presumably are the most invidious.

$\max|d|$ Maximal relative deviation. This is obtained from the 'Quotient' column in the table of the proposal. In this column, an entry of exactly 1 means that the member state in question has got exactly its equitable share of voting power. Where the entry differs from 1, the difference represents a positive or negative *relative deviation* from equitability. For example, in Table 9 the entry for Germany is 1.033, a relative deviation of +0.033 or +3.3%. This means that under Proposal A Germany has +3.3% more power than equitability would justify. For Malta the entry is 0.979, so the relative deviation is −0.021, or −2.1%, which means that Malta has 2.1% less power than equitability would require.

The value of $\max|d|$ is the largest of these relative deviations in either direction (that is, ignoring their sign). The larger the value of $\max|d|$, the more invidious is the deviation of the proposal from equitability.

$\mathrm{ran}(d)$ Range of relative deviations. This is also obtained from the 'Quotient' column. It is obtained by subtracting the smallest entry in this column from the largest. For example, in Table 9 the smallest entry is 0.979 and the largest is 1.033, so for Proposal A $\mathrm{ran}(d) = 1.033 - 0.979 = 0.054$, or 5.4%. Again, the size of $\mathrm{ran}(d)$ indicates how invidiously inequitable the proposal in question is.

The final four parameters have been defined before.

MMD The mean majority deficit of the proposal. As explained in Subsection 2.3.1, the smaller the MMD, the better the proposal is from a majoritarian viewpoint.

Σ Sensitivity. This is simply the total at the bottom of the ψ column of the proposal's table. Note that we are concerned here with sensitivity of the proposed rule for the CM regarded as a decision-making body in its own right, rather than the sensitivity of the entire two-tier system.

S Relative sensitivity. This was explained in Subsection 1.2.4. Its meaning here is the same as in Table 4 for the first five periods. High relative sensitivity (S close to 1) is a desirable feature of a decision rule, other things being equal.

R Resistance coefficient. This was explained in Subsection 1.2.5, and its meaning here is the same as in Table 4 for the first five periods. High resistance (R close to 1) is certainly very undesirable. However if one believes that the status quo ought to be easier to maintain than to change, then one would like to avoid values of R too close to 0.

Proposal A See Table 9. This proposal implements the second square-root rule (Subsection 2.3.2), as the reader can verify by comparing the 'w' column of Table 9 with the last column of Table 8, and noting that here $q = 5001$, just over half the total weight. So we should expect this proposal to be as good as possible from a majoritarian viewpoint. (It is.) Also, in view of what was said at the end of Subsection 2.3.2, we may expect it to do quite well in terms of equitability. (It does: the largest seven members—from Germany down to Poland—get a little too much power, and all the others too little; but the relative deviations are very small.) It also has very high relative sensitivity and extremely low resistance. Here are the data:

$$\rho = 0.999\,8, \qquad \chi^2 = 0.031\,07, \qquad \max|d| = 3.3\%, \qquad \mathrm{ran}(d) = 5.4\%,$$
$$\mathrm{MMD} = 1\,828,$$
$$\Sigma = 3.516 \qquad \mathrm{S} = 0.990, \qquad \mathrm{R} = 0.000\,344.$$

The only really serious criticism that can be made of this proposal is that its resistance is *too* low.

As we can see from Table 4, in the first five periods of the EU the QMV has had much higher resistance. This was mainly because the quota was kept quite high: about 71% of the total weight (see Table 2) rather than just over 50%, as in the present proposal. However, as we noted in Subsection 1.2.5, keeping the quota *pegged* at that level has resulted in pushing the resistance upwards—a very dangerous tendency, which, if allowed to continue, may lead to sclerotic immobilism.

Proposal B See Table 10. Our second proposal is a modified version of Proposal A. As before, the weight assigned to each member state is proportional to the square root of its population; but now the quota q has been raised from $5,001$ to $6,000$. As expected, this proposal is less good from a majoritarian viewpoint. Also, sensitivity is reduced, while resistance is considerably increased. On the other hand, in view of what we said in Subsection 2.2.3, we would expect this proposal to be quite equitable. In fact, it turns out to be more equitable than Proposal A; indeed, it is the most equitable of all nine proposals we consider in this booklet. Here are the data:

$$\rho = 1.000\,0, \qquad \chi^2 = 0.001\,69, \qquad \max|d| = 0.6\%, \qquad \mathrm{ran}(d) = 1.1\%,$$
$$\mathrm{MMD} = 4\,203,$$
$$\Sigma = 2.416 \qquad \mathrm{S} = 0.967, \qquad \mathrm{R} = 0.614.$$

Proposal C See Table 11. This is another modification of our Proposal A: this time we have moved the quota q up to 71% of the total weight, which has been the practice in the EU so far (see Table 2). As expected, this has an undesirable effect from a majoritarian viewpoint. Also, sensitivity is reduced, and resistance dangerously increased. But, again as expected, equitability is maintained, although this proposal does slightly less well than Proposal A in this respect. Here are the data:

$$\rho = 0.998\,5, \qquad \chi^2 = 0.214\,75, \qquad \max|d| = 8.5\%, \qquad \mathrm{ran}(d) = 14.2\%,$$
$$\mathrm{MMD} = 8\,021,$$
$$\Sigma = 0.628 \qquad \mathrm{S} = 0.887, \qquad \mathrm{R} = 0.937.$$

Proposal D See Table 12. This proposal appears in EU Presidency document [9, Annex 2.8] and is explained on p. 23 of that document. It is very similar to our Proposal C. The weight assigned to each member state is roughly proportional to the square root of its population; in fact, it is double the square root of its population expressed in millions of inhabitants, rounded off to the nearest integer. As a result of this rounding off, several members are banded together and get the same weight.[3] The quota q is set at 146, which, as a percentage of the total weight (71.57%) is just slightly higher than in Proposal C. The characteristics of this proposal are therefore not very different from those of Proposal C. The most important (and unfortunate) difference is that—no doubt due to the banding—there are much more extreme individual deviations from equitability: Cyprus' voting power is 23.6% over the odds, whereas Luxembourg is short-changed by 19%. Here are the data:

$$\rho = 0.998\,2, \qquad \chi^2 = 0.239\,01, \qquad \max|d| = 23.6\%, \qquad \mathrm{ran}(d) = 42.6\%,$$
$$\mathrm{MMD} = 8\,100,$$
$$\Sigma = 0.591 \qquad \mathrm{S} = 0.884, \qquad \mathrm{R} = 0.941.$$

[3] This kind of banding, whereby members of roughly similar size are grouped together and given the same weight, has been practised so far, in the first five periods of the EU; and is also followed by the other proposals currently discussed by the IGC. It has the practical advantage that redistribution of weights, so as to keep up with population changes, may not need to be done as frequently as under rules that prescribe a more precise relationship between weight and size.

Proposal E See Table 13. This proposal was submitted by the Italian delegation to the IGC [10].[4] In the table and in our calculations we have corrected an error that occurs in the proposal as presented in [10, p. 9]: we have set the total weight to its correct value of 354 (instead of 363); and we have accordingly adjusted the quota to 252 (rather than 258) so that the quota as percentage of the total weight should be as near as possible to what the authors of the proposal apparently intended.

From a majoritarian viewpoint, this proposal is somewhat better than Proposals C and D. It is also somewhat more sensitive and less resistant than they are. But as regards equitability it is considerably worse. It is very unfair to Lithuania, whose voting power is less than half of what it should get; while Malta gets nearly half as much again as it ought to. These are the two extreme deviations, but there are several others that are also quite large. Here are the data:

$$\rho = 0.975\,5, \qquad \chi^2 = 4.73, \qquad \max|d| = 52.3\%, \qquad \mathrm{ran}(d) = 100.8\%,$$
$$\mathrm{MMD} = 7\,552,$$
$$\Sigma = 0.782 \qquad \mathrm{S} = 0.900, \qquad \mathrm{R} = 0.906.$$

Proposal F See Table 14. This was one of three proposals submitted by the EU Presidency; it is presented in [8, Annex II, p. 6]. This proposal has little to recommend it. From a majoritarian viewpoint, it is the worst of all nine proposals; and it has also the highest resistance. In terms of equitability it is worse than Proposal E (except as measured by ρ); and its extreme individual deviations from equitability are surely unacceptable. Here are the data:

$$\rho = 0.983\,3, \qquad \chi^2 = 5.93, \qquad \max|d| = 143.1\%, \qquad \mathrm{ran}(d) = 168.2\%,$$
$$\mathrm{MMD} = 8\,191,$$
$$\Sigma = 0.584 \qquad \mathrm{S} = 0.883, \qquad \mathrm{R} = 0.946.$$

Proposal G See Table 15. This is the second of the three proposals submitted by the EU Presidency; it is presented in [8, Annex III, p. 7]. The weight of each member state is roughly proportional to its population (rather than to its square root); the weights add up to $1,000$ and the quota is set at 600. But this is a *double majority* rule: in order to pass, a resolution would need to gain the support of at least 14 members, whose joint weight is at least 600. Interestingly, this proposal is very good from a majoritarian viewpoint:

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[4] A second Italian proposal is presented in [10, p. 10]; but it lists only 27 members (it excludes Turkey as a prospective member). We have not dealt with this proposal, as it is not properly comparable with the other proposals we consider, all of which assume 28 members.

in this respect it does better than all the proposals listed so far, except of course for the Proposal A, which attains the absolute optimum. Also, the sensitivity of the present proposal is commendably high (very nearly as high as that of Proposal B) and its resistance comfortably low (lower than that of Proposal B but higher than that of Proposal A).

Unfortunately, this proposal performs very badly indeed on the equitability front: about as badly as Proposals E and F. Its individual deviations are less extreme than those of the latter, but more than those of the former. Strangely, both the very large members (from Germany down to Poland) and the very small ones (from Malta up to Estonia) get too much voting power, while all the others get too little. Finland and Sweden are particularly hard-done-by. Here are the data:

$$\rho = 0.972\,2 \qquad \chi^2 = 5.69, \qquad \max|d| = 73.3\%, \qquad \mathrm{ran}(d) = 100.1\%,$$
$$\mathrm{MMD} = 3\,865,$$
$$\Sigma = 2.370 \qquad \mathrm{S} = 0.966, \qquad \mathrm{R} = 0.486.$$

Proposal H See Table 16. This is the last of the three proposals submitted by the EU Presidency; it is presented in [8, Annex IV, p. 8]. The best that can be said about it is that in every respect it is not quite so bad as Proposal F; but this is not saying much. Here are the data:

$$\rho = 0.984\,5, \qquad \chi^2 = 2.32, \qquad \max|d| = 87.7\%, \qquad \mathrm{ran}(d) = 105.9\%,$$
$$\mathrm{MMD} = 8\,063,$$
$$\Sigma = 0.606 \qquad \mathrm{S} = 0.885, \qquad \mathrm{R} = 0.939.$$

Proposal I See Table 17. We do not put this proposal forward as a serious contender, but purely for the sake of comparison with the other eight. It goes back to a throw-away remark that Penrose makes at the very end of his little book [39, p. 73], after putting forward a detailed decision rule for the UN along the lines of his square-root rule, with quota equal to just over half the total weight—in other words, a rule analogous to our Proposal A. He then makes the rather cryptic remark that ‘a somewhat similar effect could be produced’ by a double majority rule, in which weight would be proportional to population rather than to its square root. He doesn’t state explicitly the quota and threshold number of members, but from the context it is clear that he means both of them to be just over half of the respective totals (so just over half the total weight and a simple majority of the members would be needed to pass a resolution). He does not actually recommend this system but says that it ‘would be inaccurate in that it would favour very large countries'. He leaves it at that, not offering any calculation or other evidence.

We thought we might try this idea in the case of the CM. Accordingly, we modified Proposal G: keeping the same weights (which are roughly proportional to populations) but changing the quota to 501 out of a total weight of 1,000, and the threshold to 15 members. The result vindicates Penrose's statement (or perhaps more correctly his conjecture) in part: the present proposal does very well in terms of majoritarianism, sensitivity and resistance. In these respects it is second only to Proposal A. On the other hand it is extremely bad from the viewpoint of equitability. This is what Penrose obviously meant by it being 'inaccurate'. However, rather than favouring the large countries, it is strongly biased in favour of the smaller ones. Here are the data:

$$\rho = 0.968\,9, \qquad \chi^2 = 21.05, \qquad \max|d| = 312.6\%, \qquad \mathrm{ran}(d) = 335.4\%,$$
$$\mathrm{MMD} = 3\,009,$$
$$\Sigma = 3.438 \qquad \mathrm{S} = 0.988, \qquad \mathrm{R} = 0.359.$$

3.2 Comparing the proposals

Let us now put our findings together. In Table 18 we have tabulated the various characteristics of the nine proposals. Using this table, we shall now attempt to rank the nine proposals according to our criteria.

3.2.1 Ranking by equitability Our four indicators of equitability are ρ, χ^2, $\max|d|$ and $\mathrm{ran}(d)$. Recall that from the viewpoint of equitability a high value of ρ (that is a value near 1) is desirable; but high values of the other three indicators are undesirable.

The four indicators do not completely agree about how the proposals ought to be ranked. They do agree about the top four: B, A, C and D, in this order. They also agree that I must come at the very bottom. But they are not in agreement about how ranks 5, 6, 7 and 8 are to be allocated among Proposals E, F, G and H.

In our opinion, priority should be given to the indicators $\max|d|$ and $\mathrm{ran}(d)$. As we pointed out on p. 29, ρ and χ^2 are concerned with overall or global equitability, whereas $\max|d|$ and $\mathrm{ran}(d)$ are concerned with the most extreme individual deviations from equitability. But why is equitability desirable? Arguably, the main reason is that an inequitable distribution of voting power is invidious, and in the long run may undermine the stability of the EU. For this reason it seems to us that preventing extreme deviations is more important than achieving overall equitability. Our equitability ranking is therefore:

$$\text{B, A, C, D, } \genfrac{}{}{0pt}{}{\text{E}}{\text{G}} \text{ (joint), H, F, I,}$$

in descending order of merit. Notice, however, that there is a big gap between the first four proposals in this list and the remaining five: all the latter give rise to quite large deviations from equitability.

3.2.2 Ranking by majoritarianism Here there is no difficulty; the ranking is:

A, I, G, B, E, C, H, D, F,

in descending order of merit.

3.2.3 Ranking by compliancy By the term 'compliancy' we mean high sensitivity combined with low resistance. The two do tend to go together. In fact, Table 18 shows that as far as our nine proposals are concerned they only disagree as to whether Proposal B should come third and Proposal G fourth, or vice versa. Our compliancy ranking is therefore:

$$\text{A, I, } \genfrac{}{}{0pt}{}{\text{B}}{\text{G}} \text{ (joint), E, C, H, D, F,}$$

in descending order of merit. Note, by the way, that the rankings by majoritarianism and by compliancy largely agree.

3.2.4 The bottom line The final choice is largely a matter of opinion and political preference. Our view is that equitability ought to be the prime consideration, because, as we have pointed out, inequitable distribution of voting power in the EU is invidious and a potential cause of instability. As we have shown, the other criteria are of course also important and should not be ignored.

On this basis, the choice must be made between B, A, C and D. Proposal A would have been ideal if there were no objection to a decision rule with such low resistance. But there is a very good case for making the decision rule somewhat more favourable to maintaining the status quo and less favourable to adopting any resolution that changes it. If so, Proposal A must be rejected, and the choice confined to B, C and D.

Proposals C and D, however, perform very badly according to the criteria of majoritarianism and pliancy. In particular, their resistance is dangerously high.

The leading politicians and officials of the EU do not seem to be aware of this danger. From their discussion documents, such as [8, 9, 10], it appears that they believe that by pegging the quota at a fixed proportion of the total weight (or as representing a fixed proportion of the entire EU population) they are keeping things as they are, 'steady as she goes'. But, as we argued in Subsection 1.2.5, this is a dangerous illusion. With each past enlargement of the EU, the pegging of the quota has pushed the resistance coefficient R ever higher. If the quota will stay pegged at about 71% of the total weight when the EU is enlarged to 28 members, the result will be a very resistant decision rule, that may well lead to stagnation and immobilism.

On these grounds, the clear favourite must be Proposal B.

Tables

All the tables for the first five periods of the EU are taken or adapted from [19]. In all these tables, 'Germany' in the pre-unification period denotes West Germany.

Table 1: Population of EU member states 1958–95 (1000s)

Country	1958–72	1973–80	1981–85	1986–94	1995
Germany	54 290	61 970	61 660	61 010	81 640
Italy	49 040	54 788	56 501	56 821	57 290
France	44 790	51 920	54 136	55 476	58 150
Neth'lnds	11 190	13 401	14 213	14 583	15 450
Belgium	9 050	9 740	9 853	9 876	10 140
Lux'mbrg	310	353	365	370	400
UK		55 988	55 387	56 776	58 260
Denmark		5 007	5 121	5 119	5 230
Ireland		3 086	3 431	3 542	3 580
Greece			9 701	9 994	10 460
Spain				38 632	39 210
Portugal				9 897	9 900
Sweden					8 830
Austria					8 050
Finland					5 110
Total	168 670	256 253	270 368	322 096	371 700

Note For sources for this table, see [19, p. 157].

Table 2: QMV weights and quota, first five periods

Country	1958	1973	1981	1986	1995
Germany	4	10	10	10	10
Italy	4	10	10	10	10
France	4	10	10	10	10
Neth'lnds	2	5	5	5	5
Belgium	2	5	5	5	5
Lux'mbrg	1	2	2	2	2
UK		10	10	10	10
Denmark		3	3	3	3
Ireland		3	3	3	3
Greece			5	5	5
Spain				8	8
Portugal				5	5
Sweden					4
Austria					4
Finland					3
Total	17	58	63	76	87
Quota	12	41	45	54	62
Least #	3	5	5	7	8
Quota %	70.59	70.69	71.43	71.05	71.26

Notes The penultimate row gives the least number of members whose total weight equals or exceeds the quota. The last row gives the quota as percentage of the total weight.

Table 3: QMV weight/population index, first five periods

Country	1958	1973	1981	1986	1995
Germany	0.731	0.713	0.696	0.695	0.523
Italy	0.809	0.806	0.760	0.746	0.746
France	0.886	0.851	0.793	0.764	0.735
Neth'lnds	1.773	1.648	1.510	1.453	1.383
Belgium	2.193	2.268	2.178	2.146	2.107
Lux'mbrg	32.996	25.032	23.515	22.909	21.362
UK		0.789	0.775	0.746	0.733
Denmark		2.647	2.514	2.484	2.451
Ireland		4.295	3.752	3.589	3.580
Greece			2.212	2.120	2.042
Spain				0.878	0.872
Portugal				2.141	2.158
Sweden					1.935
Austria					2.123
Finland					2.508

Note This table gives, for each member state and each of the five periods, the ratio between the member's share in the total weight (votes) under QMV and that member's share in the total population. Thus the quantity shown in this table is $(wP)/(Wp)$, where w = the given member's weight under QMV, W = the total weight of all CM members, p = the member's population and P = the total population of the EU.

Table 4: Penrose measure (ψ) under QMV, first five periods

Country	1958	1973	1981	1986	1995
Germany	0.3125	0.2070	0.1953	0.1396	0.1129
Italy	0.3125	0.2070	0.1953	0.1396	0.1129
France	0.3125	0.2070	0.1953	0.1396	0.1129
Neth'lnds	0.1875	0.1133	0.1016	0.0723	0.0594
Belgium	0.1875	0.1133	0.1016	0.0723	0.0594
Lux'mbrg	0.0000	0.0195	0.0508	0.0195	0.0229
UK		0.2070	0.1953	0.1396	0.1129
Denmark		0.0820	0.0508	0.0498	0.0363
Ireland		0.0820	0.0508	0.0498	0.0363
Greece			0.1016	0.0723	0.0594
Spain				0.1182	0.0934
Portugal				0.0723	0.0594
Sweden					0.0484
Austria					0.0484
Finland					0.0363
Total	1.3125	1.2383	1.2383	1.0850	1.0110
S	0.8450	0.8383	0.8580	0.8510	0.8607
R	0.5806	0.7098	0.7280	0.8041	0.8445

Notes S is the *relative sensitivity* (see Subsection 1.2.4) and R the *resistance coefficient* of the decision rule (see Subsection 1.2.5).

Table 5: Banzhaf index (β) under QMV, first five periods

Country	1958	1973	1981	1986	1995
Germany	0.238	0.167	0.158	0.129	0.112
Italy	0.238	0.167	0.158	0.129	0.112
France	0.238	0.167	0.158	0.129	0.112
Neth'lnds	0.143	0.091	0.082	0.067	0.059
Belgium	0.143	0.091	0.082	0.067	0.059
Lux'mbrg	0.000	0.016	0.041	0.018	0.023
UK		0.167	0.158	0.129	0.112
Denmark		0.066	0.041	0.046	0.036
Ireland		0.066	0.041	0.046	0.036
Greece			0.082	0.067	0.059
Spain				0.109	0.092
Portugal				0.067	0.059
Sweden					0.048
Austria					0.048
Finland					0.036
Total	1.000	1.000	1.000	1.000	1.000

Table 6: QMV voting power/weight index, first five periods

Country	1958	1973	1981	1986	1995
Germany	1.012	0.970	0.994	0.978	0.971
Italy	1.012	0.970	0.994	0.978	0.971
France	1.012	0.970	0.994	0.978	0.971
Neth'lnds	1.214	1.061	1.033	1.012	1.022
Belgium	1.214	1.061	1.033	1.012	1.022
Lux'mbrg	0.000	0.457	1.292	0.684	0.985
UK		0.970	0.994	0.978	0.971
Denmark		1.281	0.861	1.163	1.042
Ireland		1.281	0.861	1.163	1.042
Greece			1.033	1.012	1.022
Spain				1.035	1.005
Portugal				1.012	1.022
Sweden					1.041
Austria					1.041
Finland					1.042

Note This table gives, for each member state and each of the five periods, the ratio between the member's Banzhaf index β and that member's share in the total weight under QMV. Thus the quantity shown in this table is $\beta W/w$, where β = the given member's Banzhaf index under QMV, w = the given member's weight under QMV and W = the total weight of all CM members.

Table 7: QMV power/population-square-root index, first five periods

Country	1958	1973	1981	1986	1995
Germany	0.904	0.878	0.902	0.899	0.797
Italy	0.951	0.934	0.942	0.932	0.951
France	0.995	0.959	0.963	0.943	0.944
Neth'lnds	1.195	1.033	0.977	0.952	0.964
Belgium	1.333	1.212	1.174	1.156	1.190
Lux'mbrg	0.000	1.097	3.049	1.615	2.309
UK		0.924	0.952	0.932	0.943
Denmark		1.224	0.814	1.107	1.013
Ireland		1.559	0.994	1.339	1.224
Greece			1.183	1.149	1.171
Spain				0.956	0.952
Portugal				1.155	1.204
Sweden					1.039
Austria					1.088
Finland					1.025

Note This table gives, for each member state and each of the five periods, the ratio between the member's Banzhaf index (β) under QMV and that member's share in the sum of the square roots of populations. Thus the quantity shown in this table is $\beta S/s$, where $\beta =$ the given member's Banzhaf index under QMV, $s =$ the square root of the member's population and $S =$ the sum of these square roots.

Table 8: Population of present and prospective EU members

Country	Pop. (1000s)	%	Pop. sqrt	%
Germany	82 038	15.037	9 057.48	8.79
Turkey	64 385	11.802	8 024.03	7.79
UK	59 247	10.860	7 697.21	7.47
France	58 966	10.808	7 678.93	7.46
Italy	57 612	10.560	7 590.26	7.37
Spain	39 394	7.220	6 276.46	6.09
Poland	38 667	7.088	6 218.28	6.04
Romania	22 489	4.122	4 742.26	4.60
Netherlands	15 760	2.889	3 969.89	3.85
Greece	10 533	1.931	3 245.46	3.15
Czech Rep	10 290	1.886	3 207.80	3.11
Belgium	10 213	1.872	3 195.78	3.10
Hungary	10 092	1.850	3 176.79	3.08
Portugal	9 980	1.829	3 159.11	3.07
Sweden	8 854	1.623	2 975.57	2.89
Bulgaria	8 230	1.509	2 868.80	2.79
Austria	8 082	1.481	2 842.89	2.76
Slovakia	5 393	0.989	2 322.28	2.25
Denmark	5 313	0.974	2 304.99	2.24
Finland	5 160	0.946	2 271.56	2.21
Ireland	3 744	0.686	1 934.94	1.88
Lithuania	3 701	0.678	1 923.80	1.87
Latvia	2 439	0.447	1 561.73	1.52
Slovenia	1 978	0.363	1 406.41	1.37
Estonia	1 446	0.265	1 202.50	1.17
Cyprus	752	0.138	867.18	0.84
Luxembourg	429	0.079	654.98	0.64
Malta	379	0.069	615.63	0.60
Total	545 566	100.001	102 993.00	100.00

Note Source of population figures: [8]. The second column of figures shows the population as percentage of the total (the apparent discrepancy in the total is due to rounding errors); the next column shows the square root of the population; the last column shows the square root of the population as percentage of the total.

Table 9: Proposal A

Country	w	ψ	100β	Quotient
Germany	879	0.319 282	9.080 8	1.033
Turkey	779	0.279 160	7.939 7	1.019
UK	747	0.266 702	7.585 3	1.015
France	746	0.266 316	7.574 4	1.015
Italy	737	0.262 834	7.475 3	1.014
Spain	609	0.214 542	6.101 8	1.002
Poland	604	0.212 683	6.049 0	1.001
Romania	460	0.160 342	4.560 3	0.991
Netherlands	385	0.133 662	3.801 5	0.987
Greece	315	0.109 032	3.101 0	0.984
Czech Rep	311	0.107 630	3.061 1	0.984
Belgium	310	0.107 279	3.051 2	0.984
Hungary	308	0.106 580	3.031 3	0.984
Portugal	307	0.106 231	3.021 3	0.984
Sweden	289	0.099 935	2.842 3	0.983
Bulgaria	279	0.096 445	2.743 0	0.983
Austria	276	0.095 397	2.713 2	0.983
Slovakia	225	0.077 652	2.208 5	0.982
Denmark	224	0.077 305	2.198 6	0.982
Finland	221	0.076 261	2.169 0	0.981
Ireland	188	0.064 819	1.843 5	0.981
Lithuania	187	0.064 474	1.833 7	0.981
Latvia	152	0.052 374	1.489 6	0.980
Slovenia	137	0.047 187	1.342 0	0.980
Estonia	117	0.040 301	1.146 2	0.980
Cyprus	84	0.028 919	0.822 5	0.979
Luxembourg	64	0.022 021	0.626 3	0.979
Malta	60	0.020 653	0.587 4	0.979
Total	10 000	3.516 019	100.000 0	

$$q = 5001 \quad \min \# = 7.$$

Note For general explanations see p. 27. For details about Proposal A, see p. 30.

Table 10: Proposal B

Country	w	ψ	100β	Quotient
Germany	879	0.213 722	8.844 8	1.006
Turkey	779	0.189 107	7.826 1	1.005
UK	747	0.181 224	7.499 9	1.004
France	746	0.180 978	7.489 7	1.004
Italy	737	0.178 759	7.397 8	1.004
Spain	609	0.147 332	6.097 3	1.001
Poland	604	0.146 101	6.046 3	1.001
Romania	460	0.110 970	4.592 5	0.998
Netherlands	385	0.092 772	3.839 3	0.997
Greece	315	0.075 835	3.138 4	0.996
Czech Rep	311	0.074 869	3.098 4	0.996
Belgium	310	0.074 628	3.088 4	0.996
Hungary	308	0.074 142	3.068 3	0.996
Portugal	307	0.073 901	3.058 3	0.996
Sweden	289	0.069 556	2.878 6	0.996
Bulgaria	279	0.067 142	2.778 6	0.996
Austria	276	0.066 418	2.748 7	0.996
Slovakia	225	0.054 119	2.239 7	0.995
Denmark	224	0.053 878	2.229 7	0.995
Finland	221	0.053 155	2.199 8	0.995
Ireland	188	0.045 206	1.870 8	0.995
Lithuania	187	0.044 966	1.860 9	0.995
Latvia	152	0.036 541	1.512 2	0.995
Slovenia	137	0.032 931	1.362 8	0.995
Estonia	117	0.028 121	1.163 8	0.995
Cyprus	84	0.020 187	0.835 4	0.995
Luxembourg	64	0.015 380	0.636 5	0.995
Malta	60	0.014 424	0.596 9	0.995
Total	10 000	2.416 363	100.000 0	

$$q = 6000, \quad \min \# = 10.$$

Note For general explanations see p. 27. For details about Proposal B, see p. 30.

Table 11: Proposal C

Country	w	ψ	100β	Quotient
Germany	879	0.050 474	8.040 5	0.915
Turkey	779	0.046 406	7.392 5	0.949
UK	747	0.044 956	7.161 6	0.959
France	746	0.044 910	7.154 2	0.959
Italy	737	0.044 488	7.087 0	0.962
Spain	609	0.038 012	6.055 4	0.994
Poland	604	0.037 746	6.012 9	0.996
Romania	460	0.029 524	4.703 1	1.022
Netherlands	385	0.024 968	3.977 5	1.033
Greece	315	0.020 587	3.279 5	1.041
Czech Rep	311	0.020 332	3.238 9	1.041
Belgium	310	0.020 269	3.228 9	1.042
Hungary	308	0.020 142	3.208 7	1.042
Portugal	307	0.020 079	3.198 5	1.042
Sweden	289	0.018 935	3.016 3	1.044
Bulgaria	279	0.018 294	2.914 3	1.045
Austria	276	0.018 102	2.883 6	1.045
Slovakia	225	0.014 815	2.360 0	1.049
Denmark	224	0.014 750	2.349 6	1.049
Finland	221	0.014 555	2.318 7	1.049
Ireland	188	0.012 408	1.976 6	1.051
Lithuania	187	0.012 344	1.966 3	1.051
Latvia	152	0.010 051	1.601 1	1.053
Slovenia	137	0.009 066	1.444 3	1.054
Estonia	117	0.007 743	1.233 5	1.054
Cyprus	84	0.005 566	0.886 6	1.055
Luxembourg	64	0.004 245	0.676 2	1.057
Malta	60	0.003 977	0.633 5	1.056
Total	10 000	0.627 743	100.000 0	

$$q = 7100, \quad \min\# = 13.$$

Note For general explanations see p. 27. For details about Proposal C, see p. 31.

Table 12: Proposal D

Country	w	ψ	100β	Quotient
Germany	18	0.047 413	8.020 6	0.912
Turkey	16	0.043 772	7.404 6	0.951
UK	15	0.041 716	7.056 8	0.945
France	15	0.041 716	7.056 8	0.946
Italy	15	0.041 716	7.056 8	0.958
Spain	13	0.037 167	6.287 3	1.032
Poland	12	0.034 717	5.872 9	0.972
Romania	9	0.026 765	4.527 7	0.984
Netherlands	8	0.023 962	4.053 5	1.053
Greece	6	0.018 173	3.074 2	0.976
Czech Rep	6	0.018 173	3.074 2	0.988
Belgium	6	0.018 173	3.074 2	0.992
Hungary	6	0.018 173	3.074 2	0.998
Portugal	6	0.018 173	3.074 2	1.001
Sweden	6	0.018 173	3.074 2	1.064
Bulgaria	6	0.018 173	3.074 2	1.102
Austria	6	0.018 173	3.074 2	1.114
Slovakia	5	0.015 213	2.573 5	1.144
Denmark	5	0.015 213	2.573 5	1.149
Finland	5	0.015 213	2.573 5	1.164
Ireland	4	0.012 205	2.064 7	1.098
Lithuania	4	0.012 205	2.064 7	1.104
Latvia	3	0.009 181	1.553 1	1.022
Slovenia	3	0.009 181	1.553 1	1.134
Estonia	2	0.006 137	1.038 1	0.887
Cyprus	2	0.006 137	1.038 1	1.236
Luxembourg	1	0.003 065	0.518 5	0.810
Malta	1	0.003 065	0.518 5	0.864
Total	204	0.591 142	100.000 0	

$$q = 146 = 71.57\% \text{ of } 204, \quad \min\# = 13.$$

Note For general explanations see p. 27. For details about Proposal D, see p. 31.

Table 13: Proposal E

Country	w	ψ	100β	Quotient
Germany	33	0.070 263	8.987 0	1.022
Turkey	33	0.070 263	8.987 0	1.154
UK	33	0.070 263	8.987 0	1.203
France	33	0.070 263	8.987 0	1.205
Italy	33	0.070 263	8.987 0	1.219
Spain	26	0.057 967	7.414 2	1.217
Poland	26	0.057 967	7.414 2	1.228
Romania	10	0.022 875	2.925 8	0.636
Netherlands	10	0.022 875	2.925 8	0.760
Greece	10	0.022 875	2.925 8	0.929
Czech Rep	10	0.022 875	2.925 8	0.941
Belgium	10	0.022 875	2.925 8	0.944
Hungary	10	0.022 875	2.925 8	0.950
Portugal	10	0.022 875	2.925 8	0.953
Sweden	8	0.018 057	2.309 6	0.799
Bulgaria	8	0.018 057	2.309 6	0.828
Austria	6	0.013 914	1.779 7	0.645
Slovakia	6	0.013 914	1.779 7	0.791
Denmark	6	0.013 914	1.779 7	0.795
Finland	6	0.013 914	1.779 7	0.805
Ireland	6	0.013 914	1.779 7	0.947
Lithuania	3	0.006 968	0.891 2	0.477
Latvia	3	0.006 968	0.891 2	0.586
Slovenia	3	0.006 968	0.891 2	0.651
Estonia	3	0.006 968	0.891 2	0.762
Cyprus	3	0.006 968	0.891 2	1.061
Luxembourg	3	0.006 968	0.891 2	1.392
Malta	3	0.006 968	0.891 2	1.485
Total	354	0.781 833	100.000 0	

$$q = 252 = 71.19\% \text{ of } 354, \quad \min\# = 11.$$

Note For general explanations see p. 27. For details about Proposal D, see p. 32.

Table 14: Proposal F

Country	w	ψ	100β	Quotient
Germany	10	0.038 399	6.580 4	0.749
Turkey	10	0.038 399	6.580 4	0.845
UK	10	0.038 399	6.580 4	0.881
France	10	0.038 399	6.580 4	0.882
Italy	10	0.038 399	6.580 4	0.893
Spain	8	0.032 034	5.989 7	0.901
Poland	8	0.032 034	5.989 7	0.909
Romania	6	0.024 732	4.238 3	0.921
Netherlands	5	0.020 853	3.573 6	0.928
Greece	5	0.020 853	3.573 6	1.134
Czech Rep	5	0.020 853	3.573 6	1.149
Belgium	5	0.020 853	3.573 6	1.153
Hungary	5	0.020 853	3.573 6	1.160
Portugal	5	0.020 853	3.573 6	1.164
Sweden	4	0.016 819	2.882 2	0.997
Bulgaria	4	0.016 819	2.882 2	1.033
Austria	4	0.016 819	2.882 2	1.044
Slovakia	3	0.012 704	2.177 1	0.968
Denmark	3	0.012 704	2.177 1	0.972
Finland	3	0.012 704	2.177 1	0.985
Ireland	3	0.012 704	2.177 1	1.158
Lithuania	3	0.012 704	2.177 1	1.164
Latvia	3	0.012 704	2.177 1	1.432
Slovenia	3	0.012 704	2.177 1	1.589
Estonia	3	0.012 704	2.177 1	1.861
Cyprus	2	0.008 510	1.458 4	1.736
Luxembourg	2	0.008 510	1.458 4	2.279
Malta	2	0.008 510	1.458 4	2.431
Total	144	0.583 535	100.000 0	

$$q = 102 = 70.83\% \text{ of } 144, \quad \min\# = 14.$$

Note For general explanations see p. 27. For details about Proposal F, see p. 32.

Table 15: Proposal G

Country	w	ψ	100β	Quotient
Germany	150	0.290 492	12.259 4	1.395
Turkey	118	0.227 884	9.617 2	1.235
UK	109	0.210 810	8.896 7	1.191
France	108	0.208 945	8.817 9	1.182
Italy	106	0.205 214	8.660 5	1.175
Spain	72	0.146 144	6.167 6	1.013
Poland	71	0.144 243	6.087 4	1.008
Romania	41	0.092 785	3.915 7	0.851
Netherlands	29	0.072 342	3.053 0	0.793
Greece	19	0.055 361	2.336 4	0.742
Czech Rep	19	0.055 361	2.336 4	0.751
Belgium	19	0.055 361	2.336 4	0.754
Hungary	18	0.053 668	2.264 9	0.735
Portugal	18	0.053 668	2.264 9	0.738
Sweden	16	0.050 262	2.121 2	0.734
Bulgaria	15	0.048 567	2.049 6	0.735
Austria	15	0.048 567	2.049 6	0.743
Slovakia	10	0.040 050	1.690 2	0.751
Denmark	10	0.040 050	1.690 2	0.755
Finland	9	0.038 340	1.618 0	0.732
Ireland	7	0.034 928	1.474 1	0.784
Lithuania	7	0.034 928	1.474 1	0.788
Latvia	4	0.029 788	1.257 1	0.827
Slovenia	4	0.029 788	1.257 1	0.918
Estonia	3	0.028 080	1.185 0	1.013
Cyprus	1	0.024 640	1.039 8	1.238
Luxembourg	1	0.024 640	1.039 8	1.625
Malta	1	0.024 640	1.039 8	1.733
Total	1 000	2.369 545	100.000 0	

$q = 600, \quad \min \# = 14$ *imposed threshold.*

Note For general explanations see p. 27. For details about Proposal G, see p. 32.

Table 16: Proposal H

Country	w	ψ	100β	Quotient
Germany	23	0.045 741	7.549 2	0.859
Turkey	23	0.045 741	7.549 2	0.969
UK	23	0.045 741	7.549 2	1.011
France	23	0.045 741	7.549 2	1.012
Italy	23	0.045 741	7.549 2	1.024
Spain	19	0.039 447	6.510 5	1.069
Poland	19	0.039 447	6.510 5	1.078
Romania	11	0.023 861	3.938 1	0.856
Netherlands	9	0.019 720	3.254 7	0.845
Greece	9	0.019 720	3.254 7	1.033
Czech Rep	9	0.019 720	3.254 7	1.047
Belgium	9	0.019 720	3.254 7	1.050
Hungary	9	0.019 720	3.254 7	1.057
Portugal	9	0.019 720	3.254 7	1.060
Sweden	7	0.015 474	2.554 0	0.884
Bulgaria	7	0.015 474	2.554 0	0.915
Austria	7	0.015 474	2.554 0	0.925
Slovakia	5	0.011 153	1.840 7	0.818
Denmark	5	0.011 153	1.840 7	0.822
Finland	5	0.011 153	1.840 7	0.833
Ireland	5	0.011 153	1.840 7	0.979
Lithuania	5	0.011 153	1.840 7	0.984
Latvia	5	0.011 153	1.840 7	1.211
Slovenia	5	0.011 153	1.840 7	1.344
Estonia	5	0.011 153	1.840 7	1.573
Cyprus	3	0.006 824	1.126 2	1.341
Luxembourg	3	0.006 824	1.126 2	1.760
Malta	3	0.006 824	1.126 2	1.877
Total	288	0.605 898	99.999 5	

$$q = 206 = 71.53\% \text{ of } 288, \quad \min\# = 13.$$

Note For general explanations see p. 27. For details about Proposal H, see p. 33. The discepancy in the total of the 100β column is due to rounding error.

Table 17: Proposal I

Country	w	ψ	100β	Quotient
Germany	150	0.257 342	7.485 9	0.852
Turkey	118	0.214 255	6.232 5	0.800
UK	109	0.203 422	5.917 4	0.792
France	108	0.202 241	5.883 0	0.789
Italy	106	0.199 868	5.814 0	0.789
Spain	72	0.161 557	4.699 5	0.772
Poland	71	0.160 597	4.671 6	0.773
Romania	41	0.126 485	3.679 3	0.800
Netherlands	29	0.113 821	3.311 0	0.860
Greece	19	0.103 542	3.012 0	0.956
Czech Rep	19	0.103 542	3.012 0	0.968
Belgium	19	0.103 542	3.012 0	0.972
Hungary	18	0.102 522	2.982 3	0.968
Portugal	18	0.102 522	2.982 3	0.971
Sweden	16	0.100 463	2.922 4	1.011
Bulgaria	15	0.099 442	2.892 7	1.037
Austria	15	0.099 442	2.892 7	1.048
Slovakia	10	0.094 306	2.743 3	1.219
Denmark	10	0.094 306	2.743 3	1.225
Finland	9	0.093 287	2.713 6	1.228
Ireland	7	0.091 229	2.653 8	1.412
Lithuania	7	0.091 229	2.653 8	1.419
Latvia	4	0.088 157	2.564 4	1.687
Slovenia	4	0.088 157	2.564 4	1.872
Estonia	3	0.087 143	2.534 9	2.167
Cyprus	1	0.085 096	2.475 4	2.947
Luxembourg	1	0.085 096	2.475 4	3.868
Malta	1	0.085 096	2.475 4	4.126
Total	1 000	3.437 708	100.000 0	

$q = 501, \quad \min\# = 15$ *imposed threshold.*

Note For general explanations see p. 27. For details about Proposal I, see p. 33.

Table 18: Comparing the nine proposals

Proposal	ρ	χ^2	max$\lvert d\rvert$	ran(d)	MMD	S	R
A	0.999 8	0.03	3.3	5.4	1 828	0.990	0.000
B	1.000 0	0.00	0.6	1.1	4 203	0.967	0.614
C	0.998 5	0.21	8.5	14.2	8 021	0.887	0.937
D	0.998 2	0.24	23.6	42.6	8 100	0.884	0.941
E	0.975 5	4.73	52.3	100.8	7 552	0.900	0.906
F	0.983 3	5.93	143.1	168.2	8 191	0.883	0.946
G	0.972 2	5.69	73.3	100.1	3 865	0.966	0.486
H	0.984 5	2.32	87.7	105.9	8 063	0.885	0.939
I	0.968 9	21.05	312.6	335.4	3 009	0.988	0.359

max $\lvert d\rvert$ and ran(d) are given in percentages.

Note For general explanations see p. 27. For discussion see § 3.2.

Glossary

CM Council of Ministers of the EU.

Decision rule See Section 1.1, p. 1; Subsection 1.1.2, p. 2.

EU European Union. We use this term also for the earlier phases, before the Maastricht Treaty, when the Union was known by other names.

Equitability See Subsection 2.1.3, p. 18.

Majoritarianism See Subsection 2.1.3, p. 18.

$\max|d|$ Maximal relative deviation. See p. 29.

$\min\#$ The least number of members whose 'yes' vote is required to pass a resolution. See p. 28.

MMD Mean majority deficit. See Subsection 2.3.1, p. 24.

q The quota of a weighted decision rule.

QMV Qualified majority voting. Eurospeak for the weighted decision rule used by the CM. See p. iii.

Quotient See p. 27.

R Coefficient of resistance. See Subsection 1.2.5, p. 9.

$\mathrm{ran}(d)$ Range of relative deviations. See p. 29.

Resistance See Subsection 1.2.5, p. 9.

S Relative sensitivity index. See Subsection 1.2.4, p. 8.

Sensitivity See Subsection 1.2.4, p. 7.

Voting power See Subsection 1.2.2. p. 4.

w Weight assigned to voter. See p. 27.

Weighted decision rule See Subsection 1.1.1, p. 2.

β Banzhaf's index of voting power. See Subsection 1.2.3, p. 6.

ρ Pearson's product-moment coefficient of correlation. See p. 28.

Σ Sensitivity. See p. 29.

χ^2 Chi-squared. See p. 28.

ψ Penrose's measure of voting power. See Subsection 1.2.2, p. 5.

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