Voting Power and Parliamentary Defections: The 1953–54 French National Assembly Revisited

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Abstract

We reconsider Riker's [10] test of the hypothesis that inter-party migrations in the French National Assembly in 1953–54 can be explained by deputies' tendency to maximize their a priori voting power. However, instead of the Shapley–Shubik index used by Riker, we use the [absolute] Banzhaf measure, which we argue is more appropriate for this task. The theoretical model we use differs in some key respects from that of Riker, due to the difference in the underlying notion as to the nature of voting power. Our conclusion, however, is broadly in agreement with Riker's: the hypothesis under question is not substantiated.

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1 Introduction

In one of the earliest attempts to examine the effect of a priori voting power on actual political phenomena, Riker [10] looked at changes in party affiliation in the French National Assembly in 1953–54 (hereafter referred to briefly as 'the Assembly'), and used these data to test the hypothesis that deputies who switched parties were seeking thereby to increase their a priori voting power. His findings were negative, or at best inconclusive.[1]

By now there is a large body of literature applying considerations of a priori voting power to political institutions such as the UN, the US Congress, the US Presidential Electoral College, the US Supreme Court and its rulings on the implementation of the 'Equal Protection' clause in the 14th Amendment to the Constitution; and of course numerous writings on voting-power considerations in the European Union. But, as far as we know, no-one has followed Riker's lead in examining the phenomenon of inter-party migration from the viewpoint of voting power.

In his paper Riker uses only one measure of a priori voting power: the Shapley–Shubik (S-S) index, proposed by those authors [11] in 1954. Indeed, Riker refers to it throughout — beginning with the paper's title — as '*the* power index'. As a matter of fact, L S Penrose [9] had proposed another measure of a priori voting power in 1946; but it did not become widely known until 1965, when it was reinvented by Banzhaf [2], after whom it is generally named.[2] (Since then, other indices of a priori voting power have been proposed, but we need not dwell on them here.)

In the present paper we re-examine Riker's data using the Banzhaf measure of voting power instead of the S-S index. The motivation for this re-examination arises from the distinction between two underlying notions of a priori voting power — I-power and P-power — which we have explained in detail in our [5].[3] In what follows, we shall assume that the reader is familiar with this distinction.

[1]For a brief but useful summary of [10], see Brams [3, pp. 194–95].

[2]For further details see our [5, § 1.2].

[3]See also summary in our [6] and brief recapitulation in our [7].

As we have argued in [5], the S-S index is not a valid measure of a priori I-power, but is by far the most serious candidate for measuring a priori P-power. On the other hand, in our opinion P-power is the wrong notion of voting power for the kind of study undertaken by Riker. This notion conceptualizes a voter's a priori power as that voter's expected or estimated relative share in a fixed purse, 'the prize of power', a definite amount of transferable utility, which a winning coalition — having formed in order to push a bill through the voting body and having succeeded in doing so — lays its hands on and divides among its members. But, as Coleman [4] pointed out as long ago as 1971, this kind of scenario does not really apply to divisions on bills in a parliament.[4] It does not normally happen that the voters (in this case party-blocs rather than individual persons) who manage to push a bill through a legislature thereby obtain a fixed amount of transferable utility, which they share according to a binding agreement concluded beforehand.[5]

We believe that, in the present context, the appropriate notion of a priori voting power is that of I-power — power-as-influence — which is correctly captured and formalized by the Banzhaf (Bz) measure.

Note that we do not wish to dismiss out of hand the hypothesis that the political behaviour of politicians, and in particular the inter-party migrations of deputies in the Assembly, is motivated by cynical (or, in game-theoretic terms, 'rational') considerations of maximizing power rather than by ideological convictions.[6] Indeed, it is this very hypothesis that we, like Riker, wish to examine. The question, however, is *what is the precise nature* of that power which deputies of the Assembly supposedly sought to maximize. In our view, to the extent that defection of a deputy from one party-bloc in the Assembly to another was motivated by considerations of voting power, the power in question should be conceptualized as likelihood of influence over the outcome of a division of the Assembly (determining whether a bill is passed or blocked), rather than as expected share in some fixed prize.

Let us add that maximizing I-power need not be less mercenary (or 'ra-

[4] Cf. our [5, Comment 2.2.2].

[5] This scenario may be more credible in the context of divisions inside a multi-party government. Such is the context of Aumann's remark [1, p. 13] regarding the formation of multi-party governments in Israel; see however our critique in [5, p. 205, footnote 32].

[6] As Riker [10, p. 131] puts it: 'In the present context the maximization of power is equated with rational behavior (an abstraction analogous to the abstraction of economic man). Assuming that ideological convictions are irrelevant to considerations of power, behavior motivated by ideology must be regarded as irrational. It is difficult to imagine that over 40 members of the national legislature in a nation whose cultural leaders pride themselves on their logic would allow political ideals to divert them from *realpolitik*. One hardly expects a concern for principle to appear in *la République des camarades*.'

tional') than maximizing P-power. Indeed, influence can be sold;[7] and such practice by a politician is arguably as corrupt — though perhaps not quite as easy to detect — as embezzlement from the public purse.

While we were at it, we also examined 'collateral' changes of Bz power due to migration. Clearly, each migration from one bloc (the *bloc of origin*) to another (the *destination bloc*) affected also the voting power of the deputies remaining in the former bloc, as well as that of the old members of the latter bloc. We looked at the changes in the Bz powers of those deputies, in order to see whether we could detect any significant pattern.

Contrary to what may naïvely be thought, testing Riker's hypothesis in terms of I-power instead of P-power does not amount simply to substituting the (relative) Bz index for the S-S index. This is because, as we have argued in [7], the total amount of I-power cannot be regarded as a fixed quantity of transferable utility.

In Section 2 we describe the mathematical model we use and explain how and why it differs from Riker's. Our calculations and findings are described in Section 3, summarized in Table 1 and discussed in Section 4.

2 The Model

Riker [10] models the Assembly, at any point in time during 1953–54, as a weighted voting game (WVG), in which the voters are party-blocs rather than individual deputies. (Of course, some blocs are singletons, consisting of a sole member.) The justification for this is that party discipline in the Assembly was on the whole quite strict, so that the deputies of each party tended to vote in unison.[8]

The weight assigned to each bloc is the number of its members; and the quota is the least integer greater than $n/2$, where n is the total number of deputies. So in this model a bill is passed iff the total membership of the blocs voting for it is over half the total number of deputies.

During 1953–54, the composition of the Assembly changed 42 times. So instead of one WVG Riker has in fact 43 successive WVGs, namely $\mathcal{G}^t$, with $t = 0, 1, \ldots, 42$ in chronological order. (Not all 42 changes were due to inter-party migration of deputies; for details see Section 3.)

[7] For an interpretation of the Bz power of a voter as the price the voter may exact from a vote-buying outsider, see [5, Comment 3.2.15].

[8] For further details on this, see [10, pp. 122–23].

In what follows, we assume a given ordering of the blocs at each time t. For the list of the $\mathcal{G}^t$, see Section 3.

According to Riker, the a priori voting power of the i-th bloc-voter B_i^t of $\mathcal{G}^t$ is $\phi_i[\mathcal{G}^t]$, the value assigned by the S-S index ϕ to the i-th voter in $\mathcal{G}^t$. He then assumes that this voting power is shared equally by the deputies belonging to B_i^t, so the power of each of them is

$$\frac{\phi_i[\mathcal{G}^t]}{|B_i^t|}. \tag{2.1}$$

We shall use the same WVGs $\mathcal{G}^t$ as Riker for measuring the voting power of party-blocs. However, since we conceptualize voting power as I-power, we shall use the Bz measure β' instead of the S-S index ϕ. So we take the voting power of the i-th party-bloc B_i^t in $\mathcal{G}^t$ to be ${\beta'}_i[\mathcal{G}^t]$.

Recall that ${\beta'}_a[\mathcal{G}]$, the Bz power of voter a in a simple voting game (SVG) $\mathcal{G}$, can be defined as the prior probability of the event that voter a is *critical*; that is, the event in which the voters other than a are so divided, that if a joins the 'yes' voters the bill in question will be passed, whereas if a joins the 'no' voters the bill will be blocked. Here the a priori assumption is that voters act independently of one another, each voting 'yes' or 'no' with equal probability, $\frac{1}{2}$.

Note that we are using the Bz *measure* β' (aka 'the *absolute* Bz index') rather than the [relative] Bz index β, which is obtained from β' by normalization, so that $\sum_x \beta_x[\mathcal{G}] = 1$, where x ranges over all the voters of $\mathcal{G}$.

The reason we use the Bz measure rather than the Bz index is that the hypothesis we wish to test is that deputies of the Assembly were seeking to maximize their absolute I-power; but the total amount of I-power is determined endogenously by the SVG, and is not the same for all SVGs.

In this respect I-power differs markedly from P-power. In the case of P-power, the total payoff to be distributed among the members of a winning coalition that manages to get a bill passed is a constant (which is determined exogenously); so by a suitable choice of units this total can always be taken to be 1. There is no loss of generality in this convention, because there is no reason to suppose that merely changing the decision rule — for example, going from $\mathcal{G}^t$ to $\mathcal{G}^{t+1}$ — makes any change to the total prize available.

Another difference between our model and Riker's is that we cannot use the analogue of (2.1) as the measure of the a priori power of an individual deputy belonging to bloc B_i^t. The obvious justification for (2.1) is that $\phi_i[\mathcal{G}^t]$ is interpreted as the expected amount of *transferable utility* that bloc B_i^t will obtain as payoff; and it is reasonable to assume that each member of B_i^t will get an equal share of this expected payoff. But — as we have explained in detail in [7, § 3] — I-power, as measured by β', is not a quantity of transferable utility. Therefore we cannot treat $\beta'_B[\mathcal{G}^t]$, the voting power of bloc B_i^t in our model, as a quantity that can be divided equally among the members of B_i^t to yield the individual power of each.

Instead, as explained in [7], in order to compute the a priori power of individual deputies, we must set up the *composite* SVG:

$$\mathcal{H}^t = \mathcal{G}^t[\mathcal{H}_1^t, \ldots, \mathcal{H}_m^t]. \tag{2.2}$$

Here $\mathcal{G}^t$ is, as explained above, the Assembly (at time t) modelled as a WVG, whose voters are the party-blocs rather than individual deputies.[9] $\mathcal{H}_i^t$ is the *internal* SVG of the i-th bloc (at time t): it models the decision rule within this bloc.

For a rigorous definition of composite SVGs, see [8, Def. XI.2.7] or [5, Def. 2.3.12]. Informally speaking, the composite SVG $\mathcal{H}^t$ works as follows. When a bill is proposed in the Assembly, it is decided in a two-stage process. First, each bloc decides, according to its own internal decision rule, how to vote on the bill. Then the bloc vote of each bloc is cast (as decided within the bloc) in the plenum of the Assembly, and the bill is finally decided there according to $\mathcal{G}^t$.

A deputy a belonging to the i-th bloc B_i^t has direct 'internal' voting power $\beta'_a[\mathcal{H}_i^t]$, wielded in determining the position of the bloc; as well as indirect voting power $\beta'_a[\mathcal{H}^t]$, wielded in determining the outcome of a division of the Assembly. The hypothesis we wish to test is that deputies seek to maximize their indirect I-power, the power to influence the decisions of the Assembly as a whole.

If, for every $i = 1, \ldots, m$, the SVG $\mathcal{H}_i^t$ is such that exactly half of its coalitions are winning, then for $a \in B_i^t$,

$$\beta'_a[\mathcal{H}^t] = \beta'_a[\mathcal{H}_i^t]\beta'_i[\mathcal{G}^t]. \tag{2.3}$$

(For a proof, see [8, p. 282] or [5, p. 67].)

We shall assume that internal decisions within each bloc are made by ordinary majority; and if a bloc has an even number of members and they are evenly split, the tie is broken at random, say by flipping a coin.[10] Under these conditions, (2.3) holds. Moreover,

$$\beta'_a[\mathcal{H}_i^t] = \frac{k}{h2^{h-1}}\binom{h}{k}, \tag{2.4}$$

[9] Strictly speaking, the actual number of blocs varies in time, so that instead of a constant m we should have m^t, depending on t. But this complication can be avoided by introducing empty dummy blocs.

[10] Because of this random step, if $|B_i^t|$ is even then $\mathcal{H}_i^t$ is not, strictly speaking, an SVG. But the probabilistic definition of $\beta'_a[\mathcal{H}_i^t]$ still makes sense.

where $h := |B_i^t|$ and $k :=$ the least integer greater than $h/2$. (For a proof, see [5, pp. 55, 58–59].)

3 Findings

First, here is the list of the $\mathcal{G}^t$; the blocs are ordered in decreasing size. For the names of the parties concerned, see [10, p. 127].

$\mathcal{G}^0 = [314; 105, 100, 88, 85, 75, 56, 46, 32, 23, 13, 1, 1, 1]$

$\mathcal{G}^1 = [314; 105, 100, 88, 85, 75, 55, 47, 32, 23, 13, 1, 1, 1]$

$\mathcal{G}^2 = [314; 105, 100, 88, 84, 75, 55, 47, 32, 23, 15; 1, 1]$

$\mathcal{G}^3 = [314; 105, 100, 88, 85, 75, 55, 47, 32, 23, 14, 1, 1]$

$\mathcal{G}^4 = [314; 105, 100, 88, 85, 75, 55, 47, 32, 23, 15, 1]$

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$\mathcal{G}^5 = [314; 105, 100, 89, 85, 75, 55, 47, 32, 23, 15, 1]$

$\mathcal{G}^6 = [314; 105, 100, 89, 85, 75, 55, 47, 32, 23, 14, 1, 1]$

$\mathcal{G}^7 = [314; 105, 100, 89, 83, 75, 55, 47, 32, 23, 14, 1, 1, 1, 1]$

$\mathcal{G}^8 = [314; 105, 100, 89, 81, 75, 55, 47, 34, 23, 14, 1, 1, 1, 1]$

$\mathcal{G}^9 = [314; 105, 100, 89, 81, 75, 55, 47, 34, 23, 15, 1, 1, 1]$

$\mathcal{G}^{10} = [314; 105, 100, 89, 81, 75, 55, 47, 34, 24, 15, 1, 1]$

$\mathcal{G}^{11} = [314; 105, 100, 89, 80, 75, 55, 47, 34, 25, 15, 1, 1]$

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$\mathcal{G}^{12} = [314; 105, 100, 89, 80, 75, 55, 46, 34, 25, 15, 1, 1]$

$\mathcal{G}^{13} = [314; 105, 100, 89, 79, 75, 55, 46, 34, 26, 15, 1, 1]$

$\mathcal{G}^{14} = [314; 105, 100, 88, 79, 75, 55, 46, 34, 26, 15, 1, 1, 1]$

$\mathcal{G}^{15} = [314; 105, 100, 87, 79, 75, 55, 46, 34, 26, 15, 1, 1, 1, 1]$

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$\mathcal{G}^{16} = [314; 105, 100, 87, 79, 75, 55, 47, 34, 26, 15, 1, 1, 1, 1]$

$\mathcal{G}^{17} = [314; 105, 100, 87, 78, 75, 55, 47, 34, 26, 15, 1, 1, 1, 1, 1]$

$\mathcal{G}^{18} = [314; 105, 100, 87, 78, 76, 55, 47, 34, 25, 15, 1, 1, 1, 1, 1]$

$\mathcal{G}^{19} = [314; 105, 100, 87, 78; 76, 55, 46, 34, 25, 15, 1, 1, 1, 1, 1, 1]$

$\mathcal{G}^{20} = [314; 105, 100, 87, 78, 76, 55, 34, 28, 25, 15, 15, 3, 1, 1, 1, 1, 1, 1]$

$\mathcal{G}^{21} = [314; 105, 100; 87, 78, 76, 53, 34, 28, 25, 15, 15, 5, 1, 1, 1, 1, 1, 1]$

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$\mathcal{G}^{22} = [314; 105, 100, 87, 77, 76, 53, 34, 28, 25, 15, 15, 5, 1, 1, 1, 1, 1, 1]$

$\mathcal{G}^{23} = [314; 105, 100, 87, 77, 76, 53, 34, 27, 25, 15, 15, 6, 1, 1, 1, 1, 1, 1]$
$\mathcal{G}^{24} = [314; 105, 100, 87, 77, 76, 53, 33, 27, 25, 16, 15, 6, 1, 1, 1, 1, 1, 1]$
$\mathcal{G}^{25} = [314; 105, 100, 87, 76, 76, 53, 33, 27, 25, 16, 15, 6, 1, 1, 1, 1, 1, 1, 1]$
$\mathcal{G}^{26} = [314; 105, 99, 88, 76, 76, 53, 33, 27, 25, 16, 15, 6, 1, 1, 1, 1, 1, 1, 1]$
$\mathcal{G}^{27} = [314; 105, 99, 88, 76, 75, 53, 33, 27, 25, 16, 15, 6, 1, 1, 1, 1, 1, 1, 1, 1]$
•
$\mathcal{G}^{28} = [313; 105, 99, 88, 76, 75, 53, 33, 27, 24, 16, 15, 6, 1, 1, 1, 1, 1, 1, 1]$
$\mathcal{G}^{29} = [313; 105, 99, 86, 76, 75, 53, 33, 27, 24, 16, 15, 6, 1, 1, 1, 1, 1, 1, 1, 1, 1]$
$\mathcal{G}^{30} = [313; 105, 99, 86, 76, 75, 53, 33, 27, 24, 22, 15, 1, 1, 1, 1, 1, 1, 1, 1, 1]$
$\mathcal{G}^{31} = [313; 105, 99, 86, 76, 74, 53, 33, 27, 24, 22, 15, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]$
$\mathcal{G}^{32} = [313; 105, 99, 86, 76, 73, 54, 33, 27, 24, 22, 15, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]$
$\mathcal{G}^{33} = [313; 105, 99, 86, 76, 72, 54, 34, 27, 24, 22, 15, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]$
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$\mathcal{G}^{34} = [313; 105, 99, 86, 76, 73, 54, 34, 27, 24, 22, 15, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]$
$\mathcal{G}^{35} = [313; 105, 99, 86, 76, 72, 54, 34, 27, 24, 22, 15, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]$
$\mathcal{G}^{36} = [313; 105, 99, 86, 76, 71, 54, 34, 27, 24, 22, 15, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]$
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$\mathcal{G}^{37} = [314; 105, 99, 86, 76, 71, 54, 34, 28, 24, 22, 15, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]$
$\mathcal{G}^{38} = [314; 105, 99, 85, 76, 71, 54, 34, 28, 24, 22, 15, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]$
$\mathcal{G}^{39} = [314; 105, 98, 85, 76, 71, 54, 34, 28, 24, 22, 15, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]$
$\mathcal{G}^{40} = [314; 105, 98, 85, 76, 72, 54, 34, 28, 24, 22, 15, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]$
$\mathcal{G}^{41} = [314; 105, 98, 85, 76, 72, 54, 34, 28, 24, 22, 16, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]$
$\mathcal{G}^{42} = [314; 105, 98, 85, 76, 72, 55, 34, 28, 24, 22, 16, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]$

Some of the changes were due to causes other than migration (such as death, resignation or by-election); these are marked with •. We are only concerned with the 35 cases in which the change was due to migration of one or more deputies, who switched from existing blocs to others or formed new blocs.[11] In most cases the new blocs were singletons; but in the change from $\mathcal{G}_{19}$ to $\mathcal{G}_{20}$ the Groupe du Centre républicain, originally consisting of 46 deputies, was deserted by 18 deputies: 15 of which formed the new Groupe Paysan and 3 who formed the new Groupe du Centre démocratique.

[11]Riker lists only 34 changes as due to migration: he omits the change from $\mathcal{G}^{25}$ to $\mathcal{G}^{26}$, which we have included.

Our main findings are shown in Table 1. The data shown in the nine columns of the table are as follows.

Column 1. The symbol ${}_i^t \to{}_j^{t+1}$ denotes migration from B_i^t, the i-th bloc in $\mathcal{G}^t$, to B_j^{t+1}, the j-th bloc in $\mathcal{G}^{t+1}$. Thus the first row of the table contains data about migration from the 6th bloc of $\mathcal{G}^0$ to [what became thereby] the 7th bloc of $\mathcal{G}^1$.

Column 2 shows the number of migrants. Thus the migration of the first row involved just one deputy. From the list of the $\mathcal{G}^t$ we can see that indeed the 6th bloc in $\mathcal{G}^0$ lost one of its original 56 members, while the 7th bloc of $\mathcal{G}^1$ has one new 47th member.

Column 3 shows the indirect Bz power $\beta'_a[\mathcal{H}^t]$, of any member a of B_i^t, the bloc of origin of the migration. The value of $\beta'_a[\mathcal{H}^t]$ is given in (2.3). To calculate it we needed first the values of $\beta'_a[\mathcal{H}_i^t]$ and $\beta'_i[\mathcal{G}^t]$. We calculated $\beta'_a[\mathcal{H}_i^t]$ by hand (using a pocket calculator) from (2.4). We obtained $\beta'_i[\mathcal{G}^t]$ using the IOP 1.0 computer program.[12]

The powers are given in units of 10^{-6}. Thus, looking at the first row we see that each of the 56 members of B_6^0 had Bz power 0.020732.

Column 4 shows the indirect Bz power of each of the remaining members of the bloc of origin, after the departure of the migrant(s). These are calculated and presented in the same way as in Column 3. Thus from the first row we see that after the migration of one member from B_6^0, each of the remaining 55 members of what now became B_6^1 had Bz power 0.020924.

In cases where the bloc of origin had no remaining members, this column shows a dash (—).

Column 5 is obtained by comparing the previous two columns. It gives the percentage gain (or loss, shown as negative gain) in Bz power of the remaining members (if any) of the bloc of origin. Thus the migration of the first row caused the Bz power of each of 55 remaining members of the bloc of origin to increase by 0.9%.

Column 6 shows the Bz power of each old member of the destination bloc. These are calculated and presented in the same way as in Columns 3 and 4. Thus from the first row we see that prior to the migration, each of the 46

[12]The Indices of Power (IOP) computer program was composed in 1996–97 by Thomas Braeuninger and Thomas Koenig of the Mannheim Center of European and Social Research, University of Mannheim. It can be downloaded from the internet at URL `http://www.mzes.uni-mannheim.de/mzes-engl/arb2/pow.html`

members of B_7^0 had Bz power 0.019482.

In cases where the destination bloc did not exist before the migration, this column shows a dash (—).

Column 7 shows the Bz power of each member of the destination bloc after the migration. Thus from the first row we see that each of the 47 members of B_7^1 had Bz power 0.019682.

Column 8 is obtained by comparing the previous two columns. It gives the percentage gain (or loss, shown as negative gain) in power of the old members (if any) of the destination bloc. Thus the migration of the first row caused the Bz power of each of the 46 old members of what now became B_7^1 to increase by 1.0%.

Column 9 is obtained by comparing Columns 3 and 7. It gives the percentage gain (or loss, shown as negative gain) in power of the migrants themselves. Thus the first row shows that in this case the (single) migrant lost 5.1% of his Bz power as a result of his migration.

4 Discussion

From the last column of Table 1 we can see at once that the hypothesis that deputies who switched parties were seeking to increase their a priori voting power is unsubstantiated or, at best, doubtful. In most cases, migrants lost rather than gained Bz power. Although the positive entries in this column tend to be larger in absolute value than the negative ones, this is a somewhat misleading artefact of the use of percentages: a quadrupling of voting power would appear as a gain of 300%, whereas the reverse change would appear as a loss of 75%.

Thus our conclusion is broadly similar to that reached by Riker.[13]

Our negative conclusion does not mean that the work reported in this paper was a waste of time. Negative findings are sometime no less interesting and important than positive ones. It remains to account for this particular negative result.

We must admit that, unlike Riker, we are not at all puzzled by our negative finding. To us, the hypothesis in question did not look at all likely

[13] We cannot perform the analogues of some of Riker's tests, which involve adding up the voting powers of several deputies (for example, of all migrants). Such sums may make sense for values of the S-S index, which are interpreted as quantities of transferable utility, but not for Bz powers, which cannot be treated in this way.

to start with. This is not only because calculation of a priori voting power according to the Bz measure (or, for that matter, by the S-S index) are too complicated and opaque to be done — even approximately — by mere intuition. Rather, it is mainly because if migrating deputies were at all motivated by voting-power considerations, it is *de facto* (that is, a posteriori) voting power that concerned them. Riker [10, p. 129] claims that, in the specific conditions of the National Assembly of 1953–54, 'an empirical power index would not differ notably from the a priori index.' But we find his arguments for this claim unconvincing.

Be that as it may, there are surely various other likely motives that may drive a deputy to switch parties. Some of these motives may be ideological or even idealistic; but others may have to do with considerations of *Realpolitik* or even mercenary gain, without being directly related to a priori voting power.

The following further observations can be made from Table 1. First, in all migrations from a smaller to a larger bloc the migrant gained Bz power; and, conversely, a migrant from a larger to a smaller bloc lost Bz power.[14] Second, in most cases a migration resulted in a decrease of the Bz power of the members remaining in the bloc of origin, and an increase of the Bz power of the original members of the bloc of destination.

These facts are not difficult to account for. In a WVG with many voters, the Bz powers of the voters tend to be roughly proportional to their weights.[15] This applies, in particular, to the factor $\beta'_i[\mathcal{G}^t]$, the Bz power of the i-th bloc B^t_i, in (2.3). On the other hand, $\beta'_a[\mathcal{H}^t_i]$, the direct 'internal' Bz power of a deputy within a bloc (under majority rule) is of the same order as $\sqrt{1/|B^t_i|}$.[16] Thus, in a legislature with many blocs (such as the Assembly) the indirect Bz power of deputies belonging to larger blocs is normally greater than that of members of smaller blocs.[17]

If inter-party migrations were mainly motivated by the migrants' wish to increase their a priori voting power, then we would expect the Assembly — as well as all other legislatures that operate according to similar constitutional

[14]This finding is similar to that of Riker in general, but not in detail: there are migrations as a result of which the migrant lost S-S power but gained Bz power, and vice versa.

[15]See, for example, [5, Table 5.3.7].

[16]See [5, p. 56].

[17]Of course, it is also possible to gain Bz power by migrating from a larger to a smaller bloc. This tends to happen when there are few blocs. Thus, for example, if the internal decision rule of all blocs is that of simple majority, a move from the WVG $[51; 50, 49, 1]$ to the WVG $[51; 49, 49, 2]$ increases the indirect Bz power of the migrant from 0.08420625 to 0.25.

rules — to gravitate to an equilibrium: perhaps one in which there are very few large blocs, and no small ones. That this is by no means the case in reality can be counted as an additional piece of evidence against the hypothesis examined in the present paper.

Table 1: The Migrations

Migration	#	Power in Bloc of Origin Before ×10^{-6}	After	Gain %	Power in Destination Bloc Before ×10^{-6}	After	Gain %	Migrants' Gain %
${}^{0}_{6} \to {}^{1}_{7}$	1	20732	20924	0.9	19482	19682	1.0	−5.1
${}^{1}_{4} \to {}^{2}_{10}$	1	26806	26531	−1.0	8427	9001	6.8	−66.4
${}^{1}_{11} \to {}^{2}_{10}$	1	2686	—	—	8427	9001	6.8	235.1
${}^{2}_{10} \to {}^{3}_{4}$	1	9001	8387	−6.8	26531	26785	1.0	197.6
${}^{3}_{12} \to {}^{4}_{10}$	1	2930	—	—	8387	9001	7.3	207.2
${}^{5}_{10} \to {}^{6}_{12}$	1	9410	8796	−6.5	—	2930	—	−68.9
${}^{6}_{4} \to {}^{7}_{13}$	1	26700	26486	−0.8	—	3174	—	−88.1
${}^{6}_{4} \to {}^{7}_{14}$	1	26700	26486	−0.8	—	3174	—	−88.1
${}^{7}_{4} \to {}^{8}_{8}$	2	26486	26031	−1.7	15341	16084	4.8	−39.3
${}^{8}_{14} \to {}^{9}_{10}$	1	3147	—	—	9666	10330	6.9	225.5
${}^{9}_{13} \to {}^{10}_{9}$	1	2441	—	—	13550	13379	−1.3	448.1
${}^{10}_{4} \to {}^{11}_{9}$	1	26140	26140	0.0	13380	13380	0.0	−48.8
${}^{12}_{4} \to {}^{13}_{9}$	1	26097	26383	1.1	13301	12865	−3.3	−50.7
${}^{13}_{3} \to {}^{14}_{13}$	1	28078	27289	−0.7	—	2197	—	−92.2
${}^{14}_{3} \to {}^{15}_{14}$	1	27892	27929	0.1	—	3296	—	−88.2
${}^{16}_{4} \to {}^{17}_{15}$	1	26273	25998	−1.0	—	3052	—	−88.4
${}^{17}_{9} \to {}^{18}_{5}$	1	13300	13084	−1.6	25378	25467	0.4	91.5
${}^{18}_{7} \to {}^{19}_{16}$	1	20324	19939	−1.9	—	3296	—	−83.8
${}^{19}_{7} \to {}^{20}_{10}$	15	19939	19417	−2.6	—	11433	—	−42.7
${}^{19}_{7} \to {}^{20}_{12}$	3	19939	19417	−2.6	—	6661	—	−66.8
${}^{20}_{6} \to {}^{21}_{12}$	2	19781	19237	−2.8	6661	8120	21.9	−59.0
${}^{22}_{8} \to {}^{23}_{12}$	1	15449	15401	−0.3	8188	8075	−1.4	−47.7
${}^{23}_{7} \to {}^{24}_{10}$	1	17393	17462	0.4	11317	11252	−0.6	−35.3
${}^{24}_{4} \to {}^{25}_{19}$	1	26080	25751	−1.3	—	3601	—	−86.2
${}^{25}_{2} \to {}^{26}_{3}$	1	30789	30734	−0.2	28245	28310	0.2	−8.1
${}^{26}_{5} \to {}^{27}_{20}$	1	25709	25696	−0.1	—	3845	—	−85.0
${}^{28}_{3} \to {}^{29}_{20}$	1	28359	28068	−1.0	—	3605	—	−87.3

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Table 1 *continued from previous page*

		Power in Bloc of Origin			*Power in Destination Bloc*			*Migrants'*
		Before	*After*	*Gain*	*Before*	*After*	*Gain*	*Gain*
Migration	#	$\times 10^{-6}$		%	$\times 10^{-6}$		%	%
${}^{28}_{3} \rightarrow {}^{29}_{21}$	1	28359	28068	−1.0	—	3605	—	−87.3
${}^{29}_{12} \rightarrow {}^{30}_{10}$	6	7376	—	—	11316	13661	20.7	85.2
${}^{30}_{5} \rightarrow {}^{31}_{21}$	1	25762	25424	−1.3	—	3569	—	−86.1
${}^{31}_{5} \rightarrow {}^{32}_{6}$	1	25424	25354	−0.3	19849	19964	0.6	−21.5
${}^{32}_{5} \rightarrow {}^{33}_{7}$	1	25354	25073	−1.1	18024	17900	−0.7	−29.4
${}^{34}_{5} \rightarrow {}^{35}_{22}$	1	25414	25073	−1.3	—	3633	—	−85.7
${}^{35}_{5} \rightarrow {}^{36}_{23}$	1	25073	25074	0.0	—	3707	—	−85.2
${}^{37}_{3} \rightarrow {}^{38}_{24}$	1	28249	28270	0.1	—	3589	—	−87.3
${}^{38}_{2} \rightarrow {}^{39}_{25}$	1	30954	30663	−0.9	—	3624	—	−88.3
${}^{39}_{25} \rightarrow {}^{40}_{5}$	1	3624	—	—	25130	25121	0.0	593.1
${}^{40}_{24} \rightarrow {}^{41}_{11}$	1	3565	—	—	9831	9916	0.9	178.2
${}^{41}_{23} \rightarrow {}^{42}_{6}$	1	3801	—	—	20336	20747	2.0	445.8

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