The Treaty of Nice and Qualified Majority Voting

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ABSTRACT

We analyse and evaluate three decision rules for the Council of Ministers of the EU, which are prescribed by the 'definitive form' of the Treaty of Nice. The first will apply from 2005 to the present 15-member EU, if it will not have been enlarged by then. The second or third will apply to an enlarged 27-member EU. We conclude that the first of these is an improvement on the current decision rule; but the other two have extremely undesirable features.

The Treaty of Nice and Qualified Majority Voting

1 Introduction

The most important decision-making body of the European Union (EU) is its Council of Ministers (CM). The CM has several different rules for adopting acts, depending on the kind of issue involved. The greatest number of issues (except those concerned with the constitution of the EU itself) are decided by a rule known in EU parlance as *qualified majority voting* (QMV).

The Intergovernmental Conference on institutional reform of the EU, held at Nice in early December 2000, ended in the small hours of Monday, 11 December, after adopting the provisional text of the Treaty of Nice [5], published on the following day. The 'definitive' text of the treaty, signed on 26 February 2001 [6], was published on 28 February 2001. The treaty contains important provisions modifying QMV. Our aim in the present paper is to analyse and evaluate these modifications.

Until now, QMV has always been a pure weighted decision rule: each member state is assigned a number of bloc-votes, or *weight*, and a proposed act is adopted if the total weight of those voting for it equals or exceeds a certain *quota*.[1] The weights and quota were fixed afresh each time the EU was enlarged.[2] (See Table 1.)

The Treaty of Nice contains three new versions of QMV, which apply to two scenarios. First, Article 3 of the *Protocol on the Enlargement of the European Union* prescribes amendments to the current QMV that will take effect on 1 January 2005 in the present 15-member CM, assuming that the EU will not be enlarged by then.[3] We shall denote this amended rule by $\mathcal{N}_{15}$.

Second, Section 2 of the *Declaration on the Enlargement of the European*

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[1]This applies to the usual procedure, whereby acts are proposed by the EU's highest executive body, the Commission. In the present paper we confine our attention to voting power under this usual procedure. (For an act not proposed by the Commission to be adopted, it must not only attain the weight quota but also be supported by at least two-thirds of the member-states.)

[2]Strictly speaking, the name 'European Union' applies only following the Maastricht Treaty, which took effect in November 1993; but we shall use this name also for the earlier stages.

[3]See [6, p. 97–98].

Union lays down a version of QMV that will apply in the CM of a prospective enlarged 27-member EU.[4] We shall denote this rule by $\mathcal{N}_{27}$.

Third, a *Declaration on the qualified majority threshold and the number of votes for a blocking minority in an enlarged Union* contains a provision that contradicts the specification of $\mathcal{N}_{27}$ stated three pages earlier, and gives rise to a modified version of QMV for an enlarged 27-member EU.[5] We shall denote this modified rule by $\mathcal{N}'_{27}$.

Thus the treaty is ambiguous—perhaps deliberately so—as to which rule will apply in an enlarged 27-member CM. In this paper we shall therefore analyse and assess both variants, $\mathcal{N}_{27}$ and $\mathcal{N}'_{27}$.

Each of the three rules is presented not as a pure weighted rule, but as the conjunction or meet of three such rules.[6] In order for an act to be adopted by the CM, the member states supporting it must satisfy not only the bloc-vote quota but also a plain numerical quota and a population quota: they must constitute an ordinary majority of all member states, and represent at least 62% of the total population of the EU.

In Section 2 we analyse the structure of $\mathcal{N}_{15}$ and point out that it is in fact the meet of two (rather than three) weighted rules, because the effect of the plain numerical quota is vacuous; but we show that it cannot be recast as a single pure weighted rule. We also isolate the effect of the population quota and show that it is quite small but not negligible.

In Section 3 we analyse the structure of $\mathcal{N}_{27}$ and point out that here again the effect of the plain numerical quota is vacuous, and that of the population quota is nugatory. We show that $\mathcal{N}_{27}$ can be recast as a pure weighted rule.

In Section 4 we analyse the structure of $\mathcal{N}'_{27}$ and show that the effects of both the numerical quota and the population quota are nugatory albeit not vacuous. We show that $\mathcal{N}'_{27}$ cannot be recast as a pure weighted rule.

In Section 5 we evaluate $\mathcal{N}_{15}$ and compare it with the current QMV. We then evaluate $\mathcal{N}_{27}$ and $\mathcal{N}'_{27}$; we compare them with the current QMV as well as with an alternative 27-member QMV rule (Rule B) which we regard as a desirable benchmark. Our conclusions are summarized in Section 6.

In our calculations of voting powers we have used the Braeuninger–Koenig IOP program [2] and a program written by Dennis Leech [15]. We are grateful to Thomas Braeuninger and Dennis Leech for their invaluable help in using these programs.

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[4] See [6, p. 164].

[5] See [6, p. 167].

[6] For a general definition of the meet of several decision rules, see [10, pp. 27–29].

The basic concepts and results from the theory of voting power, as well as the criteria for evaluating a decision rule of a body such as the CM, which we shall use here are outlined and explained in a fairly non-technical style in [11]. For a more detailed rigorous exposition, see [10]. In the present paper we keep explanations of those concepts, results and criteria to a bare minimum, in order to avoid repetitiousness.

We shall in general stick here to the terminology and notation used in [11]. However, whereas in that booklet—which was addressed to a broad readership—game-theoretic terminology was strictly avoided for didactic reasons, here we shall allow ourselves to employ some terms borrowed from game theory. Thus we shall use the term *coalition* for any set of voters—that is, representatives of member states of the EU—in the CM.[7] In particular, a coalition S is said to be a *winning* coalition (under a given decision rule) if a division in which all members of S vote 'yes' has positive outcome, so that the act in question is adopted. Any other coalition—that is, one whose 'yes' votes are insufficient for an act to be adopted—is said to be a *losing* coalition.[8]

2 QMV in a non-enlarged CM

In this section we analyse a decision rule we denote by $\mathcal{N}_{15}$, which is the version of QMV that the Treaty of Nice prescribes for acts proposed by the Commission to the CM as of 1 January 2005, assuming that the EU will not have been enlarged by that date.

2.1 The official specification of $\mathcal{N}_{15}$ We quote from the 'definitive' text of the treaty [6, p. 97–98]:

> ... Where the Council is required to act by a qualified majority, the votes of its members shall be weighted as follows:

Belgium	12
Denmark	7
Germany	29
Greece	12

[7]Note that this term does not carry here the connotation it has in normal political parlance: we do not assume that a coalition is formed by prior agreement or that its members act in a co-ordinated way. Cf. [10, Com. 2.2.1].

[8]However, a 'losing' coalition may still be *successful* in a division in which all its members vote 'no', if these 'no' votes are sufficient to ensure a negative outcome, so that the act in question is blocked.

Spain	27
France	29
Ireland	7
Italy	29
Luxembourg	4
Netherlands	13
Austria	10
Portugal	12
Finland	7
Sweden	10
United Kingdom	29

> Acts of the Council shall require for their adoption at least 169 votes in favour cast by a majority of the members where this Treaty requires them to be adopted on a proposal from the Commission.[9]
>
> . . . When a decision is to be adopted by the Council by a qualified majority, a member of the Council may request verification that the Member States constituting the qualified majority represent at least 62% of the total population of the Union. If that condition is shown not to have been met, the decision in question shall not be adopted.

This text has a couple of rather strange features. First, the phrase 'cast by at least a majority of the members', which appears to impose an additional plain numerical quota (of eight members out of 15), is in fact otiose, because—as can easily be verified—it takes at least eight members to attain the vote quota of 169.[10]

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[9] In the provisional text of the treaty [5, p. 74], the vote quota was given as 170. This was changed to 169 in the 'definitive' text. No explanation is offered for this change.

[10] As we shall see in Sections 3 and 4, a similar thing applies to the identical phrase that occurs in the text that prescribes $\mathcal{N}_{27}$; but in the case of $\mathcal{N}'_{27}$ this clause is not vacuous. The only point of including this phrase in the case of $\mathcal{N}_{15}$ and $\mathcal{N}_{27}$ seems to be that it may become operative at some intermediate stage between the present 15-member EU and its enlargement to 27 members, or at some later stage of further enlargement—in which case it will not need to be inserted by a further amendment of the EU treaty. Note that such an amendment requires unanimity, which will grow exponentially harder to achieve as the number of members increases. So, while the EU membership is still relatively small, it is sound strategy to plant in the treaty desirable provisions that may only be needed at a later stage.

Second, the final paragraph, concerning the 62% population quota, is decidedly odd. The population figures of EU members, or fairly accurate estimates, are common knowledge. So the 'verification' that a member 'may request' according to that paragraph is a simple arithmetical computation that takes only a few seconds with a pocket calculator.[11]

There are two ways in which the provision of that paragraph may be interpreted and operated.

The literal interpretation is that in some cases when the coalition supporting an act meets the 169 vote quota but not the 62% population quota, the latter fact—although plain for all to see—will not be 'verified' because all the members who do not support the act will nevertheless fail formally to request verification, thus allowing the act to be adopted by default.

A simpler interpretation is that the verification will always be performed, either automatically or because in practice a member who does not support the act will always request it.

At present it is impossible to tell which interpretation will in fact apply.[12] In what follows, we shall assume the latter interpretation, for the sake of simplicity.[13]

According to our interpretation, $\mathcal{N}_{15}$ can be represented formally as the meet of two weighted rules:

$$\mathcal{N}_{15} = \mathcal{W}_{15} \wedge \mathcal{P}_{15}, \tag{1}$$

where $\mathcal{W}_{15}$ is the weighted rule with weighted votes as detailed above (p. 3f) and quota 169, and $\mathcal{P}_{15}$ is the weighted rule whose weights are the population

[11]See however footnote 34 in Subsection 5.4. By the way, it is not clear to us why the authors of the treaty chose the figure of 62% for the population quota.

[12]Experience with international treaties suggests that the literal interpretation of a provision is not always the one that operates in practice. A notorious case is the decision rule of the UN Security Council. The UN Charter says that decisions on substantive (non-procedural) matters require 'the concurring votes of all permanent members'. But in practice abstention by a permanent member is not taken as failure to 'concur'—although it plainly is that—and so does not constitute a veto. For a discussion of this in relation to voting power see [8, 12].

[13]The literal interpretation may arguably be construed as introducing—for the first time in the history of QMV in the CM—abstention as a *tertium quid*, distinct from both a 'yes' and a 'no' vote, at least for some members. The calculation of voting power in such cases is considerably more complex, and the theory underlying it is not sufficiently established. See [8] and [10, Chapter 8]. But in any case, under either interpretation the provision in question cannot make a great deal of difference to the distribution of voting power because, as we shall see in Subsection 2.3, it affects a very small number of possible divisions of the CM.

sizes of the 15 members and whose quota is equal to 62% of their total population.

This means that a division of the CM will have positive outcome under $\mathcal{N}_{15}$ just in case it would have positive outcomes under both $\mathcal{W}_{15}$ and $\mathcal{P}_{15}$.

In what follows we shall take the population sizes to be as shown in Table 2, which are the most up-to-date figures available to us. This makes the population quota equal to $232,701,500$. Naturally, all our calculations and conclusions that depend on these figures may need to be modified if the population data change while the data of $\mathcal{W}_{15}$ are still in force. But it would take relatively large disparities in the rates of population change across 15 countries to make a significant difference.

The following two subsections are concerned with methodological technicalities and may be skipped by a reader who is not interested in such matters.

2.2 Dimension of $\mathcal{N}_{15}$ Taylor and Zwicker [20, p. 35] define the *dimension* of a decision rule $\mathcal{G}$ to be the least number k such that $\mathcal{G}$ can be represented as the meet of k weighted rules. From equation (1) it follows that the dimension of $\mathcal{N}_{15}$ is at most 2. We shall now show that it is *exactly* 2; in other words, it is impossible to represent $\mathcal{N}_{15}$ by means of a *single* system of weights and quota.

We shall prove this claim using the method of [20, § 2.4]. Consider the following two coalitions:

A := {Germany, France, Italy, Spain, Netherlands, Greece, Belgium, Portugal, Ireland},

B := {UK, France, Italy, Spain, Netherlands, Portugal, Sweden, Austria, Denmark, Luxembourg}.

It is straightforward to verify that both A and B are winning coalitions under $\mathcal{N}_{15}$: in the weighting of $\mathcal{W}_{15}$ (see p. 3f), both A and B have total weight 170, which exceeds the vote quota of $\mathcal{W}_{15}$; and the total populations of A and B (see Table 2) are $288,240,000$ and $263,637,000$ respectively, both of which exceed the $232,701,500$ population quota of $\mathcal{P}_{15}$.

Now let Germany move from A to B, while Sweden, Austria, Denmark and Luxembourg move in the opposite direction. We get two new coalitions:

A' := {France, Italy, Spain, Netherlands, Greece, Belgium, Portugal, Sweden, Austria, Denmark, Ireland, Luxembourg},

B' := {Germany, UK, France, Italy, Spain, Netherlands, Portugal}.

Both of these are losing coalitions under $\mathcal{N}_{15}$, because the total population of A' is only $228,880,000$, and the total weight of B' is only 168.

But suppose it were possible to modify the weights and quota of $\mathcal{W}_{15}$, so that for the resulting modified weighted rule $\mathcal{W}^*$ we would have $\mathcal{N}_{15} = \mathcal{W}^*$. Let w^* be the weighting of $\mathcal{W}^*$ and q^* its quota.

Then it would follow that $w^*(A') + w^*(B') = w^*(A) + w^*(B)$; but also

$$\begin{array}{llll} w^*(A) \geq q^*, & w^*(B) \geq q^*, & \text{hence} & w^*(A) + w^*(B) \geq 2q^*; \\ w^*(A') < q^*, & w^*(B') < q^*, & \text{hence} & w^*(A') + w^*(B') < 2q^*; \end{array}$$

so $w^*(A') + w^*(B') < w^*(A) + w^*(B)$—a contradiction.

2.3 The effect of $\mathcal{P}_{15}$ The only difference between $\mathcal{N}_{15}$ and its first component, $\mathcal{W}_{15}$, is that the latter has 15 winning coalitions that are losing coalitions under $\mathcal{N}_{15}$ because they do not attain the population quota of $\mathcal{P}_{15}$.[14] Here is a list of these exceptional coalitions:

1. All members except Germany, UK and Denmark;
2. All members except Germany, UK and Finland;
3. All members except Germany, UK and Ireland;
4. All members except Germany, France and Denmark;
5. All members except Germany, France and Finland;
6. All members except Germany, France and Ireland;
7. All members except Germany, Italy and Denmark;
8. All members except Germany, Italy and Finland;
9. All members except Germany, Italy and Ireland;
10. All members except Germany, UK and Sweden;
11. All members except Germany, UK and Austria;

[14]Thus the population clause affects only 15 possible divisions of the CM out of a total of $2^{15} = 32,768$. Of this total, $2,707$ would have positive outcome under $\mathcal{W}_{15}$ but only $2,692$ do so under $\mathcal{N}_{15}$.

12. All members except Germany, France and Sweden;
13. All members except Germany, France and Austria;
14. All members except Germany, Italy and Sweden;
15. All members except Germany, Italy and Austria.

The first nine of these exceptional coalitions have weight 172 and the remaining six have weight 169 under $\mathcal{W}_{15}$—which makes all 15 of them *minimal* winning coalitions under this rule.[15]

This fact makes it easy to obtain the values of ψ, the Penrose measure of voting power,[16] under $\mathcal{N}_{15}$ from the values of ψ under $\mathcal{W}_{15}$.

The means for doing so is provided by Lemma 3.3.12 in [10].[17] According to this lemma, if $\mathcal{G}$ and $\mathcal{H}$ are decision rules that differ solely in that one minimal winning coalition under $\mathcal{G}$, say T, is a losing coalition under $\mathcal{H}$, then for any voter a,

$$\psi_a[\mathcal{H}] = \begin{cases} \psi_a[\mathcal{G}] - \frac{1}{2^{m-1}} & \text{if } a \in T, \\ \psi_a[\mathcal{G}] + \frac{1}{2^{m-1}} & \text{if } a \notin T, \end{cases}$$

where m is the number of voters.

Applying this to the present case we see that if a is any one of the 15 voters, then $\psi_a[\mathcal{N}_{15}]$ (a's voting power under $\mathcal{N}_{15}$) can be obtained from $\psi_a[\mathcal{W}_{15}]$ (a's voting power under $\mathcal{W}_{15}$) by subtracting from the latter $1/2^{14}$ for each of the 15 exceptional coalitions in which a is included, and adding $1/2^{14}$ for each of the 15 exceptional coalitions from which a is excluded. Thus we have

$$\psi_{\text{Germany}}[\mathcal{N}_{15}] = \psi_{\text{Germany}}[\mathcal{W}_{15}] + \frac{15}{2^{14}},$$

because Germany is excluded from all 15 exceptional coalitions;

$$\psi_a[\mathcal{N}_{15}] = \psi_a[\mathcal{W}_{15}] - \frac{5}{2^{14}},$$

where $a =$ UK, France or Italy, because each of these members is excluded from 5 exceptional coalitions and included in 10;

$$\psi_a[\mathcal{N}_{15}] = \psi_a[\mathcal{W}_{15}] - \frac{9}{2^{14}},$$

[15] A winning coalition is *minimal* if it does not include any other winning coalition; in other words, the omission of any of its members would turn it into a losing coalition.

[16] Often called the 'absolute Banzhaf measure' or 'absolute Banzhaf index' and denoted by β'.

[17] This simple result is implicit in Dubey and Shapley [7, p. 107].

where a = Sweden, Austria, Denmark, Finland or Ireland, because each of these members is excluded from 3 exceptional coalitions and included in 12; and

$$\psi_a[\mathcal{N}_{15}] = \psi_a[\mathcal{W}_{15}] - \frac{15}{2^{14}},$$

where a is any one of the remaining members (Spain, Netherlands, Greece, Belgium, Portugal, or Luxembourg), because these members belong to all 15 exceptional coalitions.

Thus the effect of the population component $\mathcal{P}_{15}$ of $\mathcal{N}_{15}$ is to boost the voting power of Germany and reduce the voting powers of all other members to varying extents: the greatest losers are Spain, Netherlands, Greece, Belgium, Portugal and Luxembourg; followed by Sweden, Austria, Denmark, Finland and Ireland; with UK, France and Italy losing least of all. But in all cases—including those of Germany and the six greatest losers—the effect is rather small.

The values of ψ under $\mathcal{W}_{15}$ and $\mathcal{N}_{15}$ are shown in Tables 5 and 6, respectively. The differences between the ψ values in the two tables are indeed quite small.

The meaning of the other data shown in Tables 5 and 6 will be discussed in Subsection 5.1.

3 QMV in a 27-member CM, first variant

We now turn to the decision rule we denote by $\mathcal{N}_{27}$, which is the first variant of QMV for the CM of the prospective enlarged 27-member EU prescribed by the Treaty of Nice.[18]

3.1 The official specification of $\mathcal{N}_{27}$ We quote the relevant text from [6, p. 164].

THE WEIGHTING OF VOTES IN THE COUNCIL

MEMBERS OF THE COUNCIL	WEIGHTED VOTES
Germany	29
United Kingdom	29
France	29
Italy	29

[18] In view of the contradictory text quoted below (p. 13), it is perhaps better to say that the treaty *appears* to prescribe $\mathcal{N}_{27}$.

Spain	27
Poland	27
Romania	14
Netherlands	13
Greece	12
Czech Republic	12
Belgium	12
Hungary	12
Portugal	12
Sweden	10
Bulgaria	10
Austria	10
Slovakia	7
Denmark	7
Finland	7
Ireland	7
Lithuania	7
Latvia	4
Slovenia	4
Estonia	4
Cyprus	4
Luxembourg	4
Malta	3
TOTAL	345

> Acts of the Council shall require for their adoption at least 258 votes in favour, cast by a majority of members, where this Treaty requires them to be adopted on a proposal from the Commission.
>
> ... When a decision is to be adopted by the Council by a qualified majority, a member of the Council may request verification that the Member States constituting the qualified majority represent at least 62% of the total population of the Union. If that condition is shown not to have been met, the decision in question shall not be adopted.

This text again has the same strange features as that specifying $\mathcal{N}_{15}$, discussed in Subsection 2.1. The phrase 'cast by a majority of the members' is again vacuous, because—as can easily be verified—it takes at least 14 members to attain the weight quota of 258. Also the same remarks we made in Subsection 2.1 regarding the last quoted paragraph, concerning the 62%

population quota, apply here as well. Moreover, as we shall see in a moment, the effect of this population quota is in the present case not just small, but quite insignificant.

As in the case of $\mathcal{N}_{15}$, we model $\mathcal{N}_{27}$ formally as the meet of two weighted rules:

$$\mathcal{N}_{27} = \mathcal{W}_{27} \wedge \mathcal{P}_{27}, \tag{2}$$

where $\mathcal{W}_{27}$ is the weighted rule with weighted votes as detailed above (p. 9f) and quota 258, and $\mathcal{P}_{27}$ is the weighted rule whose weights are the population sizes of the 27 members and prospective members and whose quota is equal to 62% of their total population.

This means that a division of the CM will have positive outcome under $\mathcal{N}_{27}$ just in case it would have positive outcomes under both $\mathcal{W}_{27}$ and $\mathcal{P}_{27}$.

In what follows we shall take the population sizes to be as shown in Table 3, which are the most up-to-date figures available to us. This makes the population quota equal to $298,332,220$. Again, all our calculations and conclusions that depend on these figures may need to be modified if the population data change while the data of $\mathcal{W}_{27}$ are still in force. But it would take relatively large disparities in the rates of population change across 27 countries to make a significant difference.

3.2 $\mathcal{N}_{27}$ as a weighted rule The only difference between $\mathcal{N}_{27}$ and its first component, $\mathcal{W}_{27}$, is that the latter has three winning coalitions that are losing coalitions under $\mathcal{N}_{27}$ because they do not attain the population quota of $\mathcal{P}_{27}$.[19] These exceptional coalitions are:

1. All members except Germany, UK and France;
2. All members except Germany, UK and Italy;
3. All members except Germany, France and Italy.

Each of these coalitions has weight 258 in the weighting of $\mathcal{W}_{27}$ (p. 9f), which is exactly the quota of that weighted rule; but their populations fall short of $298,332,220,$ the population quota of $\mathcal{P}_{27}$.

Using the method explained in Subsection 2.3, we therefore have

$$\psi_{\text{Germany}}[\mathcal{N}_{27}] = \psi_{\text{Germany}}[\mathcal{W}_{27}] + \frac{3}{2^{26}},$$

[19]Thus the population clause affects only 3 possible divisions of the CM out of a total of $2^{27} = 134,217,728$. Of this total, $2,226,794$ would have positive outcome under $\mathcal{W}_{27}$ but only $2,226,791$ do so under $\mathcal{N}_{27}$.

because Germany is excluded from all three exceptional coalitions;

$$\psi_a[\mathcal{N}_{27}] = \psi_a[\mathcal{W}_{27}] + \frac{1}{2^{26}},$$

where a = UK, France or Italy, because each of these members is excluded from two exceptional coalitions and included in one; and

$$\psi_a[\mathcal{N}_{27}] = \psi_a[\mathcal{W}_{27}] - \frac{3}{2^{26}},$$

where a is any one of the remaining members. The differences between the values of ψ under $\mathcal{N}_{27}$ and $\mathcal{W}_{27}$ are of the order of 10^{-8} (one-hundred-millionth); they are so minute as to be negligible. So, as far as considerations of voting power are concerned, we could use $\mathcal{W}_{27}$ as an excellent approximation to $\mathcal{N}_{27}$.

However, we can do better than that: $\mathcal{N}_{27}$ *itself* can be presented with absolute precision as a weighted rule.[20]

To this end, let us modify $\mathcal{W}_{27}$ as follows.

- Add 0.25 to the weights of the UK, France and Italy, making their weights 29.25 each.
- Add 0.50 to the weight of Germany, making it 29.50.
- Add 0.50 to the quota, making it 258.50.

All other weights are left unchanged. Let us denote the resulting weighted rule by $\widetilde{\mathcal{W}}$. We shall now show that the winning coalitions of $\widetilde{\mathcal{W}}$ are precisely those of $\mathcal{N}_{27}$.

For any coalition S, we denote by $w(S)$ and $\widetilde{w}(S)$ its weights under $\mathcal{W}_{27}$ and $\widetilde{\mathcal{W}}$, respectively.

Suppose first that S is a winning coalition under $\mathcal{N}_{27}$. There are two possible cases to consider.

Case 1: $w(S) \geq 259$. In this case certainly $\widetilde{w}(S) \geq 259 > 258.50$, so S wins under $\widetilde{\mathcal{W}}$ as well.

Case 2: $w(S) = 258$ exactly. If S contains Germany, or at least two of the remaining three big members—the UK, France, Italy—then $\widetilde{w}(S) \geq 258.50$, so S wins under $\widetilde{\mathcal{W}}$ as well.

[20] We are extremely grateful to William Zwicker for helping us reach this conclusion.

The only other way in which $w(S)$ can reach 258—without Germany and without two of the remaining three big members—is by consisting of exactly one of these big three and all the remaining 23 members, from Spain down. But this would mean that S is one of the three exceptional coalitions, which are losing coalitions under $\mathcal{N}_{27}$, contrary to our supposition that S wins under $\mathcal{N}_{27}$.

Conversely, suppose now that S is a losing coalition under $\mathcal{N}_{27}$. Again, there are two cases to consider.

Case 3: S is one of the three exceptional coalitions. Then $\widetilde{w}(S) = 258.25 < 258.50$, so S loses under $\widetilde{\mathcal{W}}$ as well.

Case 4: S is not one of the three exceptional coalitions. Since S loses under $\mathcal{N}_{27}$, it must lose also under $\mathcal{W}_{27}$, because the only coalitions that lose under $\mathcal{N}_{27}$ but win under $\mathcal{W}_{27}$ are the three exceptional ones. So we must have $w(S) \leq 257$. But then clearly $\widetilde{w}(S) \leq 257+0.50+3\times0.25 = 258.25 < 258.50$, so S loses under $\widetilde{\mathcal{W}}$ as well.

We have therefore established that $\mathcal{N}_{27} = \widetilde{\mathcal{W}}$, which means that $\mathcal{N}_{27}$ is indeed a weighted rule. For convenience, in order to avoid fractional weights and quota, we can now multiply all the weights and the quota of $\widetilde{\mathcal{W}}$ by 4. The values of ψ for this rule are shown in Table 8.

The meaning of the other data shown in Table 8 will be discussed in Subsection 5.1.

4 QMV in a 27-member CM, second variant

In this section we analyse the decision rule we denote by $\mathcal{N}'_{27}$, which is the second variant of QMV for the CM of the prospective enlarged 27-member EU prescribed by the Treaty of Nice.

4.1 The official specification of $\mathcal{N}'_{27}$ This rule is not stated explicitly, as were $\mathcal{N}_{15}$ and $\mathcal{N}_{27}$. Instead, the *Declaration on the qualified majority threshold and the number of votes for a blocking minority in an enlarged Union* [6, p. 167] states:

> Insofar as all the candidate countries listed in the Declaration on the enlargement of the European Union have not yet acceded to the Union when the new vote weightings take effect (1 January 2005), the threshold for a qualified majority will move, according to the pace of accessions, from a percentage below the current

> one to a maximum of $73,4\%$.[21] When all the candidate countries mentioned above have acceded, the blocking minority, in a Union of 27, will be raised to 91 votes, and the qualified majority threshold resulting from the table given in the Declaration on enlargement of the European Union will be automatically adjusted accordingly.

The 'table given in the Declaration on enlargement of the European Union' is the one we have quoted above (p. 9f). Since the weights in that table add up to 345, raising the blocking minority to 91 amounts to lowering the quota from 258 to 255. Thus, the effect of this further declaration is to replace $\mathcal{W}_{27}$ by a new weighted rule, with the same weights as $\mathcal{W}_{27}$ but with 255 as quota.[22]

In addition, this change makes the plain numerical quota, requiring 'a majority of members', non-vacuous: for example, under $\mathcal{W}_{27}$ the 13 heaviest members have total weight 257, which exceeds the new quota, as well as total population far exceeding 62% of the entire enlarged EU. We shall therefore model $\mathcal{N'}_{27}$ formally as the meet of three weighted rules:

$$\mathcal{N'}_{27} = \mathcal{W'}_{27} \wedge \mathcal{M}_{27} \wedge \mathcal{P}_{27}, \tag{3}$$

where $\mathcal{W'}_{27}$ is the weighted rule with weighted votes as detailed above (p. 9f) but quota 255; $\mathcal{M}_{27}$ is the ordinary majority rule, with weight 1 for each member and quota 14; and $\mathcal{P}_{27}$, as before, is the weighted rule whose weights are the population sizes of the 27 members and prospective members and whose quota is equal to 62% of their total population.

Moreover, unlike $\mathcal{N}_{27}$, the new rule $\mathcal{N'}_{27}$ cannot be recast as a single weighted rule. To see this, consider the following two coalitions:

A := {Germany, France, Italy, Poland, Romania, Netherlands, Greece, Czech Republic, Belgium, Hungary, Portugal, Slovakia, Denmark, Finland, Ireland, Lithuania, Slovenia, Estonia, Cyprus, Luxembourg, Malta},

B := {UK, France, Italy, Spain, Romania, Netherlands, Czech Republic, Belgium, Hungary, Portugal, Sweden, Bulgaria, Austria, Denmark, Finland, Ireland, Lithuania, Cyprus, Luxembourg}.

[21] Sic. This is French for 73.4%. This sentence may indicate that a tacit agreement has been reached regarding the interim stages of enlargement and the quota to be set at each stage.

[22] We refrain from speculating as to what convoluted reasoning led the authors of the treaty to specify $\mathcal{N'}_{27}$ in such tortuous fashion.

It is straightforward to verify that both A and B are winning coalitions under $\mathcal{N}'_{27}$: in the weighting of $\mathcal{W}'_{27}$ both A and B have total weight 255, which is exactly the vote quota of this rule; both contain a majority of the members; and their total populations (see Table 3) are $354,935,000$ and $338,308,000$ respectively, both of which exceed the $298,332,220$ population quota of $\mathcal{P}_{27}$.

However, if Germany swaps places with Sweden, Bulgaria and Austria, we get two new coalitions:

A' := {France, Italy, Poland, Romania, Netherlands, Greece, Czech Republic, Belgium, Hungary, Portugal, Sweden, Bulgaria, Austria, Slovakia, Denmark, Finland, Ireland, Lithuania, Slovenia, Estonia, Cyprus, Luxembourg, Malta},

B' := {Germany, UK, France, Italy, Spain, Romania, Netherlands, Czech Republic, Belgium, Hungary, Portugal, Denmark, Finland, Ireland, Lithuania, Cyprus, Luxembourg}.

Both of these are losing coalitions under $\mathcal{N}'_{27}$, because the total population of A' is only $298,063,00$, and the total weight of B' is only 254.

So the same argument as that used in Subsection 2.2 proves that $\mathcal{N}'_{27}$ cannot be represented by a single system of weights and quota.

4.2 The effect of $\mathcal{M}_{27}$ and $\mathcal{P}_{27}$ The only difference between $\mathcal{N}'_{27}$ and its first component, $\mathcal{W}'_{27}$, is that the latter has 23 winning coalitions that are losing coalitions under $\mathcal{N}'_{27}$. Of these, 16 coalitions do not attain the quota of $\mathcal{M}_{27}$ because they do not contain a majority of the members; and another 7 coalitions do not attain the population quota of $\mathcal{P}_{27}$.[23]

To list the 16 exceptional coalitions of the first kind, let us denote by B the coalition consisting of the 8 biggest members, from Germany down to the Netherlands. Thus:

B := {Germany, UK, France, Italy, Spain, Poland, Romania, Netherlands}.

The 16 exceptional coalitions that are winning under $\mathcal{W}'_{27}$ (as well as under $\mathcal{P}_{27}$) but not under $\mathcal{M}_{27}$ are:

1. $B \cup$ {Greece, Czech Republic, Belgium, Hungary, Portugal};
2. $B \cup$ {Greece, Czech Republic, Belgium, Hungary} $\cup$ {Sweden};

[23] Thus the plain numerical majority clause affects only 16 possible divisions of the CM out of a total of $2^{27} = 134,217,728$. Of this total, $2,718,774$ would have positive outcome under $\mathcal{W}'_{27}$ but only $2,718,758$ do so under $\mathcal{W}'_{27} \wedge \mathcal{M}_{27}$. The population clause affects a further 7 divisions, reducing the number of winning coalitions under $\mathcal{N}'_{27}$ to $2,718,751$.

3. $B \cup \{$Greece, Czech Republic, Belgium, Hungary$\} \cup \{$Bulgaria$\}$;
4. $B \cup \{$Greece, Czech Republic, Belgium, Hungary$\} \cup \{$Austria$\}$;
5. $B \cup \{$Greece, Czech Republic, Belgium, Portugal$\} \cup \{$Sweden$\}$;
6. $B \cup \{$Greece, Czech Republic, Belgium, Portugal$\} \cup \{$Bulgaria$\}$;
7. $B \cup \{$Greece, Czech Republic, Belgium, Portugal$\} \cup \{$Austria$\}$;
8. $B \cup \{$Greece, Czech Republic, Hungary, Portugal$\} \cup \{$Sweden$\}$;
9. $B \cup \{$Greece, Czech Republic, Hungary, Portugal$\} \cup \{$Bulgaria$\}$;
10. $B \cup \{$Greece, Czech Republic, Hungary, Portugal$\} \cup \{$Austria$\}$;
11. $B \cup \{$Greece, Belgium, Hungary, Portugal$\} \cup \{$Sweden$\}$;
12. $B \cup \{$Greece, Belgium, Hungary, Portugal$\} \cup \{$Bulgaria$\}$;
13. $B \cup \{$Greece, Belgium, Hungary, Portugal$\} \cup \{$Austria$\}$;
14. $B \cup \{$Czech Republic, Belgium, Hungary, Portugal$\} \cup \{$Sweden$\}$;
15. $B \cup \{$Czech Republic, Belgium, Hungary, Portugal$\} \cup \{$Bulgaria$\}$;
16. $B \cup \{$Czech Republic, Belgium, Hungary, Portugal$\} \cup \{$Austria$\}$.

Under $\mathcal{W'}_{27}$, all these are *minimal* winning coalitions: the first has weight 257 and each of the remaining 15 has weight 255.

The exceptional coalitions of the second kind—those that are winning under $\mathcal{W'}_{27}$ (as well as under $\mathcal{M}_{27}$) but not under $\mathcal{P}_{27}$—are:

1. All members except Germany, UK, France and Malta;
2. All members except Germany, UK, Italy, Malta;
3. All members except Germany, France, Italy and Malta;
4. All members except Germany, UK, Spain and Latvia.
5. All members except Germany, UK and France;
6. All members except Germany, UK and Italy;
7. All members except Germany, France and Italy.

Under $\mathcal{W}'_{27}$ the first four of these are *minimal* winning coalitions: the first three have weight 255 each and the fourth has weight 256. The remaining three have weight 258 each and are not minimal, as they contain Malta, whose weight is 3; however, they do become minimal after the first three coalitions are removed (that is, turned into losing coalitions).

Using the method explained in Subsection 2.3 we can now obtain the Penrose power ψ of each member under $\mathcal{N}'_{27}$ from its value under $\mathcal{W}'_{27}$. For any member a we have

$$\psi_a[\mathcal{N}'_{27}] = \psi_a[\mathcal{W}'_{27}] + \frac{\mathrm{Ex}_a - \mathrm{In}_a}{2^{26}},$$

where Ex_a is the number of exceptional coalitions from which a is excluded and In_a is the number of exceptional coalitions in which a is included. For convenience, we have collected the values of In_a, Ex_a and $\mathrm{Ex}_a - \mathrm{In}_a$ in Table 10.

From this table we can see that the combined effect of the plain majority and population clauses is to boost the voting powers of the 11 smallest members, from Malta up to Slovakia, and to reduce the voting power of the 16 big and middle-sized members, from Germany down to Austria. However, the effect is rather erratic: Malta's voting power is boosted by $15/2^{26}$ and Latvia's by $11/2^{26}$, while all the remaining small members gain $9/2^{26}$. The greatest losers are Poland, Romania and the Netherlands, whose voting power under $\mathcal{N}'_{27}$ is $23/2^{26}$ less than under $\mathcal{W}'_{27}$; followed by Spain, who loses $21/2^{26}$; and by Greece, the Czech Republic, Belgium, Hungary and Portugal, who lose $17/2^{26}$.

But in any case all these losses and gains are of the order of 10^{-7} or 10^{-8}; smaller than the rounding errors in our figures for voting power, which are calculated to 6 decimal places. Thus, while the plain majority and population clauses—embodied in the components $\mathcal{M}_{27}$ and $\mathcal{P}_{27}$ of $\mathcal{N}'_{27}$—add a great deal of formal complexity, they do not significantly affect the voting powers of members: for all practical purposes these powers are the same under $\mathcal{N}'_{27}$ as under $\mathcal{W}'_{27}$.

The values of ψ for $\mathcal{N}'_{27}$ are shown in Table 9.

The meaning of the other data shown in Table 9 will be discussed in Subsection 5.1.

5 Evaluation of the rules

In this section we assess and evaluate the QMV rules laid down in the Treaty of Nice, which we have denoted by $\mathcal{N}_{15}$, $\mathcal{N}_{27}$ and $\mathcal{N}'_{27}$ and whose structure we have analysed in Sections 2, 3 and 4, respectively. For comparison, we consider also two other decision rules.

Since $\mathcal{N}_{15}$ is intended as a direct replacement for the QMV rule currently operated by the CM, which will remain in force until the end of 2004 if the EU is not enlarged by then, it is instructive to compare these two rules in order to see whether $\mathcal{N}_{15}$ is an improvement on the existing rule.

As a benchmark for evaluating $\mathcal{N}_{27}$ and $\mathcal{N}'_{27}$ we use Rule B, an adaptation of Proposal B introduced in [11] and judged there to be the best of nine proposals of QMV rules for the CM of an enlarged EU. We also compare $\mathcal{N}_{27}$ and $\mathcal{N}'_{27}$ with the current QMV rule.

The relevant extensive data are presented in tables whose structure is explained in Subsection 5.1. From these data are distilled certain synoptic parameters, which are used for evaluating the rules according to several criteria. These criteria and the relevant parameters are explained briefly in Subsection 5.2. (A reader who is familiar with [11] may skip these two subsections.) Following that, we discuss each rule in turn.

5.1 How to read Tables 4–9 The extensive data needed for evaluating the five rules are presented in Tables 4, 6, 7, 8 and 9. Table 5 provides data for $\mathcal{W}_{15}$, which is not itself one of the rules under evaluation, but which is the main component of $\mathcal{N}_{15}$.

The first column in each of the six tables lists the member states in descending order of population size.

The next column, except in Tables 6 and 9, lists the weight of each member. This column is missing in Tables 6 and 9 because, as we proved in Subsections 2.2 and 4.1, the corresponding rules, $\mathcal{N}_{15}$ and $\mathcal{N}'_{27}$, cannot be represented as weighted rules. The weights shown in Table 8 are the adjusted weights as explained on p. 13.

The next column, headed 'ψ', gives the voting power of each member as quantified by the Penrose measure ψ. Justifications for using this measure in a context such as the present one are outlined in [11, Introduction] and explained in detail in [10]; so we need not rehearse them here.

The sum of the ψ values of all voters is not constant, but varies from one decision rule to another; so in order to see at a glance the *relative* distribution of voting power it is convenient to 'normalize' ψ. This is done by dividing the value of ψ for each voter by the sum total of the ψ values of all the voters. The resulting quantity is the *[relative] Banzhaf index* β.[24] Consequently, the

[24] Thus

$$\beta_a[\mathcal{G}] = \frac{\psi_a[\mathcal{G}]}{\sum_{x \in V} \psi_x[\mathcal{G}]},$$

values of β for all the voters add up to 1. In each of the Tables 4–8, the column headed '100β' gives the values of the Banzhaf index multiplied by 100, so that they are expressed in percentage terms, adding up to 100 (though in some cases there are small rounding errors). The reader must be warned that the Banzhaf index can only give an indication of the *relative* powers of voters under a given decision rule. Unlike the Penrose measure, the Banzhaf index does not provide a reliable comparison of powers—even powers of the same voter—under two different rules. For example, comparing Table 6 and Table 5, we find that the β values of the UK, France and Italy are higher under $\mathcal{N}_{15}$ than under $\mathcal{W}_{15}$. However, this does not mean that these members are better off under the former rule. On the contrary, all members except Germany have less power under $\mathcal{N}_{15}$ than under $\mathcal{W}_{15}$. It is only the *relative* position of the three members that is better under $\mathcal{N}_{15}$.

The final column in Tables 4, 6, 7, 8 and 9, headed 'Quotient', is obtained by dividing the 100β figure for each member by the population-square-root index of that member. The latter index equals the square root of the member's population as a percentage of the sum of the square roots of the populations of all members; it is given in the last column of Table 2 (for the present 15-member EU) and Table 3 (for the prospective enlarged 27-member EU). The Quotient figures will be used in assessing the equitability of a decision rule. Since we do not propose to evaluate $\mathcal{W}_{15}$, we have not included them in Table 5.

Under each table we state the values of the following parameters. First, in the case of the weighted rules (that is, all except $\mathcal{N}_{15}$ and $\mathcal{N}'_{27}$) we state the value of the quota q both in absolute terms and as a percentage of the sum of all weights. Second, we state the value of $\min \#$, which is the least number of members whose 'yes' votes are sufficient for an act to be adopted; in other words, the size of the smallest possible winning coalition. Third, we state the value of ω, the number of divisions with positive outcome (that is, the number of winning coalitions).

5.2 Criteria and synoptic parameters We evaluate the rules according to four criteria.

The first criterion for assessing a decision rule for a body of representatives such as the CM is its *equitability.* Assuming that in each division of the council every representative votes in accordance with the view of a majority of the citizens of his or her constituency,[25] the citizens have *indirect* voting power whereby they exert influence on decisions made by the council.

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where $\mathcal{G}$ is the decision rule, a is any voter and V is the set of all voters (the assembly).

[25] This assumption is referred to in [11] as a 'democratic idealization'.

An equitable decision rule for the council is one that equalizes the indirect powers of the citizens across all constituencies. This approach goes back to Penrose [17, 18],[26] who found that a decision rule is equitable just in case the voting powers (as measured by ψ) of the representatives are proportional to the square root of the size of their respective electorates.[27]

It follows that in a table for a perfectly equitable decision rule for the CM, the figures in the '100β' column would be identical to those given in the last column of the relevant population table: Table 2 (for the present 15-member EU) or Table 3 (for the prospective 27-member EU). And consequently in the 'Quotient' column all the entries would be 1.[28]

Thus, for example, the figures 0.867 and 1.010 shown for Germany and the UK respectively in the last column of Table 6 indicate that under $\mathcal{N}_{15}$ Germany has 13.3% less power than equitability would require, while the UK has 1.0% more than equitability would justify.

In order to assess the degree to which a given rule is equitable, we therefore gauge how close its '100β' column is to the ideal presented by the last column of the relevant population table. We use the following four parameters for this purpose.

ρ Pearson's product-moment coefficient of correlation between the '100β' column in the table of the given rule and the last column of the relevant population table. This coefficient is designed to detect *linear* relationships between two series of data. The closer ρ is to 1, the better the linear fit between these two columns.

χ^2 Chi-squared: a standard way of measuring the closeness of fit between the '100β' column of the given rule and the last column of the relevant population table. The *smaller* the value of χ^2, the closer the fit. Note that χ^2 cannot be used for comparing two decision rules with different numbers of voters; so we can only use it for comparing $\mathcal{N}_{15}$ with the current decision rule; and $\mathcal{N}_{27}$, $\mathcal{N}'_{27}$ and our Rule B with one another.

$\max|d|$ Maximal relative deviation. This is obtained from the 'Quotient' column in the table of the rule. It is the largest difference between a figure in this column and 1.

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[26] See also Banzhaf [1]. For more recent papers using this approach in the context of the CM, see [9], Laruelle and Widgrén [14] and Sutter [19].

[27] For a proof, see [10, § 3.4]. For a discussion of the very widespread but erroneous idea that equitability is achieved by making the voting weights of representatives proportional to the size of the respective electorates, see [11, §§ 2.1, 2.2].

[28] Here we are assuming, as seems reasonable, that the ratio of electorate to population size is fairly constant across the EU.

ran(d) Range of relative deviations. This is also derived from the 'Quotient' column. It is obtained by subtracting the smallest entry in this column from the largest.

While ρ and χ^2 measure the *overall* equitability of a rule, $\max|d|$ and ran(d) focus on the most extreme *individual* deviations from equitability, which presumably are the most invidious.

The second criterion we use is the degree to which a given rule conforms to the prescription of *majoritarianism* or *majority rule*. Arguably, a rule should come as close as possible to producing decisions that conform to the wishes of a majority of the entire electorate.

In a two-tier decision-making system, consisting of a council of representatives who vote on behalf of their respective constituencies, it is always possible that—although each representative votes in accordance with a majority opinion in his or her own constituency—the outcome may nevertheless be opposed by a majority of the entire electorate.[29] When this occurs, the difference between the size of the majority opposing the outcome and the minority supporting it is the *majority deficit* of that particular decision. (If the outcome is supported by a majority of the entire electorate, the majority deficit is 0.)

The *mean majority deficit (MMD)* of a decision rule is the statistical mean (or a priori expected value) of the majority deficit that may occur under that rule. We use it as a measure of the degree to which the given rule deviates from majoritarianism. The MMD can only be used for comparing decision rules that apply to councils whose entire electorates have the same size.

According to [10, pp. 54–67], the MMD of a decision rule $\mathcal{G}$ of the CM is given, to an extremely good approximation, by

$$\mathrm{MMD}[\mathcal{G}] \approx \frac{\sqrt{n} - \sum_i \psi_i[\mathcal{G}]\sqrt{n_i}}{\sqrt{2\pi}}, \tag{4}$$

where n is the size of the entire electorate of the EU, n_i is the size of the electorate of the i-th member state, and $\psi_i[\mathcal{G}]$ is the voting power of that state's representative on the CM under $\mathcal{G}$. In what follows we shall take population as proxy for electorate.

The third criterion for assessing a decision rule of the CM is its *relative sensitivity*. The [absolute] sensitivity of a decision rule is the sum of the

[29] An example of this is the fact, which is by now well known, that a US president may be—and on several occasions was—elected by a majority of the Electoral College, although another candidate won a majority of the popular vote.

voting powers (as measured by ψ) of all members of the CM. It measures the degree to which the CM collectively is empowered as a decision-making body, the ease with which an average member can make a difference to the outcome of a division. It is thus a good indicator of effectiveness.

The *relative sensitivity index*, denoted by S, measures the sensitivity of the given rule on a logarithmic scale, on which $\mathrm{S} = 0$ holds for the least sensitive rule (unanimity) with the same number of voters, and $\mathrm{S} = 1$ holds for the most sensitive rule (the ordinary majority rule) with that number of voters.[30] Note that S cannot be used for comparing the sensitivities of decision rules with different numbers of voters.

The fourth and final criterion we use for assessing a decision rule for the CM is its *resistance*. By this we mean its propensity to resist changes to the status quo by favouring a negative outcome of a division (blocking of a proposed act) rather than a positive one (adopting a proposed act). The *resistance coefficient* of a decision rule $\mathcal{G}$, denoted by $\mathrm{R}[\mathcal{G}]$, is given by

$$\mathrm{R}[\mathcal{G}] := \frac{2^{m-1} - \omega[\mathcal{G}]}{2^{m-1} - 1}, \tag{5}$$

where m is the number of voters and $\omega[\mathcal{G}]$ is the number of divisions with positive outcome (in other words, the number of winning coalitions) under $\mathcal{G}$.

For the ordinary majority rule, which (with an odd number of voters, as is the case with the rules under consideration) gives equal a priori probabilities to positive and negative outcomes, we have $\mathrm{R} = 0$. For the unanimity rule, which is the most resistant, $\mathrm{R} = 1$.

Arguably, a decision rule for the CM should not have very low resistance (R too close to 0), because the status quo ought to be protected to some extent, and changing it not made too easy. But a high value of R (too close to 1) can be disastrous because it may lead to immobilism.

A very important fact, which is apparently not widely realized, about weighted decision rules is that if the quota is kept pegged at a constant percentage of the sum of the weights, and if that percentage is greater than 50%, then as the number of voters increases the resistance tends to grow at an increasingly steep rate. This is indeed what happened to the QMV rule of the CM during its first five periods: the quota was kept more or less pegged at about 71% of the total weight; and with each successive enlargement the resistance went up (see Table 1).[31]

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[30] For further details see [10, p. 61] and [11, pp. 7–9].

[31] The figures given in [10, p. 169] for R in the third and fifth periods contain slight errors, which are corrected here.

The resistance coefficient R is closely connected with another parameter, which Coleman [3] called 'the power of a [decision-making] collectivity to act' and denoted by A: it equals the a priori probability that a proposed act will be adopted by the decision-making body rather than rejected by it. For a given decision rule $\mathcal{G}$, this probability is given by

$$\mathrm{A}[\mathcal{G}] := \frac{\omega[\mathcal{G}]}{2^m}, \tag{6}$$

where m and $\omega[\mathcal{G}]$ are as above. For large values of m, A is given approximately by

$$\mathrm{A} \approx \frac{1 - \mathrm{R}}{2}. \tag{7}$$

For the decision rules we shall examine here, with $m = 15$ or 27, the approximation is excellent.

5.3 Characteristics of the present QMV rule For extensive data of this rule, see Table 4.

An oft-heard complaint is that at present the smaller member states wield too much power in the CM and the larger ones too little. Most of those who have voiced this complaint make the very common error of assuming that equitability would require the voting weight (rather than voting power) of a member state to be proportional to its population size (rather than to its square root).[32] If that assumption were correct, then the present QMV would have been grossly inequitable.

However, the complaint itself is not entirely unfounded, even from the viewpoint of Penrose's square-root rule. This is evident from the 'Quotient' figures in Table 4.[33]

Indeed, the smaller members, from Luxembourg up to Greece, have more relative voting power than equitability would justify. The most extreme case is Luxembourg (124.1% over the odds!), followed by the much less extreme cases of Portugal, Ireland, Belgium and Greece (20.6%, 20.5%, 19.1% and 17.5%, respectively) and the very slightly over-endowed Austria, Sweden, Finland and Denmark (9.3%, 4.3%, 2.6% and 1.2%, respectively).

On the other hand, the bigger members, from Germany down to the Netherlands, have less power than equitability demands. But the only really

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[32]Cf. footnote 27 above. A typical example of this complaint, articulating the common error, is quoted in [10, pp. 156–57].

[33]Note, by the way, that these figures, as well as the value of ρ given below, differ slightly from those in [10, Table 5.3.9]. This is because the latter rely on older population data.

bad case is that of Germany (20.1% too little). The relative power deficits of the other five—the UK, France, Italy, Spain and the Netherlands—are quite small (6.0%, 5.7%, 4.6%, 4.5% and 4.0%, respectively).

Note that the figures in the 'Quotient' column display a see-saw pattern: for obvious reasons, within each group of members with the same weight, the figures increase as the population decreases.

The synoptic parameters of the present QMV rule are as follows.

$$\rho = 0.9855, \qquad \chi^2 = 2.97, \qquad \max|d| = 124.1\%, \qquad \mathrm{ran}(d) = 144.2\%,$$
$$\mathrm{MMD} = 5\,519, \qquad \mathrm{S} = 0.861, \qquad \mathrm{R} = 0.844.$$

These will be useful for comparing the other four rules with the present one. For the moment we would only like to take a look at the present rule's resistance coefficient, R = 0.844. As we noted above, Table 1 makes it clear that this is the highest value of R in the history of the CM so far. While it cannot be regarded as excessively high, it approaches the danger mark. The corresponding value of Coleman's index A—the a priori probability of an act being adopted by the present CM—is 0.078. In betting terms, the odds against an act being adopted are just under 12 : 1. Of course, this does not mean that in reality about 12 out of 13 acts proposed by the Commission are rejected by the CM. The Commission does not table proposals at random, nor do members of the CM decide how to vote by flipping a coin. In practice, the Commission normally makes sure before tabling a proposal that it is going to be approved. This is where political bargaining and diplomacy come in. Representatives of the member states negotiate with one another, and in many cases they do come to an agreement. However, the value of R is a good indication of the steepness of the gradient up which the engine of diplomacy must climb. So far, it has managed to do so, with some huffing and puffing. But if the odds become too steep, the engine may not be able to labour successfully against them.

5.4 Characteristics of $\mathcal{N}_{15}$ Extensive data of this rule are shown in Table 6. Comparing this table with Table 4, we see that $\mathcal{N}_{15}$ will give the five biggest members—from Germany down to Spain—more voting power, both absolutely (the 'ψ' column) and in relative terms (the '100β' column), than they have at present.

Moreover, for the first time Germany will get more voting power than the UK, France and Italy. This is entirely due to the contribution of the component $\mathcal{P}_{15}$ in equation (1), because in the other component, $\mathcal{W}_{15}$, these four members have equal weights.[34]

[34] This way of giving Germany more power seems to have solved a tricky diplomatic

All the remaining member states, from the Netherlands down, will lose voting power, both absolutely and relatively.[35] However, a glance at the 'Quotient' column of Tables 6 and 4 suggests that $\mathcal{N}_{15}$ is on the whole significantly more equitable than the present rule. Also, quite unlike the latter, $\mathcal{N}_{15}$ will not discriminate between members on a simple small-versus-big basis. Here the see-saw pattern, which we observed in the case of the present QMV rule, is more pronounced. As we go down the 'Quotient' column of Tables 6, the figures rise above the 1 mark, then plunge below it, only to rise above it again; and this is repeated four times—the four peaks being Spain, Portugal, Austria and Luxembourg. The smallest member, Luxembourg, is still the most over-endowed (with 93.8% over the odds); but it is now followed by Spain, the fifth biggest (14.7%). The biggest member, Germany, is still the least favoured (13.3% deficit); but it is now followed by Denmark, the fourth-smallest (12.9% deficit).

These observations are reflected in the synoptic parameters of $\mathcal{N}_{15}$:

$$\rho = 0.9832, \qquad \chi^2 = 1.59, \qquad \max|d| = 93.8\%, \qquad \text{ran}(d) = 107.1\%,$$
$$\text{MMD} = 5\,422, \qquad \text{S} = 0.861, \qquad \text{R} = 0.836.$$

Referring to Table 11, we see that χ^2, $\max|d|$ and $\text{ran}(d)$ are all significantly smaller here than under the present QMV; and for these parameters smaller is better. True, ρ here is somewhat smaller than for the present QMV, which appears to suggest that the latter is more equitable. But this is an artefact of the mathematical behaviour of the correlation coefficient, which makes it adversely affected by *non-linearity*; so its smaller value here reflects the pronounced see-saw pattern in the 'Quotient' column of Table 6.

Table 11 also shows that the improvement in equitability achieved by $\mathcal{N}_{15}$ compared to the present QMV rule is accompanied by an improvement according to two other criteria: lower MMD and resistance. The value $\text{R} = 0.836$ corresponds to $\text{A} = 0.082$, which means that the a priori betting odds against a proposed act being adopted under $\mathcal{N}_{15}$ are just above 11 : 1. The

problem. According to press reports, Germany demanded to have more power, because after re-unification it has a much larger population. The leaders of the other big three—especially those of the French delegation, who were facing presidential elections not long after Nice—did not want to seem to be too 'soft' by giving up the long-standing parity of weights with Germany. The component $\mathcal{P}_{15}$ squared this circle, as its population numbers are not called 'weights' (although in effect that is what they are). This may partly explain the odd formulation of the population paragraph in the text of the treaty. In addition, Germany was also compensated in other ways, mainly by increasing its relative representation, as of 1 January 2004, in the European Parliament.

[35] In this, the Netherlands has been treated somewhat unfairly, as its voting power under the present rule is already smaller than equitability requires.

sensitivity of $\mathcal{N}_{15}$ is slightly lower than that of the present QMV rule, but the difference is too small to affect the value of S to three decimal places.

So $\mathcal{N}_{15}$ seems to us a considerable improvement on the present QMV rule.

5.5 Characteristics of Rule B This rule is adapted from Proposal B, which was the overall winner of the beauty contest conducted in [11] between nine proposed rules for QMV in a 28-member enlarged CM. The adaptation we have made was to eliminate Turkey, which does not feature in the Nice text as a prospective member for the time being.

This is a benchmark decision rule that follows Penrose's prescription for equitability of a decision rule for a council of representatives. In accordance with this prescription, we aimed to make the voting power of each member state proportional to the square root of its population (which is taken as proxy for its electorate). But as Penrose [18, Appendix] pointed out, under a weighted voting rule with many voters, voting powers tend to be very nearly proportional to the respective weights, provided certain conditions are satisfied, as they are in the present case. So we have used as weights the percentage figures shown in the last column of Table 3, simply multiplying them by 100 in order to get whole numbers. Unfortunately, due to rounding errors these weights add up to $10,002$ rather than a neat $10,000$.

Setting the quota at just over 50% of the total weight would have produced a decision rule with the least possible MMD,[36] and high sensitivity. But this would also produce a resistance coefficient equal or very close to 0, which would not give privileged position to the status quo, and thus would arguably make adopting acts too easy. So we have set the quota at $6,000$, which is very nearly 60% of the total weight, and produces (as we shall see in a moment) a value of R very slightly greater than it used to be in the first period of the EU, but smaller than in subsequent periods.

Extensive data of Rule B are presented in Table 7. As can be seen at once from the 'Quotient' column of the table, Rule B is almost perfectly equitable. Its synoptic parameters are as follows.

$$\rho = 1.0000, \qquad \chi^2 = 0.00264, \qquad \max|d| = 0.8\%, \qquad \mathrm{ran}(d) = 1.5\%,$$
$$\mathrm{MMD} = 3\,869, \qquad \mathrm{S} = 0.966, \qquad \mathrm{R} = 0.604.$$

The values of ρ, χ^2, $\max|d|$ and $\mathrm{ran}(d)$ here are just about as good as one can hope for. Note also that the value of MMD, for a total population of some 481m, is considerably lower even than that of the present QMV rule, with a total population of only some 375m.

[36] See [11, pp. 25–26]; for a proof see [10, pp. 72–74].

The value R = 0.604 corresponds to a value of approximately 0.198 of Coleman's index A. In betting terms this means that the odds against an act being adopted by the CM would be about 4 : 1, three times shorter than at present.

5.6 Characteristics of $\mathcal{N}_{27}$ Extensive data of this rule are shown in Table 8. Comparing the 'ψ' column of this table with that of Table 7, we observe that under $\mathcal{N}_{27}$ each and every member has much less voting power than under Rule B: the difference is nearly a whole order of magnitude.

We saw in Subsection 5.4 that in $\mathcal{N}_{15}$ the Treaty of Nice managed to give Germany more power than to the other three big members without officially giving it more weight than them. The intention behind $\mathcal{N}_{27}$ must have been similar, but here it has failed utterly: although according to our adjusted weighting Germany has greater weight than these three members, the resulting boost in voting power is too small to show in our table, which gives ψ to 'only' six decimal places.

The 'Quotient' column of Table 8 reveals that $\mathcal{N}_{27}$ is rather inequitable. But, as in the case of $\mathcal{N}_{15}$, the see-saw pattern is pronounced and there is no discrimination along small-versus-big as under the present QMV rule. The resulting inequalities in the distribution of voting power seem somewhat haphazard: Luxembourg is again the most over-endowed (with 82.7% over the odds), followed by Malta and Cyprus (46.4% and 38.5%, respectively). But Latvia, the sixth-smallest member, is the least favoured (with a deficit of 23.2%), followed by Germany, Slovenia and Romania (deficits of 19.1%, 14.8% and 14.3%, respectively).

The synoptic parameters of $\mathcal{N}_{27}$ are as follows.

$$\rho = 0.9793, \qquad \chi^2 = 1.83, \qquad \max|d| = 82.7\%, \qquad \mathrm{ran}(d) = 105.9\%,$$
$$\mathrm{MMD} = 8\,054, \qquad \mathrm{S} = 0.847, \qquad \mathrm{R} = 0.967.$$

Comparing these values with those of the other rules (see Table 11), we observe that the value of ρ under $\mathcal{N}_{27}$ is quite low (reflecting, no doubt, the see-saw pattern of the 'Quotient' column). The values of $\max|d|$ and $\mathrm{ran}(d)$ are much worse here than under Rule B, though not quite so bad as under the present QMV rule.

The MMD of $\mathcal{N}_{27}$ is quite high compared to that of the benchmark Rule B, and the sensitivity of the former is considerably lower than that of the latter.

But what is truly alarming about $\mathcal{N}_{27}$ is the high value of the resistance coefficient R. This value, 0.967, corresponds to a value of approximately 0.017 of Coleman's index A. As we saw in Subsection 5.3, the present value of A is about 0.078. The a priori probability of an action being adopted by the CM under $\mathcal{N}_{27}$ will be about 4.7 times smaller than at present. In betting terms, the odds against an act being adopted will be just under 60 : 1, five times longer than the present 12 : 1.

This is entirely due to the high level of the quota. Clearly, the politicians and officials who drew up the Treaty of Nice did not realize that when the EU is enlarged from 15 to 27 members, the quota of the QMV rule ought to be *reduced* from its present level of some 71% of the total weight to something nearer 60%. Instead, they have *raised* it to 74.78%.

The resulting rule, if it is ratified in its present form, is likely to lead to a sclerosis of decision-making at the CM, which may in turn endanger the functioning of the whole EU. Unfortunately, when these effects will begin to be noticed, it may well be too late to do much about them. A change in the basic rules will require unanimity among the members, who would all have not only to admit that $\mathcal{N}_{27}$ is bad, but also to agree on how to amend it. But unanimity among the 27 members will be extremely difficult to achieve.

5.7 Characteristics of $\mathcal{N}'_{27}$ Extensive data of this rule are shown in Table 9. Comparing the 'ψ' column of this table with that of Table 8, we observe that under $\mathcal{N}'_{27}$ every member has somewhat greater voting power than under $\mathcal{N}_{27}$, but still a great deal less than under Rule B.

As in the case of $\mathcal{N}_{27}$, the difference between the voting power of Germany and those of the other big three members under $\mathcal{N}'_{27}$ is too small to show in our table. This is because, as we saw in Subsection 4.2, the combined effect of $\mathcal{M}_{27}$ and $\mathcal{P}_{27}$ is insignificant.

A glance at the 'Quotient' column of Table 9 reveals that $\mathcal{N}'_{27}$ is somewhat more equitable than $\mathcal{N}_{27}$ (Table 8), but much less than Rule B (Table 7).

The synoptic parameters of $\mathcal{N}'_{27}$ are as follows.

$$\rho = 0.9797, \qquad \chi^2 = 1.78, \qquad \max|d| = 81.2\%, \qquad \text{ran}(d) = 105.0\%,$$
$$\text{MMD} = 7\,922, \qquad \text{S} = 0.911, \qquad \text{R} = 0.959.$$

As can be seen from Table 11, all these show an improvement compared to $\mathcal{N}_{27}$. As in both cases the plain majority and population clauses have no significant effect, the improvement is entirely due to the difference between the quotas of $\mathcal{W}_{27}$ and $\mathcal{W}'_{27}$: in the former it is 74.78% of the total weight, while in the latter it is 73.91%. But the improvement is not very great. In particular, the value R = 0.959 corresponds to A = 0.020; in betting terms, the a priori odds against an act being adopted under $\mathcal{N}'_{27}$ are longer than 48 : 1, as compared with the present 12 : 1. This is not quite so bad as the 60 : 1 odds under $\mathcal{N}_{27}$, but is dire enough.

Thus, what we have said about $\mathcal{N}_{27}$ applies to almost the same extent to $\mathcal{N}'_{27}$ as well.

6 Conclusions

The Treaty of Nice contains provisions for three versions of QMV, the main decision rule of the CM of the EU, for adopting acts proposed by the Commission.

The first version, denoted here by $\mathcal{N}_{15}$, will take effect on 1 January 2005 in case the present 15-member EU will not have been enlarged by then.

$\mathcal{N}_{15}$ is a meet (or conjunction) of two weighted voting rules: $\mathcal{W}_{15}$ and $\mathcal{P}_{15}$. $\mathcal{W}_{15}$ is summarized in Table 5. $\mathcal{P}_{15}$ is a weighted rule in which each member state has weight equal to the size of its population, and the quota needed is 62% of the entire EU population.

We have shown in Subsection 2.2 that (given the most up-to-date population statistics available to us) $\mathcal{N}_{15}$ cannot be presented as a pure weighted rule.

We have found $\mathcal{N}_{15}$ to be significantly more equitable than the present QMV rule. Judged by two other criteria—majoritarianism and resistance—$\mathcal{N}_{15}$ performs better than the present QMV rule. According to a fourth criterion—sensitivity—there is no significant difference between $\mathcal{N}_{15}$ and the present rule. Overall, we therefore judge $\mathcal{N}_{15}$ to be a definite improvement.

The second and third versions, denoted here by $\mathcal{N}_{27}$ and $\mathcal{N}'_{27}$, are two alternative variants designed for an enlarged 27-member CM.

$\mathcal{N}_{27}$ is on the face of it also a meet of two weighted rules: $\mathcal{W}_{27}$ and $\mathcal{P}_{27}$. For the definition of $\mathcal{W}_{27}$, see p. 9f. $\mathcal{P}_{27}$ is like $\mathcal{P}_{15}$, but with 27 members instead of 15. However, we have shown in Subsection 3.2 that (given the most up-to-date population statistics available to us) $\mathcal{N}_{27}$ itself can be presented as a pure weighted rule, as displayed in Table 8. Moreover, our analysis in Subsection 3.2 shows that the effect of the population component, $\mathcal{P}_{27}$, on the members' voting powers is nugatory: the difference between $\mathcal{N}_{27}$ and $\mathcal{W}_{27}$ is so minute as to make no significant difference. Consequently, $\mathcal{N}_{27}$ does not give Germany measurably more voting power than to the UK, France and Italy—contrary to the apparent intention of the authors of the Nice Treaty.

$\mathcal{N}'_{27}$ is the meet of three weighted rules: $\mathcal{W}'_{27}$, $\mathcal{M}_{27}$ and $\mathcal{P}_{27}$. $\mathcal{W}'_{27}$ has the same weighting as $\mathcal{W}_{27}$, but with quota 255 (instead of 258); $\mathcal{M}_{27}$ is an ordinary majority rule, requiring an act to be supported by at least 14 of the 27 members; $\mathcal{P}_{27}$ is as before. We have shown in Subsection 4.1 that—like $\mathcal{N}_{15}$ and unlike $\mathcal{N}_{27}$—$\mathcal{N}'_{27}$ cannot be presented as a pure weighted rule. However, in Subsection 4.2 we have shown that the combined effect of the plain majority and population components, $\mathcal{M}_{27}$ and $\mathcal{P}_{27}$, on the members' voting powers is nugatory: the difference between $\mathcal{N}'_{27}$ and $\mathcal{W}'_{27}$ is so minute as to make no significant difference. Consequently, $\mathcal{N}'_{27}$, like $\mathcal{N}_{27}$, does not give Germany measurably more voting power than to the UK, France and Italy—contrary to the apparent intention of the authors of the Nice Treaty.

We have found $\mathcal{N}_{27}$ and $\mathcal{N}'_{27}$ to be somewhat more equitable than the present QMV rule.

However, $\mathcal{N}_{27}$ and $\mathcal{N}'_{27}$ score very badly on our other criteria. In particular, their resistance coefficient is extremely high: getting acts adopted under them will be a priori about 4.7 times and 3.8 times, respectively, as hard as it is at present. This is entirely due to the quotas of $\mathcal{W}_{27}$ and $\mathcal{W}'_{27}$, which—at 258 or even 255 out of a total weight of 345—are much too high.

This may well get the CM bogged down in immobilism, and pose a serious threat to the functioning of the EU.

If the 27-member decision rule is not amended before the EU is enlarged, its adverse effects will surely be noticed after the enlargement. But then it will be extremely difficult to amend it, because the amendment would have to be approved unanimously by all 27 members, which would be very hard to achieve.

Our best hope is that the rule will be amended and replaced by something much more like Rule B before the Treaty of Nice is ratified by the present 15 member states. One cannot be too optimistic about the likelihood of this. To our knowledge, the report [11]—which uses the same tools of analysis as the present paper and contains a recommendation of Proposal B, from which the present Rule B has been adapted—was available to the parties concerned before the Nice conference, but had no effect on the outcome. However, perhaps this was because there was not enough time to digest the analysis of [11]. If so, there may still be some hope that an amendment will be made before $\mathcal{N}_{27}$ or $\mathcal{N}'_{27}$ is set in stone.

Addendum (June 2001) Before the Treaty of Nice can come into force, it must be ratified by all 15 current member states; but Ireland is the only member which, viewing the ratification of every fundamental EU treaty as an amendment to its constitution, submits it to a referendum. In a referendum held on 7 June 2001, the Irish electorate rejected the Treaty of Nice by 529,478 votes to 456,461.

As less than one-third of the Irish electorate participated in the referendum, and as all major Irish parties as well as the Irish Catholic Church support the Nice Treaty, it is expected that the Irish government will conduct a second referendum in which the Nice Treaty will be approved. However, the implementation of the treaty is likely to be delayed; and in the meantime it may perhaps be slightly amended so as to allay some of the objections of Irish public opinion. Although these have little or nothing to do with the treaty's provisions on QMV, the delay offers a further opportunity for amending these provisions. Whether this opportunity will be taken up is quite another matter.

In any case, the result of the 7 June referendum is a vivid illustration of the difficulty of achieving unanimity even among the existing 15 EU members: half a million voters were able to throw into disarray the laboriously negotiated plans for enlarging the EU to half a billion citizens.

7 Tables

Table 1: QMV weights and quota, first five periods

Country	1958	1973	1981	1986	1995
Germany	4	10	10	10	10
Italy	4	10	10	10	10
France	4	10	10	10	10
Neth'lnds	2	5	5	5	5
Belgium	2	5	5	5	5
Lux'mbrg	1	2	2	2	2
UK		10	10	10	10
Denmark		3	3	3	3
Ireland		3	3	3	3
Greece			5	5	5
Spain				8	8
Portugal				5	5
Sweden					4
Austria					4
Finland					3
Total	17	58	63	76	87
Quota	12	41	45	54	62
Quota %	70.59	70.69	71.43	71.05	71.26
min #	3	5	5	7	8
R	0.581	0.710	0.728	0.804	0.844

Notes The '*Quota* %' row gives the quota as percentage of the total weight. The penultimate row gives the least number of members whose total weight equals or exceeds the quota. R is the resistance coefficient (see Subsection 5.2).

Table 2: Population of present EU members

Country	Pop. (1000s)	%	Pop. sqrt	%
Germany	82 038	21.858	9 057.48	13.97
UK	59 247	15.786	7 697.21	11.87
France	58 966	15.711	7 678.93	11.84
Italy	57 612	15.350	7 590.26	11.70
Spain	39 394	10.496	6 276.46	9.68
Netherlands	15 760	4.199	3 969.89	6.12
Greece	10 533	2.806	3 245.46	5.00
Belgium	10 213	2.721	3 195.78	4.93
Portugal	9 980	2.659	3 159.11	4.87
Sweden	8 854	2.359	2 975.57	4.59
Austria	8 082	2.153	2 842.89	4.38
Denmark	5 313	1.416	2 304.99	3.55
Finland	5 160	1.375	2 271.56	3.50
Ireland	3 744	0.998	1 934.94	2.98
Luxembourg	429	0.114	654.98	1.01
Total	375 325	100.001	64 855.51	99.99

Notes Source of population figures: [4]. The second column of figures shows the population as percentage of the total; the next column shows the square root of the population; the last column shows the square root of the population as percentage of the total. The apparent discrepancies in the totals of the second and last columns are due to rounding errors.

Table 3: Population of present and prospective EU members

Country	Pop. (1000s)	%	Pop. sqrt	%
Germany	82 038	17.049	9 057.48	9.54
UK	59 247	12.313	7 697.21	8.10
France	58 966	12.254	7 678.93	8.09
Italy	57 612	11.973	7 590.26	7.99
Spain	39 394	8.187	6 276.46	6.61
Poland	38 667	8.036	6 218.28	6.55
Romania	22 489	4.674	4 742.26	4.99
Netherlands	15 760	3.275	3 969.89	4.18
Greece	10 533	2.189	3 245.46	3.42
Czech Rep	10 290	2.138	3 207.80	3.38
Belgium	10 213	2.122	3 195.78	3.37
Hungary	10 092	2.097	3 176.79	3.35
Portugal	9 980	2.074	3 159.11	3.33
Sweden	8 854	1.840	2 975.57	3.13
Bulgaria	8 230	1.710	2 868.80	3.02
Austria	8 082	1.680	2 842.89	2.99
Slovakia	5 393	1.121	2 322.28	2.45
Denmark	5 313	1.104	2 304.99	2.43
Finland	5 160	1.072	2 271.56	2.39
Ireland	3 744	0.778	1 934.94	2.04
Lithuania	3 701	0.769	1 923.80	2.03
Latvia	2 439	0.507	1 561.73	1.64
Slovenia	1 978	0.411	1 406.41	1.48
Estonia	1 446	0.301	1 202.50	1.27
Cyprus	752	0.156	867.18	0.91
Luxembourg	429	0.089	654.98	0.69
Malta	379	0.079	615.63	0.65
Total	481 181	99.998	94 968.74	100.02

Notes Source of population figures: [4]. The second column of figures shows the population as percentage of the total; the next column shows the square root of the population; the last column shows the square root of the population as percentage of the total. The apparent discrepancies in the totals of the second and last columns are due to rounding errors.

Table 4: QMV in the CM at present

Country	w	ψ	100β	Quotient
Germany	10	0.112854	11.1621	0.799
UK	10	0.112854	11.1621	0.940
France	10	0.112854	11.1621	0.943
Italy	10	0.112854	11.1621	0.954
Spain	8	0.093445	9.2424	0.955
Netherlands	5	0.059387	5.8738	0.960
Greece	5	0.059387	5.8738	1.175
Belgium	5	0.059387	5.8738	1.191
Portugal	5	0.059387	5.8738	1.206
Sweden	4	0.048401	4.7872	1.043
Austria	4	0.048401	4.7872	1.093
Denmark	3	0.036316	3.5919	1.012
Finland	3	0.036316	3.5919	1.026
Ireland	3	0.036316	3.5919	1.205
Luxembourg	2	0.022888	2.2638	2.241
Total	87	1.011047	99.9999	

$$q = 62 = 71.26\% \text{ of } 87, \quad \min\# = 8, \quad \omega = 2\,549.$$

Notes For general explanations see Subsection 5.1. For assessment of this rule see Subsection 5.3.

Table 5: QMV under $\mathcal{W}_{15}$

Country	w	ψ	100β
Germany	29	0.121399	11.9209
UK	29	0.121399	11.9209
France	29	0.121399	11.9209
Italy	29	0.121399	11.9209
Spain	27	0.113098	11.1058
Netherlands	13	0.056458	5.5439
Greece	12	0.053040	5.2083
Belgium	12	0.053040	5.2083
Portugal	12	0.053040	5.2083
Sweden	10	0.044006	4.3212
Austria	10	0.044006	4.3212
Denmark	7	0.031799	3.1226
Finland	7	0.031799	3.1226
Ireland	7	0.031799	3.1226
Luxembourg	4	0.020691	2.0318
Total	237	1.018372	100.0000

$$q = 169 = 71.31\% \text{ of } 237, \quad \min\# = 8, \quad \omega = 2\,707.$$

Notes For general explanations see Subsection 5.1. $\mathcal{W}_{15}$ is not assessed in this paper. It is a component of $\mathcal{N}_{15}$.

Table 6: QMV under $\mathcal{N}_{15}$

Country	ψ	100β	Quotient
Germany	0.122314	12.1088	0.867
UK	0.121094	11.9879	1.010
France	0.121094	11.9879	1.012
Italy	0.121094	11.9879	1.025
Spain	0.112183	11.1057	1.147
Netherlands	0.055542	5.4985	0.898
Greece	0.052124	5.1601	1.032
Belgium	0.052124	5.1601	1.047
Portugal	0.052124	5.1601	1.060
Sweden	0.043457	4.3021	0.937
Austria	0.043457	4.3021	0.982
Denmark	0.031250	3.0937	0.871
Finland	0.031250	3.0937	0.884
Ireland	0.031250	3.0937	1.038
Luxembourg	0.019775	1.9577	1.938
Total	1.010132	100.0000	

$$\min \# = 8, \quad \omega = 2\,692.$$

Notes For general explanations see Subsection 5.1. For assessment of this rule see Subsection 5.4.

Table 7: Rule B (benchmark QMV rule for enlarged CM)

Country	w	ψ	100β	Quotient
Germany	954	0.231746	9.6184	1.008
UK	810	0.196224	8.1441	1.005
France	809	0.195976	8.1338	1.005
Italy	799	0.193503	8.0312	1.005
Spain	661	0.159548	6.6219	1.002
Poland	655	0.158071	6.5606	1.002
Romania	499	0.120019	4.9813	0.998
Netherlands	418	0.100388	4.1665	0.997
Greece	342	0.082039	3.4050	0.996
Czech Rep	338	0.081075	3.3649	0.996
Belgium	337	0.080834	3.3549	0.996
Hungary	335	0.080351	3.3349	0.995
Portugal	333	0.079868	3.3149	0.995
Sweden	313	0.075056	3.1151	0.995
Bulgaria	302	0.072406	3.0051	0.995
Austria	299	0.071683	2.9751	0.995
Slovakia	245	0.058705	2.4365	0.994
Denmark	243	0.058225	2.4166	0.994
Finland	239	0.057261	2.3766	0.994
Ireland	204	0.048855	2.0277	0.994
Lithuania	203	0.048611	2.0176	0.994
Latvia	164	0.039270	1.6299	0.994
Slovenia	148	0.035433	1.4706	0.994
Estonia	127	0.030394	1.2615	0.993
Cyprus	91	0.021785	0.9042	0.994
Luxembourg	69	0.016515	0.6854	0.993
Malta	65	0.015558	0.6457	0.993
Total	10 002	2.409396	100.0000	

$$q = 6\,000 = 59.99\% \text{ of } 10\,002, \quad \min\# = 10, \quad \omega = 26\,586\,251.$$

Notes For explanations see Subsection 5.1. For assessment of this rule see Subsection 5.5.

Table 8: QMV under $\mathcal{N}_{27}$

Country	$\widetilde{w}$	ψ	100β	Quotient
Germany	118	0.027356	7.7145	0.809
UK	117	0.027356	7.7145	0.952
France	117	0.027356	7.7145	0.954
Italy	117	0.027356	7.7145	0.966
Spain	108	0.026145	7.3732	1.115
Poland	108	0.026145	7.3732	1.126
Romania	56	0.015166	4.2771	0.857
Netherlands	52	0.014148	3.9900	0.955
Greece	48	0.013153	3.7092	1.085
Czech Rep	48	0.013153	3.7092	1.097
Belgium	48	0.013153	3.7092	1.101
Hungary	48	0.013153	3.7092	1.107
Portugal	48	0.013153	3.7092	1.114
Sweden	40	0.011037	3.1126	0.994
Bulgaria	40	0.011037	3.1126	1.031
Austria	40	0.011037	3.1126	1.041
Slovakia	28	0.007796	2.1984	0.897
Denmark	28	0.007796	2.1984	0.905
Finland	28	0.007796	2.1984	0.920
Ireland	28	0.007796	2.1984	1.078
Lithuania	28	0.007796	2.1984	1.083
Latvia	16	0.004469	1.2603	0.768
Slovenia	16	0.004469	1.2603	0.852
Estonia	16	0.004469	1.2603	0.992
Cyprus	16	0.004469	1.2603	1.385
Luxembourg	16	0.004469	1.2603	1.827
Malta	12	0.003374	0.9514	1.464
Total	1 385	0.354598	100.0002	

$\widetilde{q} = 1\,034 = 74.66\%$ of $1\,385$, $\min\# = 14$, $\omega = 2\,226\,791$.

Notes For explanations see Subsection 5.1. The $\widetilde{w}$ column gives the adjusted weights as explained on p. 13. For assessment of this rule see Subsection 5.6.

Table 9: QMV under $\mathcal{N}'_{27}$

Country	ψ	100β	Quotient
Germany	0.032688	7.7827	0.816
UK	0.032688	7.7827	0.961
France	0.032688	7.7827	0.962
Italy	0.032688	7.7827	0.974
Spain	0.031164	7.4198	1.123
Poland	0.031164	7.4198	1.133
Romania	0.017889	4.2591	0.854
Netherlands	0.016691	3.9740	0.951
Greece	0.015474	3.6843	1.077
Czech Rep	0.015474	3.6843	1.090
Belgium	0.015474	3.6843	1.093
Hungary	0.015474	3.6843	1.100
Portugal	0.015474	3.6843	1.106
Sweden	0.012989	3.0925	0.988
Bulgaria	0.012989	3.0925	1.024
Austria	0.012989	3.0925	1.034
Slovakia	0.009160	2.1809	0.890
Denmark	0.009160	2.1809	0.897
Finland	0.009160	2.1809	0.913
Ireland	0.009160	2.1809	1.069
Lithuania	0.009160	2.1809	1.074
Latvia	0.005251	1.2502	0.762
Slovenia	0.005251	1.2502	0.845
Estonia	0.005251	1.2502	0.984
Cyprus	0.005251	1.2502	1.374
Luxembourg	0.005251	1.2502	1.812
Malta	0.003958	0.9422	1.450
Total	0.420010	100.0002	

$$\min \# = 14 \text{ (imposed)}, \quad \omega = 2\,718\,751.$$

Notes For explanations see Subsection 5.1. For assessment of this rule see Subsection 5.7.

Table 10: Membership in exceptional coalitions under $\mathcal{W}'_{27}$

Country	In	Ex	Ex−In
Germany	16	7	-9
UK	18	5	-13
France	19	4	-15
Italy	19	4	-15
Spain	22	1	-21
Poland	23	0	-23
Romania	23	0	-23
Netherlands	23	0	-23
Greece	20	3	-17
Czech Rep	20	3	-17
Belgium	20	3	-17
Hungary	20	3	-17
Portugal	20	3	-17
Sweden	12	11	-1
Bulgaria	12	11	-1
Austria	12	11	-1
Slovakia	7	16	9
Denmark	7	16	9
Finland	7	16	9
Ireland	7	16	9
Lithuania	7	16	9
Latvia	6	17	11
Slovenia	7	16	9
Estonia	7	16	9
Cyprus	7	16	9
Luxembourg	7	16	9
Malta	4	19	15

Note For explanations see Subsection 4.2.

Table 11: Synoptic comparison

Rule	ρ	χ^2	max$\|d\|$	ran(d)	MMD	S	R
Present	0.9855	2.97	124.1	144.2	5 519	0.861	0.844
$\mathcal{N}_{15}$	0.9832	1.59	93.8	107.1	5 422	0.861	0.836
Rule B	1.0000	2.64×10^{-3}	0.8	1.5	3 869	0.966	0.604
$\mathcal{N}_{27}$	0.9793	1.83	82.7	105.9	8 054	0.847	0.967
$\mathcal{N}'_{27}$	0.9797	1.78	81.2	105.0	7 922	0.911	0.959

max $|d|$ and ran(d) are given in percentages.

Note For general explanations of the seven parameters shown here, see Subsection 5.2. For discussion see Subsections 5.3–5.7.

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