Annexations and Alliances: When Are Blocs Advantageous A Priori?

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ABSTRACT

For any simple voting game (SVG), we consider the question posed in the title from two different points of view as to what voting power means. We also distinguish between blocs imposed by annexation and those formed voluntarily, and present some general theoretical results concerning these notions. We illustrate our theoretical findings with examples using both toy SVGs and the Qualified Majority Voting rule of the Council of Ministers of the European Community (CMEC). We show that when voting power is understood as influence (I-power), forming a voluntary bloc may be advantageous even if its voting power is smaller than the sum of the original powers of its members; and it may be disadvantageous even if its voting power is greater than that sum.

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1 Introduction

Let $\mathcal{W}$ be a simple voting game (SVG). A *coalition* of $\mathcal{W}$ is simply an arbitrary set S of $\mathcal{W}$'s voters (cf. [8, p. 418]). The term 'coalition', borrowed from current usage in cooperative game theory, is not meant to imply that the members of S always vote in the same way; indeed, they may never do so. Some writers on voting power use the term 'coalition' when referring to a set of voters who combine into a single entity, always voting as one body. However, the term we shall use for such an entity is *bloc*.

Note that when a coalition S of $\mathcal{W}$ fuses into a bloc, $\mathcal{W}$ ceases to exist: it is transformed into a new SVG $\mathcal{V}$ whose voters are all those voters of $\mathcal{W}$ who do not belong to S, as well as a new voter, $\&_S$ say, who inherits, so to speak, the voting mandates of all the members of S; but the members of S themselves are no longer voters of $\mathcal{V}$. (For a rigorous definition see Section 2.)

The question we address in this paper is: *When is it advantageous to form a bloc?* Before this question can be answered correctly, it must however be made more precise; several clarifications are called for.

For a start, we must make it clear that the 'advantage' we are referring to is to be reckoned in terms of *voting power*. But now one might wonder: Advantage *to whom*? The point here is that—leaving aside external intervention—there are two ways in which a bloc can be thought to arise: annexation or voluntary consent.

Annexation occurs when one voter takes over the voting mandates of other voters, in order to use them in his or her own interest—as when a shareholder buys up the voting shares of other shareholders. In this case, it is only the annexer's advantage that counts; and common sense suggests that annexing non-dummy voters must *always* increase the voting power of the annexer and is advantageous in this respect (while annexing a dummy obviously makes no difference). In our opinion, this common-sense view is sound. However, more needs to be said about this issue, and we shall return to it in Section 3.

But if the members of a coalition S combine into a bloc voluntarily, by mutual consent, then clearly they must all derive some voting-power advantage from this. It turns out that in order to analyse this issue correctly, we must be more precise about what we mean by 'voting power'.

First, we must state that the voting power we are concerned with in this paper is *a priori*—rather than *a posteriori* or *actual*—voting power. This is the power that each voter derives from the structure of the decision rule, the SVG itself. In taking this stance, we go 'behind a veil of ignorance' regarding each voter's likes and dislikes, affinities and disaffinities among voters, and the nature of the bills to be voted on. (Cf. [1, Com. 2.2.3] and references cited there; see also [2], [5] and [6].) Of course, in reality when deciding whether to form a bloc, voters will take into account the kind of information that we ignore here, in so far as it is available. But such information may not be reliably available; and even if it is, the a priori theory developed here can serve as a benchmark, against which considerations using this information may better be appraised.

Second, it turns out that in assessing the voting-power advantages of forming a bloc, the distinction between two underlying—intuitive and pre-formal—notions of what voting power is all about makes a major difference. We are referring here to the distinction between *I-power* and *P-power*, drawn and explained in detail in [1] and summarized in [2]. We shall not repeat those explanations here, but merely recapitulate the essence of the distinction.

I-power is *power as influence*: a voter's a priori I-power is the voter's a priori ability to influence the outcome of a vote-division: whether a bill is passed or defeated. The notion of I-power presupposes *policy-seeking* voting behaviour: each voter simply votes for or against a given bill on what s/he considers to be the merit of this bill. The considerations that lead a voter to vote one way or another are completely exogenous to the decision rule (here modelled as an SVG); and they differ from voter to voter and from bill to bill. The passage or failure of a bill is regarded as a public good (or public bad), which affects all voters, irrespective of how they have voted on that bill. This notion has nothing to do with cooperative game theory.

All serious attempts to formalize and quantify a priori I-power have led—and in our opinion must lead—in one direction: to the so-called *Banzhaf (Bz) measure* of voting power, or to variants closely related to it. (For a detailed study of the Bz measure, see [1, Ch. 3] and references cited there. To set the historical record straight, the Bz measure should have been called, instead, the *Penrose measure*, because it was L S Penrose [9] who first proposed it. However, we continue to use here the term 'Bz measure' simply because it is the term commonly used in the literature.)

P-power—*power as prize*—on the other hand, is rooted in cooperative game theory: a voter's a priori P-power is that voter's a priori expected *relative share in some prize*, which a winning coalition can put its hands on by the very act of winning. It presupposes *office-seeking* voting behaviour aimed at winning, for the sake of obtaining part of the prize, which is available *only* to the winners and therefore cannot be a public good in the true sense. It also assumes bargaining and binding agreements.

Since there is no known realistic and generally accepted solution to the bargaining problem involving an arbitrary number of players, even for simple games, attempts to formalize and quantify the notion of a priori P-power have led in various directions, to several competing indexes of voting power. By far the most serious contender—as well as the most widely used—among them is the *Shapley–Shubik (S-S) index*, which is just the restriction to simple games of the Shapley value for cooperative games. Other indices whose underlying notion is that of P-power suffer from severe pathologies of behaviour, which in our view disqualify them from serving as reasonable measures of voting power. (For a discussion of the S-S index and of indices proposed by Deegan and Packel and by Johnston, see [1, Ch. 6]; for various paradoxes displayed by measures of voting power, including the severe pathologies of the two last mentioned, see [1, Ch. 7]. Another index, proposed by Holler [4], is easily seen to display the same pathologies as the Deegan–Packel index, to which it is closely related; except that unlike the latter it satisfies the Added Blocker Postulate proposed in [1, § 7.9].)

Section 2 presents some preliminary definitions. In Section 3, after addressing (from both viewpoints, P-power and I-power) the easy problem of a bloc formed by annexation, we turn to the problem of a voluntary bloc, from the viewpoint of P-power. This also turns out to be easy (assuming, of course, that we have a reasonable index of P-power): forming a bloc can be of advantage to all the prospective partners, iff the bloc's expected share in the fixed prize is greater than the sum of the shares that the partners expect to obtain when acting as separate individuals.

We deal with those easy matters mainly for the sake of contrast with the main issue of this paper, to which we turn in Section 4: the problem of a voluntary bloc from the viewpoint of I-power. The solution to this problem is far less obvious than those of Section 3, because—unlike the payoffs considered under the notion of P-power, rooted in cooperative game theory—influence is not an additive quantity. Forming a bloc can be of advantage to all the prospective partners iff each of them can obtain via the bloc greater indirect influence than s/he has directly, when they all act as separate individuals. But it is a fallacy to suppose that the influence of the bloc is in general equal to the sum of these indirect influences that the partners obtain via the bloc. In fact, these indirect influences depend crucially on the mechanism fixed by the partners for determining the voting behaviour of the bloc. This leads us to the notions of *alliance*, *expedient bloc* and *feasible bloc*. We present some general theoretical results concerning these notions, and a few simple illustrative examples. It turns out that a voluntary bloc may be expedient even if its voting power is smaller than the sum of the original powers of its members; and it may not be feasible even if its voting power is greater than that sum.

In Section 5 we apply the tools developed in the preceding section to the Council of Ministers of the European Community (CMEC).

The Appendix contains an alternative definition of the composite SVG introduced in Section 4, showing that it is indeed obtained by composition of SVGs. The Appendix also contains proofs of theorems stated in Section 4.

We do not provide definitions of concepts that we assume to be familiar to most readers of this journal; a reader who is in doubt about these is advised to consult [1].

2 Preliminaries

Here and in the next two sections, $\mathcal{W}$ is some arbitrary simple voting game (SVG); N is the assembly (set of all voters) of $\mathcal{W}$; and S is a coalition of $\mathcal{W}$ (in other words, $S \subseteq N$). To avoid trivialities, we assume that S has at least two members.

We let $\mathcal{W}|\&_S$ be the SVG that results from $\mathcal{W}$ when S fuses into a bloc. Informally, this means that the members of S now vote as a single body. Formally, the assembly of $\mathcal{W}|\&_S$ is $(N - S) \cup \{\&_S\}$, obtained from N by removing all the members of S and adding a new voter, $\&_S$, the *bloc of S*. We denote this assembly by '$N|\&_S$'. The winning coalitions of $\mathcal{W}|\&_S$ are all those $X \subseteq N - S$ such that X is winning in $\mathcal{W}$, as well as all $X \cup \{\&_S\}$ such that $X \subseteq N - S$ and $X \cup S$ is winning in $\mathcal{W}$. (Cf. [1, Def. 2.3.23].)

If $\mathcal{W}$ is a weighted voting game (WVG), then so is $\mathcal{W}|\&_S$: take the weight of $\&_S$ to be the sum of the weights that the members of S had in $\mathcal{W}$, while the weights of all other voters as well as the quota are kept the same as in $\mathcal{W}$.

If ξ is a measure of voting power, we denote by '$\xi_a[\mathcal{W}]$' the value that ξ assigns to voter a in $\mathcal{W}$. Following [1], we reserve the term *index of voting power* for a measure whose values for all voters of any SVG always add up to 1: so $\sum_{a \in N} \xi_a[\mathcal{W}] = 1$ for any $\mathcal{W}$. An index is thus a measure of *relative* voting power.

Note that all measures of P-power are in fact indices. This is because, by definition, the P-power of a voter is the expected *share* of that voter in a fixed prize, whose total value can always be fixed, by convention, as 1.

In contrast, when it comes to a priori I-power, the primary notion is the *absolute* amount of influence that a voter can exert on the outcome of a division; and the sum of these, for all voters of an SVG, cannot be taken as a fixed quantity, independent of the SVG. Indeed, the Bz measure β'—the only serious contender as a measure of a priori absolute I-power—is not an index in the strict sense used here.

The value $\beta'_a[\mathcal{W}]$—the [absolute] Bz power of voter a in $\mathcal{W}$—can be characterized probabilistically as follows. Suppose voters act independently of one another, each voting 'yes' or 'no' with equal probability of $\frac{1}{2}$. Then $\beta'_a[\mathcal{W}]$ is equal to the probability that the voters other than a are so divided that a is in a position to decide the outcome: by joining the 'yes' voters a will give rise to a winning coalition, so that the proposed bill will be passed; but if a joins the 'no' voters the bill will be defeated. (Cf. [1, Thm. 3.2.4].) The formula for $\beta'_a[\mathcal{W}]$ is

$$\beta'_a[\mathcal{W}] = \frac{\eta_a[\mathcal{W}]}{2^{n-1}},$$

where $\eta_a[\mathcal{W}]$ is the *Bz score* of a in $\mathcal{W}$—equal to the number of coalitions in which a is critical—and $n = |N|$ is the number of voters of $\mathcal{W}$.

The Bz *index* β, which can be used to measure *relative* a priori I-power, is obtained from β' by *normalization*:

$$\beta_a[\mathcal{W}] = \frac{\beta'_a[\mathcal{W}]}{\sum_{x \in N} \beta'_x[\mathcal{W}]} = \frac{\eta_a[\mathcal{W}]}{\sum_{x \in N} \eta_x[\mathcal{W}]}.$$

3 Annexations; voluntary blocs and P-power

Now let us suppose that the bloc $\&_S$ arises by annexation: a particular voter $a \in S$ takes over the voting mandates of all other members of S. Under what circumstances will this be of advantage to the annexer a?

First, let us approach this question from the viewpoint of P-power. (Here we are obviously assuming that the notion of P-power is coherent.) The annexation is a priori advantageous to a iff it gives a a greater expected share in the prize than a had originally. To formalize this condition is very easy—once we decide on a reasonable index of P-power, which may be quite a controversial matter. *Assuming* that ξ is such an index, the condition is expressed by the inequality

$$\xi_{\&_S}[\mathcal{W}|\&_S] > \xi_a[\mathcal{W}]. \tag{1}$$

Similarly, the condition that the annexation is a priori disadvantageous to a is expressed by the reverse inequality:

$$\xi_{\&_S}[\mathcal{W}|\&_S] < \xi_a[\mathcal{W}]. \tag{2}$$

If ξ satisfies (2) for some S and $a \in S$, then ξ is said to display thereby the *bloc paradox*.

Common sense suggests that annexing the voting mandates of other voters cannot possibly worsen the bargaining position of the annexer and will therefore never be disadvantageous. Anyone who, like us, finds this common-sense view compelling must consequently reject as invalid any purported index ξ of P-power that displays the bloc paradox.

Of all indices of a priori P-power known to us, the only one that does not suffer from the bloc paradox is the S-S index. In fact, this index always satisfies (1), provided S has at least one member, other than a, who is not a dummy. For a proof of this fact, and for instances in which the Deegan–Packel and Johnston indices display the bloc paradox see [1, pp. 256–7]. The index proposed by Holler [4] can also easily be shown to display the paradox. Anyone who, for some reason, prefers one of those other indices for measuring P-power must be prepared to live with the bloc paradox and accept the counter-intuitive consequence that in some cases annexation will reduce the a priori expected payoff of the annexer.

Now let us consider the same question from the viewpoint of I-power. Here again common sense suggests that annexing the voting mandates of other voters cannot diminish the a priori influence of the annexer; and if at least one of those other voters is not a dummy, this influence must actually increase. This common-sense view is vindicated by the behaviour of the Bz measure, the only serious contender for measuring absolute a priori I-power. To see this, it is enough to consider the case where S has just two members, a and one other voter, say b. In fact, we have

$$\beta'_{\&_{\{a,b\}}}[\mathcal{W}|\&_{\{a,b\}}] = \beta'_a[\mathcal{W}] + \beta'_b[\mathcal{W}]. \tag{3}$$

So in any case $\beta'_{\&_{\{a,b\}}}[\mathcal{W}|\&_{\{a,b\}}] \geq \beta'_a[\mathcal{W}]$; and if b is not a dummy in $\mathcal{W}$ then $\beta'_{\&_{\{a,b\}}}[\mathcal{W}|\&_{\{a,b\}}] > \beta'_a[\mathcal{W}]$. (For a proof, see [1, pp. 47–8].) Note that this strictly additive property of Bz power does not extend to three or more voters. This is because the Bz power of a third voter, say c, in $\mathcal{W}|\&_{\{a,b\}}$ need not be the same as in $\mathcal{W}$; for example, c may be a non-dummy in $\mathcal{W}$ and become a dummy in $\mathcal{W}|\&_{\{a,b\}}$. Still, if a annexes the voting mandates of several voters successively, one at a time, starting with one who is initially not a dummy, then in the first step the annexer's Bz power increases, and in subsequent steps it never decreases.

We must point out that, unlike the Bz measure, the Bz index does display the bloc paradox: for instances of this see [1, pp. 256–7]. However, this does not mean that annexation can reduce the annexer's I-power. The Bz index does not measure voters' absolute I-powers but their respective shares in the total I-power, which varies from one SVG to another. Cases in which the Bz index displays the bloc paradox occur where annexation, while increasing the influence of the annexer, also causes, as a by-product, a sufficiently great increase in the influence of other voters. That this can indeed happen may seem paradoxical; but is a fact all the same. (And this is one of the reasons why the Bz index cannot be used also to measure voters' a priori P-power.)

Now let us consider a bloc $\&_S$ formed voluntarily, by consent of all the members of S. Leaving the viewpoint of I-power to the next section, we adopt here the viewpoint of P-power. As is commonly done in cooperative game theory—in which the notion of P-power is rooted—we must assume that the payoffs received by voters who carry a vote-division to a successful outcome consist of quantities of transferable utility that behave in an additive way.

Since all members of S must consent to forming the bloc, we must now ask under what condition the bloc may be advantageous to all of them. Clearly, the answer is: iff the expected share of the bloc in the [fixed] prize is greater than the sum of the expected shares that the members of S receive when acting as separate individuals. Presumably, when forming the bloc the partners will agree to divide its payoff in such a way as to leave each of them better off than before. In the reverse case, where the expected share of the bloc is smaller than that sum, the bloc must be disadvantageous to at least one of the voters in S, and will therefore not be formed. (In the remaining case, when the two quantities happen to be equal, the bloc can at best leave all the prospective partners in the same position as before.) Again, formalizing these conditions is easy—leaving aside the controversial issue of selecting a reasonable index of P-power. If ξ is such an index, then the bloc is a priori advantageous iff

$$\xi_{\&_S}[\mathcal{W}|\&_S] > \sum_{x\in S} \xi_x[\mathcal{W}], \tag{4}$$

and a priori disadvantageous iff

$$\xi_{\&_S}[\mathcal{W}|\&_S] < \sum_{x\in S} \xi_x[\mathcal{W}]. \tag{5}$$

Cases where (5) holds are displayed by any half-way reasonable measure ξ of voting power. They have been dubbed *the paradox of large size*. But there is nothing genuinely paradoxical about them. It stands to reason that in some cases voters can achieve more (more payoff or, for that matter, more influence), and in some cases less, by acting as one body than they can achieve in total by acting separately. (For a more detailed discussion, see [1, § 7.2].)

4 Alliances and expedient blocs

Now let us consider the problem of voluntary blocs from the viewpoint of I-power. Clearly, the bloc $\&_S$ will be advantageous to all the members of S iff after forming $\&_S$ every one of them will be able to exercise more influence over the outcome of a division than s/he was able to exercise originally, in $\mathcal{W}$. So in formalizing the present problem we should use the Bz measure β' rather than the Bz index β. Using the latter makes little sense in the present context because, as we saw in Section 2, this index only measures a voter's *relative* I-power, which may wane even when the voter's absolute I-power waxes.

At first sight, it seems as though, in analogy with (4), the condition for the bloc being advantageous from the I-power viewpoint should be formalized as

$$\beta'_{\&_S}[\mathcal{W}|\&_S] > \sum_{x \in S} \beta'_x[\mathcal{W}]. \tag{6}$$

But this does not stand up to closer examination. For one thing, whereas the left-hand side of (4) represents the bloc's expected share of transferable utility, which can be directly portioned out among the partners, just as coffee can be portioned out from a jug into several cups, the left-hand side of (6) represents the influence of the bloc, quantified as probability. How is influence-as-probability to be portioned out?

And whereas the right-hand side of (4) can be regarded as the total expected share of S in the prize in $\mathcal{W}$, when its members act as separate individuals, the right-hand side of (6) does not have an analogous meaning. The terms of this sum are probabilities, and in general they are probabilities of events that are not disjoint from one another. So the sum does not represent anything like the 'total influence of S in $\mathcal{W}$'—a concept that is in fact rather meaningless. The apparent analogy between the problems of forming a bloc from the viewpoint of P-power and that of I-power is a false one.

Nevertheless, it is intuitively clear that, from the latter viewpoint, if the bloc $\&_S$ is to be advantageous to all the partners, then each of them, each $a \in S$, must expect to obtain via $\&_S$ greater influence over the outcome of a division than s/he had individually. How can a obtain influence 'via $\&_S$' over the outcome? *Surely, the only way is for a to influence the way* $\&_S$ *votes in a division of the assembly* $N|\&_S$ *of* $\mathcal{W}|\&_S$.

We reach a similar conclusion by approaching the problem from a somewhat different direction. The notion of I-power presupposes policy-seeking voting behaviour, whereby each voter votes on any given bill according to his or her own interests, which are totally exogenous to the SVG. But what might the 'interests' of the bloc $\&_S$ be? What meaning can be ascribed to this concept? Let us take an example that is at least potentially realistic. In [7], Lane and Mæland consider a scenario in which four members of the CMEC—Italy, Spain, Greece and Portugal—form a *Mediterranean bloc*. (Here 'Mediterranean' is obviously used as a geo-political rather than strictly geographical term: Portugal does not have a Mediterranean shore, whereas France does.) This scenario implies that the four representatives of the *Medbloc* (as we may dub it) will always vote in the same way. They may even delegate their mandates to a single Medbloc representative. But Lane and Mæland hardly mean to suggest that, even under this hypothetical scenario, the four member-states would merge completely and cease to exist as separate countries, or that the interests of Italy on every single issue that may ever come before the CMEC will always coincide with those of Portugal. Surely, when forming the bloc the prospective partners must come to some binding agreement as to how to instruct the Medbloc delegate(s) to vote on any given bill.

These considerations suggest that in order to analyse the problem of bloc formation from the present viewpoint, that of I-power, we ought to postulate that when a bloc $\&_S$ is formed, the partners also fix a particular SVG $\mathcal{W}_S$, whose assembly is S. The job of this *internal* SVG is to decide, for each bill that comes before the 'top' SVG $\mathcal{W}|\&_S$, how the bloc $\&_S$ (or its delegate) will vote in $\mathcal{W}|\&_S$.

We shall call such a structure—a bloc $\&_S$ together with an internal SVG $\mathcal{W}_S$—an *alliance*.

(Note, by the way, that from the rival viewpoint, that of P-power, an internal SVG $\mathcal{W}_S$ was not needed. The notion of P-power presupposes office-seeking voting behaviour, whereby each voter bargains with other voters, striving to reach an agreement that will maximize his or her payoff. In forming the bloc $\&_S$, the partners must agree how to split its payoff between them; but they need not agree as to how the delegate of $\&_S$ should act. This delegate will turn for instructions not to the partners, but to experts in the theory of bargaining and cooperative game theory, hoping to receive from them advice—for what it's worth—as to what optimal bargaining and voting strategy s/he ought to use.)

When the members of S form an alliance whose internal SVG is $\mathcal{W}_S$, this gives rise to a new *composite* SVG, which we shall denote by '$\mathcal{W}\|\mathcal{W}_S$'. This is in fact a special case of the general operation of composition of SVGs. In the Appendix (Subsection 6.1) we present a rigorous definition of $\mathcal{W}\|\mathcal{W}_S$ in that format. Here we shall just define $\mathcal{W}\|\mathcal{W}_S$ directly, in its own terms.

The assembly of $\mathcal{W}\|\mathcal{W}_S$ is N, the same as that of $\mathcal{W}$. The winning coalitions of $\mathcal{W}\|\mathcal{W}_S$ are all sets of the form $X \cup Y$, with $X \subseteq S$ and $Y \subseteq N - S$, satisfying at least one of the following two conditions:

- Y is a winning coalition of $\mathcal{W}$;
- X is a winning coalition of $\mathcal{W}_S$ and $S \cup Y$ is a winning coalition of $\mathcal{W}$.

Informally speaking, $\mathcal{W}\|\mathcal{W}_S$ works as follows. When a bill is proposed, the members of S decide about it using $\mathcal{W}_S$, the internal SVG of their alliance. Then, when the bill is brought before the plenary, the assembly of $\mathcal{W}$, all the members of S vote as a bloc, in accordance with their internal decision; so that now the final outcome is the same as it would have been in $\mathcal{W}|\&_S$ with the bloc voter $\&_S$ voting according to the internal decision.

Note that each member of S now has *direct* I-power in the SVG $\mathcal{W}_S$, as well as *indirect* I-power in $\mathcal{W}\|\mathcal{W}_S$, which s/he exercises via the bloc $\&_S$.

Clearly, when the members of S consider forming an alliance, they are well advised to compare their prospective indirect I-powers with the I-powers they have in the original SVG $\mathcal{W}$. We shall therefore say that an alliance with internal SVG $\mathcal{W}_S$ is *feasible* [*relative to a given SVG* $\mathcal{W}$] if

$$\beta'_a[\mathcal{W}\|\mathcal{W}_S] \geq \beta'_a[\mathcal{W}] \quad \text{for all } a \in S; \tag{7}$$

and we shall say that the alliance is *expedient* [*relative to a given SVG* $\mathcal{W}$] if

$$\beta'_a[\mathcal{W}\|\mathcal{W}_S] > \beta'_a[\mathcal{W}] \quad \text{for all } a \in S. \tag{8}$$

Moreover, we shall say that a bloc is *feasible* or *expedient* [*relative to a given SVG* $\mathcal{W}$] if there exists some internal SVG such that the resulting alliance is feasible or expedient, respectively.

We shall soon illustrate these concepts with some simple toy examples, and in Section 5 we shall present some potentially realistic examples relating to the CMEC. But first we state some general theorems, whose proofs are given in the Appendix.

Theorem 4.1 *For every $a \in S$*

$$\beta'_a[\mathcal{W}\|\mathcal{W}_S] = \beta'_a[\mathcal{W}_S] \cdot \beta'_{\&_S}[\mathcal{W}|\&_S].$$

Thus, to obtain the indirect Bz power of a in $\mathcal{W}\|\mathcal{W}_S$, multiply the direct Bz power of a in $\mathcal{W}_S$ by the Bz power of the bloc $\&_S$ in $\mathcal{W}|\&_S$.

As for the Bz powers of voters $b \in N - S$ in $\mathcal{W}\|\mathcal{W}_S$, it is tempting to jump to the conclusion that they are the same as in $\mathcal{W}|\&_S$. But this is not generally true. The reason for this is that in the probabilistic characterization of $\beta'_b[\mathcal{W}|\&_S]$ it is assumed a priori that the bloc voter $\&_S$ votes 'yes' or 'no' with equal probability of $\frac{1}{2}$ (see Section 2). But in $\mathcal{W}\|\mathcal{W}_S$ the members of S, although they vote 'as a bloc', do not in general vote 'yes' or 'no' with equal a priori probability of $\frac{1}{2}$. They do so only in the special case where the number of winning coalitions of the internal SVG $\mathcal{W}_S$ is $2^{|S|-1}$, exactly half of the number of all coalitions. In this special case the Bz powers of voters $b \in N - S$ in $\mathcal{W}\|\mathcal{W}_S$ are indeed the same as in $\mathcal{W}|\&_S$.

Theorem 4.2 *A bloc made up of two voters is never expedient. It is feasible iff originally the two voters have equal Bz powers, or at least one of them is a dummy.*

Theorem 4.3 *Let a, b and c be distinct voters of $\mathcal{W}$ such that $\beta'_a[\mathcal{W}] = \beta'_b[\mathcal{W}] \geq \beta'_c[\mathcal{W}]$. Then the bloc $\&_{\{a,b,c\}}$ is feasible. This bloc is expedient iff c is not a dummy in $\mathcal{W}|\&_{\{a,b\}}$.*

Now for some toy examples. The details of the calculations are easy and are left to the reader.

Example 4.1 Let $\mathcal{W}$ be the majority WVG with assembly $\{a, b, c, d, e, f\}$; thus

$$\mathcal{W} \cong [4; 1, 1, 1, 1, 1, 1].$$

That is, each voter has weight 1, and the quota is 4. Here the Bz power of each voter is $\frac{5}{16}$.

Now suppose that the first three voters form a bloc $\&_{\{a,b,c\}}$. We get a new WVG,

$$\mathcal{W}|\&_{\{a,b,c\}} \cong [4; 3, 1, 1, 1].$$

Here the bloc voter has Bz power $\frac{7}{8}$ and each of the remaining ones has $\frac{1}{8}$. Note that the Bz power of the bloc is *smaller* than the sum of the original Bz powers of the three partners. An observer who believes that I-power behaves like transferable utility might conclude that the bloc cannot be advantageous to all three partners. But this is an error. Put

$$\mathcal{W}_{\{a,b,c\}} \cong [2; 1, 1, 1].$$

In this internal WVG the [direct] Bz power of each partner is $\frac{1}{2}$, which by Theorem 4.1 gives each of them [indirect] Bz power $\frac{7}{16}$ in the composite SVG $\mathcal{W}\|\mathcal{W}_{\{a,b,c\}}$. Thus each partner has gained absolute power, and the bloc is expedient—just as Theorem 4.3 says.

The Bz power in $\mathcal{W}\|\mathcal{W}_{\{a,b,c\}}$ of each of the partners can also be calculated directly, without using Theorem 4.1. From the definition of this composite SVG it follows that its winning coalitions are those containing at least two of the voters a, b, c and at least one of the remaining voters d, e, f. From this it is easy to see that the Bz power of each of the voters a, b, c is indeed $\frac{7}{16}$.

Also, the Bz power in $\mathcal{W}\|\mathcal{W}_{\{a,b,c\}}$ of each of the remaining voters is $\frac{1}{8}$, which, as it happens, is the same as in $\mathcal{W}|\&_{\{a,b,c\}}$. This is because in $\mathcal{W}_{\{a,b,c\}}$ exactly half of the coalitions are winning.

On the other hand, if we were to choose

$$\mathcal{W}_{\{a,b,c\}} \cong [3; 1, 1, 1],$$

so that the internal decisions of the bloc are taken by the unanimity rule, then the direct Bz power of each partner would be $\frac{1}{4}$. This would give each of them indirect Bz power $\frac{7}{32}$ in the composite $\mathcal{W}\|\mathcal{W}_{\{a,b,c\}}$, making such an alliance infeasible.

Note also that in this case each of the voters d, e, f would have Bz power $\frac{1}{32}$ in the composite $\mathcal{W}\|\mathcal{W}_{\{a,b,c\}}$, which is much less than they have in $\mathcal{W}|\&_{\{a,b,c\}}$.

Example 4.2 Let $\mathcal{W}$ be the WVG with assembly $\{a, b, c, d\}$ such that

$$\mathcal{W} \cong [4; 2, 1, 1, 1],$$

in alphabetical order. Here a has Bz power $\frac{1}{2}$, and each of the other three has $\frac{1}{4}$. The values of the Bz index are therefore $\frac{2}{5}$ and $\frac{1}{5}$ respectively.

Let b, c, d form a bloc. Then in $\mathcal{W}|\&_{\{b,c,d\}}$ the bloc voter $\&_{\{b,c,d\}}$ has Bz power of $\frac{1}{2}$, which is less than the sum of the Bz powers that the three partners had in $\mathcal{W}$. Moreover, the Bz *index* of $\&_{\{b,c,d\}}$ in $\mathcal{W}|\&_{\{b,c,d\}}$ is also $\frac{1}{2}$, which is less than the sum of the values of the Bz index of the three partners in $\mathcal{W}$. So on the face of it forming the bloc seems to cause a loss of I-power, both absolutely and relatively. But this is not the case.

If two of b, c, d form a bloc, the third becomes a dummy; so we know from Theorem 4.3 that the three cannot form an expedient alliance, but they can form a feasible one. Indeed, if we put

$$\mathcal{W}_{\{b,c,d\}} \cong [2; 1, 1, 1],$$

then the direct Bz power of each partner in $\mathcal{W}_{\{b,c,d\}}$ is $\frac{1}{2}$, giving each of them indirect Bz power $\frac{1}{4}$ in the composite $\mathcal{W}\|\mathcal{W}_{\{b,c,d\}}$; so they do not lose power.

Example 4.3 Let $\mathcal{W}$ be the WVG with assembly $\{a, b, c, d, e, f, g\}$ such that

$$\mathcal{W} \cong [6; 2, 1, 1, 1, 1, 1, 1],$$

in alphabetical order. Here each of the lighter voters (those with weight 1) has Bz power $\frac{11}{64}$. If four of them, say b, c, d, e, form a bloc, we get:

$$\mathcal{W}|\&_{\{b,c,d,e\}} \cong [6; 2, 4, 1, 1],$$

in which the new bloc voter $\&_{\{b,c,d,e\}}$ has Bz power $\frac{5}{8}$. This is less than the sum of the original Bz powers of the four partners. So again at first glance they seem to have lost power. However, if they choose

$$\mathcal{W}_{\{b,c,d,e\}} \cong [3; 1, 1, 1, 1]$$

as their internal WVG, then each will have direct Bz power $\frac{3}{8}$ in this WVG, hence indirect Bz power $\frac{15}{64}$ in the composite $\mathcal{W}\|\mathcal{W}_{\{b,c,d,e\}}$; so they actually gain power.

Our final example in this section displays a phenomenon opposite to that of Examples 4.1 and 4.3.

Example 4.4 Let $\mathcal{W}$ be the WVG with assembly $\{a, b, c, d, e, f, g\}$ such that

$$\mathcal{W} \cong [11; 6, 5, 1, 1, 1, 1, 1],$$

in alphabetical order. Here the heaviest voter, a, has Bz power $\frac{33}{64}$ and each of the voters with weight 1 has Bz power $\frac{1}{64}$. Now let a form a bloc with c and d. Then

$$\mathcal{W}|\&_{\{a,c,d\}} \cong [11; 8, 5, 1, 1, 1],$$

in which the new bloc voter $\&_{\{a,c,d\}}$ has Bz power $\frac{9}{16} = \frac{36}{64}$. This is greater than $\frac{35}{64}$, the sum of the original Bz powers of the partners. Nevertheless, the bloc is infeasible: any internal SVG will either make c and d dummies, or give a direct Bz power $\leq \frac{3}{4}$, hence indirect Bz power $\leq \frac{27}{64}$.

5 Expedient blocs in the CMEC

Our interest in the topic of this paper was aroused by a critical comment of Garrett and Tsebelis [3] on Lane and Mæland [7]. In [7], Lane and Mæland use the Bz index to investigate, among other things, the power distribution in the CMEC, with its so-called *Qualified Majority Voting* decision rule, under various scenarios of bloc formation.

The first scenario they consider is the formation of a Medbloc, consisting of Italy, Spain, Portugal and Greece. They do not raise the question as to whether the formation of this bloc (or indeed any of the other blocs they consider) would be advantageous. Their critics—who are vehemently opposed in general to the application of power indices to the European Union—rebuke them for this.

> According to Lane and Mæland, pooling the Mediterranean governments' votes would lead to a reduction in their combined power. If each voted separately in a 15-member Council, their combined power (using the Banzhaf normalized index) would be $0.112 + 0.092 + 0.059 + 0.059 = 0.332$ [*sic*] … . Voting as a bloc, however, their index would be reduced to 0.247 … . One should immediately ask the question: Why would these governments ever choose to vote as a bloc if in so doing they lose power? [3, p. 296]

Apart from the slight arithmetical or typographical error—0.332 instead of 0.322—this critique contains two fallacies.

First, the figures quoted—as all the figures in [7]—are those for the [relative] Bz index rather than the [absolute] Bz measure. (This may be justified, since the issue studied there is potential changes in the *distribution* of voting power.) However, as we argued in the beginning of Section 4, it makes no sense to use the Bz index when enquiring whether a bloc is advantageous. Surely, when considering the formation of a Medbloc, the four potential partners are primarily interested in the consequent changes in their *absolute* I-power.

Second, while the figures given in [7] are those for [relative] I-power, measured by the Bz index, the criticism in [3] treats them as though they were values of an index of P-power, and applies the criterion of our inequality (5) in Section 3 to imply that the Medbloc would be disadvantageous. This is another fallacy against which we warned in the beginning of Section 4.

Actually, as we shall see in a moment (Example 5.1), the [absolute] Bz power of the Medbloc would be greater than the sum of the present Bz powers of the four partners. But, as Example 4.4 shows, this in itself does not guarantee that the Medbloc is expedient or even feasible: it all depends on whether the four partners can find a suitable internal SVG. (See also Example 5.3 below.) On the other hand, as we saw in Examples 4.1 and 4.3, a bloc can be expedient even if its Bz power is smaller than the sum of the original Bz powers of the partners.

Before proceeding to look at the Medbloc and other potential blocs in the CMEC, we would like to point out what we regard as a real error in [7]. When comparing the power distributions in the CMEC before and after the formation of a bloc $\&_S$, the authors present the values of the Bz index for the non-members of S in the SVG $\mathcal{W}|\&_S$ as though they were the relative voting powers of these voters in the CMEC after the formation of the bloc. And they compare these values with those for the same voters in $\mathcal{W}$. In our opinion this is mistaken, as it does not compare like with like. In computing the Bz index in $\mathcal{W}$ one implicitly makes the a priori assumption that its voters—including the members of S—vote 'yes' or 'no' with equal probability of $\frac{1}{2}$. Similarly, in computing the Bz index in $\mathcal{W}|\&_S$ one likewise assumes a priori that the voters in this SVG—including the bloc voter $\&_S$—vote 'yes' or 'no' with equal probability of $\frac{1}{2}$. But these two assumptions are not in general compatible. As we explained in Section 4, if the members of S vote in the way just described, then $\&_S$ will not do so, unless the internal SVG $\mathcal{W}_S$ happens to be such that exactly half of its coalitions are winning. (In the case of the Medbloc this may be ruled out, because the only SVGs with four voters having the required property are improper: there is a winning coalition whose complement is also winning.) For this reason, the values of the Bz index in $\mathcal{W}$ should be compared with those in the composite $\mathcal{W}\|\mathcal{W}_S$, not with those in $\mathcal{W}|\&_S$. Of course, this presupposes a particular choice of $\mathcal{W}_S$, the internal rule used by the bloc to determine how it should use its bloc vote—an issue not raised in [7].

Let us now look at a few examples of bloc formation in the CMEC. In what follows, $\mathcal{W}$ is the present CMEC under its weighted voting rule, known as 'Qualified Majority Voting' (QMV). The present weights, quota and Bz scores (η) of the members are given in Table 1. (Because the number of members is 15, the Bz powers (β') of the members are obtained by dividing their respective scores by 2^{14}.) The values of $\beta'_{\&_S}[\mathcal{W}|\&_S]$ for each bloc $\&_S$ have been newly calculated by us.

$\boxed{\text{Place Table 1 about here}}$

Table 1: QMV weights and Bz scores

Member state	Weight	η
Austria	4	793
Belgium	5	973
Britain	10	1849
Denmark	3	595
Finland	3	595
France	10	1849
Germany	10	1849
Greece	5	973
Ireland	3	595
Italy	10	1849
Luxembourg	2	375
Netherlands	5	973
Portugal	5	973
Spain	8	1531
Sweden	4	793
Total	87	16565

Quota: In order to pass, a proposed resolution must be supported by member-states whose weights add up to at least 62 (71.26% of the total).

Example 5.1 (The Medbloc) At present, the Bz powers of the four prospective partners are

Italy	0.1129
Spain	0.0934
Portugal	0.0594
Greece	0.0594
Total	0.3251

If these four members were to form a bloc $\&_S$, its Bz power in $\mathcal{W}|\&_S$ would be 0.3555. While this is greater than the sum of the original Bz powers of the four members, it does not yet guarantee that the bloc is expedient. However, let us choose

$$\mathcal{W}_S \cong [4; 2, 2, 1, 1],$$

in the order of the members as listed above. In this internal WVG, Italy and Spain have Bz power $\frac{1}{2}$, and Portugal and Greece have $\frac{1}{4}$. By Theorem 4.1, this would give the four partners the following Bz powers in the resulting composite SVG

Italy	0.1777
Spain	0.1777
Portugal	0.0889
Greece	0.0889

Thus each partner would gain Bz power, making the alliance expedient.

Another internal WVG that would do the trick is the majority rule

$$\mathcal{W}_S \cong [3; 1, 1, 1, 1].$$

Each partner has Bz power $\frac{3}{8}$ in this internal WVG, and consequently Bz power 0.1333 in the resulting composite SVG.

Which of these two suitable internal decision rules would they choose? Italy and Spain would obviously prefer the first one, and the other two members the second. They could of course alternate, choosing one rule or the other by tossing a coin or by some deterministic method.

Example 5.2 (The Nordbloc) Another scenario considered in [7] is the formation of what may be dubbed a *Nordbloc*, consisting of the three Nordic members: Sweden, Denmark and Finland. Their present Bz powers are

Sweden	0.0484
Denmark	0.0363
Finland	0.0363
Total	0.1210

If these three members were to form a bloc $\&_S$, its Bz power in $\mathcal{W}|\&_S$ would be 0.1221. If they choose

$$\mathcal{W}_S \cong [2; 1, 1, 1],$$

then each of them will have direct Bz power $\frac{1}{2}$ in this internal WVG, yielding Bz power 0.0610 in the composite SVG, thus making an expedient alliance. In this case, no other proper SVG will serve as the internal SVG.

We have checked several other hypothetical blocs in the present CMEC that figure in the scenarios of [7]: the Nordbloc plus The Netherlands; the Nordbloc plus The Netherlands and Belgium; the Nordbloc plus all three Benelux countries; the Nordbloc plus Benelux and Austria; the Nordbloc plus Benelux, Austria and Germany. All these blocs turn out to be expedient.

In the following example, which goes the other way, we consider a bloc that is not envisaged in [7].

Example 5.3 (Deunelux) A hypothetical bloc $\&_S$ formed by Germany, The Netherlands and Luxembourg may be dubbed *Deunelux*. Their present Bz powers are

Germany	0.1129
Netherlands	0.0594
Luxembourg	0.0229
Total	0.1951

(The apparent error in the total is due to rounding.) In the resulting $\mathcal{W}|\&_S$, the power of the bloc voter $\&_S$ would be 0.1975. This is greater than the sum of the partners' original Bz powers, so on the face of it the bloc seems to be advantageous. However, in order to produce a feasible alliance, the internal SVG $\mathcal{W}_S$ must give Germany direct Bz power $> \frac{1}{2}$, The Netherlands $> \frac{1}{4}$ and Luxembourg > 0. But as the reader can verify, no SVG with three voters can do this; so the bloc is infeasible.

6 Appendix

6.1 The composite $\mathcal{W}\|\mathcal{W}_S$

For a general definition of a composite SVG $\mathcal{V}[\mathcal{W}_1, \mathcal{W}_2, \ldots, \mathcal{W}_m]$, the reader is referred to [1, p. 27]. Note that according to that definition, $\mathcal{V}$ must be a *canonical* SVG, whose assembly is $I_m = \{1, 2, \ldots, m\}$, and $\mathcal{W}_1, \mathcal{W}_2, \ldots, \mathcal{W}_m$ may be any SVGs.

Now let $m = |N - S| + 1$; thus, $m - 1$ is the number of voters of $\mathcal{W}$ who do not belong to S and m is exactly the number of voters of $\mathcal{W}|\&_S$. Let $a_2, \ldots, a_m$ be an enumeration of all the members of $N - S$.

Next, let $\mathcal{V}$ be an SVG with assembly I_m, which is isomorphic to $\mathcal{W}|\&_S$ under the mapping f such that $f(\&_S) = 1$ and $f(a_i) = i$ for $i = 2, \ldots, m$.

Put $\mathcal{W}_1 := \mathcal{W}_S$; so $\mathcal{W}_1$ is the internal SVG of the alliance. Finally, for each $i = 2, \ldots, m$ let $\mathcal{W}_i$ be the SVG whose sole voter is a_i. Then

$$\mathcal{W}\|\mathcal{W}_S := \mathcal{V}[\mathcal{W}_1, \mathcal{W}_2, \ldots, \mathcal{W}_m].$$

6.2 Proof of Theorem 4.1

$\beta'_a[\mathcal{W}\|\mathcal{W}_S]$ is equal to $\mathrm{P}(\mathbf{A})$, the a priori probability of the event $\mathbf{A}$ that a is in a position to decide the outcome in $\mathcal{W}\|\mathcal{W}_S$. This event is the conjunction of two events, $\mathbf{B}$ and $\mathbf{C}$, where $\mathbf{B}$ is the event that a is in a position to decide the outcome in the internal SVG $\mathcal{W}_S$; and $\mathbf{C}$ is the event that the members of S, voting together as a bloc, are in a position to decide the outcome in $\mathcal{W}\|\mathcal{W}_S$.

The events $\mathbf{B}$ and $\mathbf{C}$ are a priori independent, because $\mathbf{B}$ depends only on how members of S vote in the internal SVG, and $\mathbf{C}$ depends only on how members of $N - S$ vote. Hence $\beta'_a[\mathcal{W}\|\mathcal{W}_S] = \mathrm{P}(\mathbf{B}) \cdot \mathrm{P}(\mathbf{C})$.

Now, by the definition of $\mathbf{B}$ we have $\mathrm{P}(\mathbf{B}) = \beta'_a[\mathcal{W}_S]$. To obtain $\mathrm{P}(\mathbf{C})$ note that the probabilistic characterization of $\beta'_{\&_S}[\mathcal{W}|\&_S]$ does not actually depend on the probability with which the bloc $\&_S$ itself votes 'yes' or 'no': all that really matters is that all *other* voters of $\mathcal{W}|\&_S$ act independently of one another, each voting 'yes' or 'no' with equal probability of $\frac{1}{2}$, which they are assumed to do in $\mathcal{W}\|\mathcal{W}_S$ as well as in $\mathcal{W}|\&_S$. This implies that $\mathrm{P}(\mathbf{C}) = \beta'_{\&_S}[\mathcal{W}|\&_S]$, which completes the proof.

6.3 Proof of Theorem 4.2

Let $S = \{a, b\}$, where a and b are distinct voters of $\mathcal{W}$. By equation (3) in Section 3,

$$\beta'_{\&_S}[\mathcal{W}|\&_{\{a,b\}}] = \beta'_a[\mathcal{W}] + \beta'_b[\mathcal{W}].$$

Hence by Theorem 4.1 in order for the bloc to be expedient, there must exist $\mathcal{W}_S$ such that

$$\begin{aligned} \beta'_a[\mathcal{W}_S](\beta'_a[\mathcal{W}] + \beta'_b[\mathcal{W}]) &> \beta'_a[\mathcal{W}], \\ \beta'_b[\mathcal{W}_S](\beta'_a[\mathcal{W}] + \beta'_b[\mathcal{W}]) &> \beta'_b[\mathcal{W}]. \end{aligned} \tag{9}$$

Since $\mathcal{W}_S$ has just two voters, either both $\beta'_a[\mathcal{W}_S]$ and $\beta'_b[\mathcal{W}_S]$ are equal to $\frac{1}{2}$, or else one of these values is 1 and the other 0. But it is easy to see that (9) is not satisfied in any of these cases.

In order for an alliance based on this bloc to be feasible, it is necessary and sufficient that $\mathcal{W}_S$ is such that

$$\begin{aligned} \beta'_a[\mathcal{W}_S](\beta'_a[\mathcal{W}] + \beta'_b[\mathcal{W}]) &\geq \beta'_a[\mathcal{W}], \\ \beta'_b[\mathcal{W}_S](\beta'_a[\mathcal{W}] + \beta'_b[\mathcal{W}]) &\geq \beta'_b[\mathcal{W}]. \end{aligned} \tag{10}$$

It is possible to choose such a $\mathcal{W}_S$ iff $\beta'_a[\mathcal{W}] = \beta'_b[\mathcal{W}]$ (choose the unanimity SVG or its dual) or at least one of the values $\beta'_a[\mathcal{W}]$ and $\beta'_b[\mathcal{W}]$ is 0 (choose a dictatorial SVG).

6.4 Proof of Theorem 4.3

Let $r := \beta'_a[\mathcal{W}] = \beta'_b[\mathcal{W}]$. Then by equation (3) in Section 3,

$$\beta'_{\&_{\{a,b\}}}[\mathcal{W}|\&_{\{a,b\}}] = 2r.$$

Let $s := \beta'_c[\mathcal{W}|\&_{\{a,b\}}]$. Then $s = 0$ iff c is a dummy in $\mathcal{W}|\&_{\{a,b\}}$; otherwise $s > 0$.

Now let c join the bloc. Using equation (3) once more, we have

$$\beta'_{\&_{\{a,b,c\}}}[\mathcal{W}|\&_{\{a,b,c\}}] = 2r + s.$$

Let us choose

$$\mathcal{W}_S \cong [2; 1, 1, 1].$$

In this WVG, the Bz power of each of the three voters is $\frac{1}{2}$. Therefore by Theorem 4.1

$$\beta'_x[\mathcal{W} \| \mathcal{W}_S] = r + \frac{s}{2} \geq \beta'_x[\mathcal{W}] \quad \text{for } x = a, b, c. \tag{11}$$

Thus the resulting alliance is feasible. Moreover, if c is a not dummy in $\mathcal{W}|\&_{\{a,b\}}$ then the inequalities in (11) are sharp, so the alliance is expedient.

On the other hand, if c is a dummy in $\mathcal{W}|\&_{\{a,b\}}$ then

$$\beta'_{\&_{\{a,b,c\}}}[\mathcal{W}|\&_{\{a,b,c\}}] = 2r,$$

and it is easy to check that no choice of $\mathcal{W}_S$ can produce an expedient alliance: there is no SVG with three voters in which two voters have Bz powers $> \frac{1}{2}$.

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