Soc Choice Welfare (2004) 22: 17–28
DOI: 10.1007/s00355-003-0274-2

Social Choice and Welfare

A new approach to rights in social choice theory which incorporates utilitarianism

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Abstract. The paper axiomatises a generalised utilitarian aggregation rule, under which different weights are assigned to utilities depending on the different rights involved. The relationship between actions, rights and the evaluation of utilities is investigated. Application is made to a famous example, Edwin-Angelina-the Judge, which appears in the social choice literature.

1 Introduction

'Social welfare' is a concept increasingly seen by economists as essential to a proper evaluation of economic policies. It is, however, a rather elusive concept. How does one arrive at an idea of what is meant by social welfare? We lack a clear idea even of what is meant by 'individual welfare' (or 'individual well-being'); yet we seek to arrive at a concept of social welfare by a process of transference of ideas from the individual to the group. 'Welfarism' is the branch of social choice theory which seeks to deal with this difficult problem, 'utilitarianism' the particular philosophical approach which proposes that we measure social welfare by summing over individuals. (For a discussion of utilitarianism, see Smart and Williams 1973; Griffin 1986; Goodin 1995; Scarré 1996). In accordance with the tradition from which utilitarianism gets its name, we will refer to individual welfare as ''utility''.

More recently, the focus in social choice theory has been on the role of rights, but just as the earlier welfaristic literature neglected rights, so the more recent literature on rights has neglected utility. Following Sen 1970a and 1970b, rights in this non-welfaristic literature are viewed as constraints on social choice. The implication is that without these constraints social welfare could be higher - rights in this context have a ''nuisance'' value as far as social

welfare is concerned. The interesting subsequent debate has concerned whether rights are best portrayed by a social choice rule as in Sen 1970a and 1970b, or in terms of freedom of, or restrictions on, individual behaviour (''positive'' and ''negative'' rights) in the framework of a game form. (See Bernholz 1974; Gibbard 1974; Nozick 1974; Gaertner et al. 1992). Within this framework, Gardenfors 1981 and Peleg 1998 have extended the exercising of, and respect for, rights to coalitions (and so to the distribution of power). Under some conditions, the social choice rule and game form approaches are equivalent (Hammond 1982, 1997).

Rights can either be viewed as legal rights or as exemplifying a moral code. Unlike the social choice literature, moral philosophers have for long been concerned with theories of rights, in particular the question, ''What is a right?''. In this vein, Hohfeld 1919 develops a theory of rights as 'claims'. In recent years there has been an upsurge in the philosophical literature, Almond 1991 distinguishing 'claims', 'powers', 'liberties' and 'immunities' as different kinds of right. However, rights may also involve 'duties' or 'obligations', and the idea that rights are rather complex is developed in Hart (1976), Sumner (1987), and elsewhere. Here, rights are embodied in 'actions', actions are the source of utility (see Sect. 3), and our aim is to examine how society might evaluate this utility. We do not for this purpose distinguish different kinds of right.

It seems that absorption in the debate (over whether social choice rules or game forms are best suited to portray rights) has led to the neglect of important weaknesses inherent in the non-welfaristic approach. First, it does not permit trade-offs which would allow respect for rights to be balanced against benefits which are thereby lost. To take an example, suppose the parents of two children can afford to educate only one child. Traditionally, this will be the eldest. Thus suppose, for the sake of argument, that both children have a right to education, but the older (in this traditional culture) has the stronger right. However, the younger, let us suppose, is the more intelligent and educating the younger will give far higher returns in terms of future income, etc. In this case, the benefits education will give the younger child may outweigh those it will give the older, even when account is taken of their respective rights. The corollary is that, without such trade-offs, rights must be treated in a hierarchical fashion should they conflict. A second weakness of the non-welfaristic approach is that treating rights as constraints is no help in making a comparative evaluation of two situations in both of which rights have been violated.

Our concern then is to try to combine the welfaristic and non-welfaristic approaches. As in traditional utilitarianism and its modern extensions (see, for example, the work on population ethics by Blackorby et al. 1995), we assume that individual welfare can in a strong sense be measured. Thus we are opposed to the position that utility should be treated, as it is in much of the literature, as necessarily constrained to be ordinal. An amusing rejection of the ordinal view, attributed there to the subject's ''high priests'', is Robertson 1952, and of interest also is Harsanyi 1955. We do, however, accept that the concept of utility is essentially fuzzy, and that an interesting extension of our results would be via application of the techniques of fuzzy set theory.

In its early days, utilitarianism was propounded as a guide to practical decision making, the aim being "the greatest happiness" (Goodin 1995). Modern utilitarianism diminishes this "hedonism" aspect, and also pays attention to the place of rules in dealing with uncertainty and co-ordinating behaviour, while maintaining a 'consequentialist' viewpoint. Harsanyi 1955 emphasises the role of empathy in making 'interpersonal comparisons of utility'. Indeed, such comparisons, even if rough and ready, create no philosophical problem, as their rejection can be dismissed as a form of 'solipsism' (Goodin 1995). (This is the belief that we cannot be sure of the existence of other minds.) Thus, for philosophers, the serious attack on utilitarianism has come in the form of a series of counter-examples, the aim of which has been to show that utilitarianism can lead to ridiculous conclusions, and which seem always to involve its neglect of rights. A colourful example is:

> "Utilitarianism might recommend feeding of Christians to lions, if it so happens that the utilities of spectators enjoying the show (plus that of the lions enjoying the meal) exceed the disutilities of the Christians being sacrificed". (Goodin 1995, p. 22)

Thus the way forward would seem to be the extension of utilitarianism to incorporate rights. To give an example, suppose a society consist of two individuals, a and b. Total income is x, derived solely from a's effort and not at all from b's effort, so that a has a property right to all of x and b to none of it. To quote Locke:

> "...every Man has a *Property* in his own *Person*....The *Labour* of his Body, and the *Work* of his Hands, are properly his." (Locke 1690, p. 305–306)

Each individual, however, has a basic right to 4 units of income. Let a and b receive the income allocations x_a and x_b, and let their utility functions be $u_a = x_a^{1/2}$ and $u_b = x_b^{1/2}$. Suppose that the weights attached to utilities, according to the rights which govern these utilities, are:

Basic and property right	1
Basic right	3/4
Property right	1/2
No right	1/4

Social welfare is represented by the weighted sum of individual utilities with the given weights. Thus, for up to 4 units of income, a's utility is weighted by 1 and b's utility by 3/4, but for additional units the respective weights are 1/2 and 1/4. It is easy to calculate that, for the values of x given in the table below, the social optimum is, correspondingly:

x	x_a	x_b
5	3.2	1.8
10	6	4
50	40	10

In the table, when $x = 5$, a's marginal utility receives a weight of 1 (basic and property right) and b's marginal utility a weight of 3/4 (basic right), etc. When $x = 10$, there is a corner solution. If however rights were neglected, and social welfare represented by the unweighted sum of individual utilities, the social optimum would correspond to the egalitarian solution, $x_a = x_b = (1/2)x$.

In the following sections, we present these ideas rather more formally. In Sect. 2, we axiomatise a generalised utilitarian aggregation rule, under which different weights are assigned to utilities depending on the different rights involved. In Sect. 3, we discuss the relationship between actions, rights and the utility obtained from actions, in implementing the aggregation rule developed in Sect. 2. The essential issue is how socially to evaluate an individual's utility when this is jointly determined by several actions, some "more right" than others. We then, in Sect. 4, apply our approach to a famous example of the literature, "Edwin-Angelina-the Judge", bringing into focus both the violation of rights and joint rights. Section 5 concludes.

2 The Model

Assume a finite set of individuals, $N = \{1, \ldots, n\}$, where $n \geq 2$. These constitute the society. Assume also a finite set of 'rights', $\{R_1, \ldots, R_{m-1}\}$ $(m \geq 2)$, and let R_m denote 'no right'. As illustrated in Sect. 1, rights may be simple, e.g., 'basic right', or compound, e.g., 'basic and property right'. In the example of Sect. 4, R_m arises whenever there is the violation of a right. The rights we have in mind are diverse. They can be economic, such as the right to medical care or environmental rights; or non-economic, such as the right to vote or minority rights. Rights also vary over time and across cultures, and differ in importance depending on the strength of society's attachment. For the moment, however, we need not assume they are ranked.

For each $i \in N$, let individual i have a utility vector, $U_i = (U_{i1}, \ldots, U_{im})$, where for each $j = 1, \ldots, m$ $U_{ij} \in \mathbb{R}_+$, the set of non-negative real numbers. Different components of utility vectors are associated with different rights. Thus R_j describes individual i's right to U_{ij}. Call $U = (U_1, \ldots, U_n)$ a 'utility profile', and let U belong to a space of utility profiles denoted by $\mathfrak{U}$; that is, we let U vary within $\mathfrak{U}$.

Assume that the society has a social preference relation over $\mathfrak{U}$. We are supposing that each individual's welfare depends only on the sum of the components of the individual's utility vector, but that the society discriminates among these components in carrying out an evaluation of the contribution of the individual's utility vector to social welfare.

We now apply axioms that restrict the social preference relation over $\mathfrak{U}$.

Axiom 1 (restricted domain). There exist positive bounds, L_i, such that $U \in \mathfrak{U}$ if and only if, for all $i \in N$, $0 \leq \Sigma U_{ij} \leq L_i$.

According to *restricted domain*, an individual's total utility varies between a lower bound of zero and an upper bound. Thus we take it that there are

limits to the heights of individual experience. The meaning of zero utility is not context free, but will in some contexts refer to a life that is just worth living.

Axiom 2 (ordering). The social preference relation is a continuous ordering. The effect of this axiom is to ensure that the social preference relation is representable by a continuous utility function, $F(U)$ $(U \in \mathfrak{V})$.

According to *ordering*, society is 'rational'.

Axiom 3 (monotone preferences). For all $i \in N$, $F(U)$ is increasing in U_{ij} for all $j = 1, \ldots, m-1$, and non-decreasing in U_{im}.

According to *monotone preferences*, social welfare is raised when individual utility governed by R_j $(j = 1, \ldots, m-1)$ increases, and not lowered when individual utility governed by R_m increases. Social welfare responds positively to the welfare of its individual members, except possibly where 'no right' is concerned.

Axiom 4 (anonymity). For all $i, k \in N$, for all $j = 1, \ldots, m$, for all $U, U' \in \mathfrak{V}$, and for all $\delta > 0$, suppose U' is derived from U by subtracting δ from U_{ij} and adding δ to U_{kj}. Then $F(U') = F(U)$.

According to *anonymity*, an amount of utility allocated to one individual is, in terms of social welfare, equivalent to the same amount of utility allocated to a different individual when they are governed by the same right. It is worth emphasising that this does not imply that society is ordinarily indifferent whether an additional unit of utility is allocated to someone whose level of utility is high or to someone whose level of utility is low. The very fact that the first has high utility and the second low utility may be a factor influencing the relative strengths of their rights to this unit of utility. We take it as axiomatic that, if allocating a unit of utility to one individual improves social welfare more than allocating it to another, then the first individual has a stronger right to the unit of utility than the second.

Axiom 5 (independence). There exists a real-valued function,

$$\varepsilon = h(\delta, j, s, V),$$

defined for $\delta > 0$, $j < s$ (j,s = 1,. . .,m) and $V = U_i$ (where $U \in \mathfrak{V}$ and $i \in N$), with the properties:

(a) $\varepsilon = 0$ if $\delta = 0$.
(b) h is differentiable with respect to δ and V.
(c) Given $V = U_i$, if U^a is derived from U by subtracting δ from U_{is} and adding ε to U_{ij}, and $U^a \in \mathfrak{V}$, then $F(U^a) = F(U)$.

When an amount δ is subtracted from the s'th component of individual i's utility vector, it may be possible to compensate for this (in terms of social welfare) by adding an amount ε to the j'th component of the same individual's utility vector. If so, then in general ε depends on δ, j, s, U and i, and the effect

of the axiom is to restrict this dependence. According to *independence*, ε does not depend on which individual's utility vector is varying, nor on the other individuals' utility vectors. Note too that (a) and (b) ensure that ε is small when δ is small.

Lemma. Given Axioms 1–5, there exists a function, h*(j,s), defined for $j < s$ $(j,s = 1,\ldots,m)$, such that, for all $\delta > 0$, for all $U \in \mathfrak{V}$ and for all $i \in N$, if U^a is derived from U by subtracting δ from U_{is} and adding ε to U_{ij}, $U^a \in \mathfrak{V}$ and $F(U^a) = F(U)$, then

$$\varepsilon = h^*(j,s)\delta.$$

Proof. By *monotone preferences*, ε is unique. Choose $k \in N$, $k \neq i$. By *independence*, we can, without loss of generality, set $U_k = 0$. Let e_s denote the unit m-vector all of whose components are 0, except for the s'th component which is 1. Suppose $U^a \in \mathfrak{V}$ (specifying U^a as above), and choose q, a positive integer. Let U^b be derived from U by subtracting δ/q from U_{is} and adding δ/q to U_{ks}; let U^c be derived from U^b by subtracting δ/q from U_{ks} and adding $\varepsilon = h(\delta/q,j,s,[\delta/q]e_s)$ to U_{kj}; and let U^d be derived from U^c by subtracting ε from U_{kj} and adding ε to U_{ij}. By *independence*, when q is large ε is small, and consequently, by *restricted domain* and *monotone preferences*, when q is large $U^b, U^c, U^d \in \mathfrak{V}$. Thus, by *ordering*, *anonymity* and *independence*,

$$\begin{aligned} F(U^d) &= F(U^c) \\ &= F(U^b) \\ &= F(U) \\ &= F(U^a). \end{aligned}$$

Notice that, by *monotone preferences*, both feasibility and equivalence in terms of social welfare are maintained when the three-step process of obtaining U^d from U is iterated q times, starting each successive iteration from the utility profile obtained in the previous iteration. Thus, since by *independence* h is differentiable, there exists a function h* such that

$$\begin{aligned} h(\delta, j, s, U_i) &= qh(\delta/q, j, s, [\delta/q]e_s) \\ &= qh^*(j,s)\delta/q + o(\delta/q) \\ &= h^*(j,s)\delta, \end{aligned}$$

on letting q tend to infinity. ■

According to the lemma, for $j < s$, a decrease of δ in utility governed by R_s is compensated by an increase of $h^*(j,s)\delta$ in utility governed by R_j. Suppose $j < s < m$, so that by *monotone preferences* $h^*(j,s) > 0$. Define

$$h^*(s,j) = 1/h^*(j,s).$$

Except for R_m, we can now dispense with the restriction $j < s$. For all $j, s < m$, a decrease of δ in utility governed by R_s is compensated by an increase of $h^*(j,s)\delta$ in utility governed by R_j.

Axiom 6 (regularity). For all s = 1,...,m-1, h*(s,m) < 1.

According to *regularity*, R_m is ranked below each of $R_1, \ldots, R_{m-1}$.

Where different rights are involved, we refer to decreases in utility combined with compensating increases as 'weighted transfers'. Transfers of utility under the same right we refer to as 'simple transfers'.

Proposition 2.1. Given *restricted domain*, *ordering*, *monotone preferences*, *anonymity*, *independence* and *regularity*, there exist positive numbers $\theta_1, \ldots, \theta_{m-1}$, and a smaller, non-negative number θ_m, such that, for all $U, U' \in \mathfrak{U}$, $F(U) \geq F(U')$ if and only if

$$\Sigma\Sigma\theta_j U_{ij} \geq \Sigma\Sigma\theta_j U'_{ij}.$$

Proof. Let $\theta_1 = 1$ and, for j = 2,...,m, let $\theta_j = h^*(1,j)$. Let $\theta_s = \max\{\theta_j; j = 1, \ldots, m\}$. Notice that by *regularity* s≠m. Defining h*(1,1) = 1, we have, for j≠s,

$$\begin{aligned} h^*(s,j) &= h^*(s,1)h^*(1,j) \\ &= h^*(1,j)/h^*(1,s) \\ &= \theta_j/\theta_s \\ &\leq 1. \end{aligned}$$

By repeated weighted and simple transfers, transforming U into U^a,

$$F(U) = F(U^a),$$

where $U^a_{ij} = 0$ for j≠s and i∈N, $U^a_{is} = L_i$ for i = 1,...,z − 1 and $U^a_{is} = 0$ for i = z + 1,...,n. Since, for all j≠s, h*(s,j) ≤ 1, such transfers are feasible. Similarly, by repeated weighted and simple transfers, transforming U′ into U^b,

$$F(U') = F(U^b),$$

where $U^b_{ij} = 0$ for j≠s and i∈N, $U^b_{is} = L_i$ for i = 1,...,z′-1 and $U^b_{is} = 0$ for i = z′ + 1,...,n. Clearly, $\Sigma\Sigma\theta_j U_{ij} \geq \Sigma\Sigma\theta_j U'_{ij}$ if and only if z > z′, or z = z′ and $U_{zs} \geq U'_{zs}$, and so, by *monotone preferences*, $\Sigma\Sigma\theta_j U_{ij} \geq \Sigma\Sigma\theta_j U'_{ij}$ if and only if

$$\begin{aligned} F(U) &= F(U^a) \\ &\geq F(U^b) \\ &= F(U'). \end{aligned}$$

■

3 Actions and utility payoffs

Assume that the weights $(\theta_1, \ldots, \theta_m)$, which according to the theory of Sect. 2 are applied to utility, are arranged in descending order. Without loss of generality, assume this ordering is strict. (If, for example, $\theta_j = \theta_{j+1}$ initially, R_j and R_{j+1} are amalgamated.) Thus, R_1 is the highest right, R_2

is the next highest, etc. Assume next that each individual $i \in N$ chooses a set of actions. Amalgamating actions if necessary, we can suppose that distinct rights apply to distinct actions. Thus we can represent individual i's choice as an action vector, $a_i = (a_{i1},...,a_{im})$, where for $j = 1,...,m$ $a_{ij} = \phi$ if R_j does not apply to any of i's actions and ϕ denotes 'no action'. Note that, as our paper concerns measurement, the basis for individual choice is immaterial.

An individual's total utility is jointly determined by the action vectors of all the individuals in the society. For each $i \varepsilon N$, our object is to construct a utility vector, $U_i = (U_{i1},...,U_{im})$, where U_{i1} is attributed to a_{i1}, etc. Let i have total utility U^i. We assume that, for $j = 1,...,m$, if $a_{ij} \neq \phi$ is substituted for $a_{ij} = \phi$, i.e., an action is substituted for no action, U^i does not fall. For simplicity, we limit discussion to actions which produce just own benefits. Thus we assume:

Axiom 7 (normalisation). For all $i \in N$ and for all $j = 1,...,m$, $U_{ij} = 0$ if $a_{ij} = \phi$.

A simple case is where the benefit which individual i obtains from any action is independent of i's other actions, and i's total utility, U^i, is an 'additively separable' function of such benefits. There is then no problem in allocating U^i to the different actions and so to the different components of i's utility vector, U_i. The order in which actions are taken does not then affect the result.

When this is not the case, it is natural to assume that the utility obtained by an individual under a given right is independent of the actions adopted by this individual under *lower rights*. This means that, in the process of evaluating the individual's utility derived from various actions, the action with the highest right is considered first, then that with the next highest, etc. We can express this in the form of an axiom:

Axiom 8 (properness). For all $i \in N$ and for all $j = 1,...,m$, U_{ij} is independent of a_{ik} for all $k = j+1,...,m$.

Axiom 8 is appealing. Suppose a strong right applies to action α, and a weaker (or no) right to action β. If adopting β increased the utility attributed to α, to which a high weight is attached, this would seem to encourage β too much. If however adopting β decreased the utility attributed to α, this would seem to discourage β too much. β has then to compensate for a loss of social welfare, due to the decrease in utility attributed to α, before socially beneficial gains are achieved. The result could be negative.

An immediate consequence of Axioms 7 and 8 is:

Proposition 4.1. For each individual $i \in N$, given $a_1,...,a_n$, but suppressing $a_1,...,a_{i-1},a_{i+1},...a_n$ as arguments, individual i's utility vector, U_i, is described by

$$U_{i1} = U^i(a_{i1}, \phi, \ldots, \phi)$$
$$U_{i2} = U^i(a_{i1}, a_{i2}, \phi, \ldots, \phi) - U^i(a_{i1}, \phi, \ldots, \phi)$$
$$\vdots$$
$$U_{im} = U^i(a_{i1}, \ldots, a_{im}) - U^i(a_{i1}, \ldots, a_{i,m-1}, \phi).$$

As noted in Sect. 1, it has not seemed necessary for our analysis to distinguish different kinds of right. Neither in this section have we considered actions with more than one beneficiary. In principle, in the case of any action, a judgement is required of who the beneficiaries are and an evaluation made of the advantages obtained. In practice, of course, determining which rights apply to which benefits may raise difficult questions which are only settled in the courts.

By way of illustration, consider the well-known "Shirts" example (Gibbard 1974), where there are two individuals, each of whom may finish up wearing either a white or blue shirt. Individual 1 might adopt the strategy of choosing both to wear a white shirt and to compel individual 2 to do the same (supposing this feasible). This behaviour involves two actions, one acceptable, but the other disapproved of in a liberal society. Thus, individual 1's strategy gives a pay-off to individual 1 which may be decomposed into utility from the first action, which may receive a high weight in social evaluation, and additional utility from the second, which may receive a low weight. If now we introduce government, the action of the government might be to compel both players to wear a white shirt, and then it is the right (or lack thereof) of the government to carry out such an action that needs to be clarified in determining which right applies to the resulting pay-offs.

4 An application

A famous and fascinating example which has been discussed in the literature is "Edwin-Angelina-the Judge" (Gibbard 1974). On the basis of seemingly weak conditions, the paradoxical result is obtained that social preferences are cyclical. The conditions involved attempt to capture the notions that the individuals concerned should be individually decisive within their "personal spheres", and in concert decisive everywhere. In this section, our object is to see if we can fit this example into our formal framework.

A simple form of the model of Sect. 2 will suffice. This is the case of just two individuals, $N = \{1,2\}$ and a single right, R_1. Utility is governed by R_1, unless obtained through the violation of another individual's right, in which case it is governed by R_2 ('no right'). We set $\theta_1 = 1$ and $\theta_2 = 0$.

In the literature, the process by which a right is violated is often not made explicit. Here we suppose that, if an individual chooses to take some action and has the right to take that action, but is prevented from doing so, then the

individual's right is violated. We will also be concerned with joint rights. We suppose that, if two or more individuals agree to take some action and have a joint right to take that action, but are prevented from doing so, then their joint right is violated.

The example of Edwin, Angelina and the Judge is derived by Gibbard 1974, with "philosophical licence", from Gilbert and Sullivan's operetta, "Trial by Jury". The story of the operetta is that Edwin, a highly eligible batchelor but of inconstant affections, breaks a promise of marriage to Angelina. The beautiful Angelina takes him to court, where she succeeds in winning the hearts alike of members of the jury, spectators and the Judge. When in desperation Edwin argues that if forced to marry Angelina he would constantly beat her when "tipsy", the Judge proposes making him "tipsy" as an experiment. This proposal is rejected by the court, but resolution comes with the Judge's decision to marry Angelina himself.

In Gibbard's example, denote Angelina by A and Edwin by E. Let w_0 denote both Angelina and Edwin remain single, w_E Angelina marries Edwin and w_J Angelina marries the Judge. Preferences are described by the following table, where from top to bottom alternatives are in descending order of preference and preferences are strict:

A	E
w_E	w_0
w_J	w_E
w_0	w_J

Thus Angelina would like most to marry Edwin and next to marry the Judge. Edwin would like most for them both to remain single. However, he would rather marry Angelina himself than bear the thought of her being happily married to the Judge.

In applying our model, let us take Angelina's utility to be zero under w_0 and Edwin's zero under w_J. Angelina obtains utility from marriage, and we suppose for the sake of argument that her utility is 5 if she marries the Judge and 10 if she marries Edwin. For Edwin, utility is 5 if Angelina marries him instead of the Judge, and 10 if they both remain single. Total utilities under the different social alternatives are then:

	A	E
w_0	0	10
w_E	10	5
w_J	5	0

It will be observed that these utilities are consistent with the preference orderings given above. By our earlier assumptions, they receive a weight of one in calculating social welfare, unless obtained through the violation of a right, in which case they receive a weight of zero.

It turns out that this calculation of social welfare is not independent of the feasible set. Consider the different cases. Let the feasible set be $\{w_0,w_E\}$. Then if w_E is the outcome, we can reasonably suppose that Angelina has forced

Edwin to marry her. Thus Angelina has violated Edwin's right to remain single, her utility of 10 receives a weight of zero and w_0 is socially preferred to w_E. Suppose, however, the feasible set is $\{w_J,w_0\}$. Then, if w_0 is the outcome, we can reasonably take it that Edwin has prevented Angelina from marrying the judge. Edwin has violated Angelina's right to marry the judge, his utility of 10 receives a weight of zero and w_J is socially preferred to w_0.

Suppose Edwin and Angelina have a joint right to get married. Then, if the feasible set is $\{w_E,w_J\}$, Edwin offers marriage, Angelina accepts, no rights are violated by w_E, and w_E is therefore socially preferred to w_J. As in the Gibbard 1974 paper, we have a cycle, but obtaining the cycle has depended on varying the feasible set! If in fact the feasible set is $\{w_0,w_E,w_J\}$, Angelina and Edwin will rationally choose w_E. w_E gives Angelina her best outcome, and strategically choosing w_E makes sense for Edwin, since he knows that if he does not marry Angelina she will marry the Judge. Thus, w_E is freely chosen by both Angelina and Edwin, does not violate any right and is the social optimum. The cycle describes social preferences only in the case of "unsophisticated, irrational voters" (Saari 1998).

Society may of course deny Angelina and Edwin the right to get married on the grounds that Edwin's reason for marrying Angelina is unethical (Blau 1975). (There will surely be some attempt to dissuade them from marriage.) In this case, the utility received by Edwin and Angelina when w_E is the outcome receives a zero weight, and it follows that w_J is the social optimum.

We conclude that paradoxes can be resolved in our framework. Social alternatives are ordered by their social evaluation, and this evaluation is unambiguous once it is clear what the feasible set is, since it is then also clear whether or not violations of rights take place when different social alternatives are realised.

5 Conclusion

The discussion of Edwin-Angelina-the Judge in Sect. 4 shows that once rights are introduced, it is not enough in comparing social states just to consider outcomes; one must also take account of the actions which lead to those outcomes. A right to act is also a right to the utility generated by the act, and in Sect. 3 it is shown that this relationship may be quite complex. However, there are important instances where it is not so, in which the rights of individuals may be thought of as 'claims' and the focus is on the actions of a 'social planner'. A case in point, redistribution, is discussed via a numerical example in Sect. 1, while other cases include the allocation of public services such as education or health services.

Concerning extensions of the theory, one would like to see benefits that "spill over" discussed. What weight, for example, should be given to fortunes inherited from ancestors active in the slave trade or tobacco industry? The optimal size of population might also be profitably discussed in our framework.

Importantly, as mentioned in Sect. 1, greater realism might be introduced through 'fuzzification'. A difficulty here is that it is not altogether clear how to "fill boxes" via weighted and simple transfers of utility, as required in the proof of Proposition 2.1, when both the sizes of the boxes and the quantities transferred are fuzzy.

Thus, while there is a 'possible world' to which Proposition 2.1 applies, and this provides insight into more complex worlds, interesting questions remain.

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