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Coalition formation theories revisited : an empirical investigation of Aumann's hypothesis

Forthcoming in:
Braham, Matthew and Steffen, Frank (Eds), ***Power freedom and voting : conceptual, formal, and applied dimensions.*** **Springer, 2007.**
www.**springer**.com/

COALITION FORMATION THEORIES REVISITED:
AN EMPIRICAL INVESTIGATION OF AUMANN'S HYPOTHESIS*

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May 2006

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* The authors are grateful to Moshé Machover for his helpful comments.

ABSTRACT

Robert J Aumann, the 2005 Nobel laureate in Economics, hypothesized in 1995 that, in forming a majority coalition government in real life, the party charged with forming the coalition will choose to form the coalition that maximizes its Shapley-Shubik index. We subjected this hypothesis to empirical testing in nine countries. It was found that for the data sets investigated, this hypothesis produces the smallest number of correct predictions. Three variations of this hypothesis appear to perform somewhat better: restricting the maximization process to the set of closed majority coalitions, or likewise but with a further requirement that the coalition selected be of minimal size or of minimal range. However, none of these variations achieves a level of predictive performance comparable to the Leiserson-Axelrod closed minimal range theory or to the Gamson-Riker minimum size principle. We therefore conclude that Aumann's hypothesis should be rejected, and that considerations of maximizing a priori voting power do not seem to account for the actual behavior of political parties in forming governmental coalitions.

KEY WORDS • closed minimal range coalitions • coalitions of minimum size • coalition-formation theories • Shapley-Shubik index • voting power

1. Introduction

In one of the earliest attempts to examine the effect of a priori voting power on actual political phenomena, Riker (1959) looked at changes in party affiliation in the French National Assembly in 1953-54, and used these data to test the hypothesis that deputies who switched parties were seeking thereby to increase their a priori voting power. His findings were negative, or at best inconclusive.

In his paper Riker used the voting power index proposed by Shapley and Shubik (1954) – which was the only measure of a priori voting power known to him at that time.[1]

By now there is a large body of literature applying considerations of a priori voting power to political institutions such as the UN, the US Congress, the US Presidential Electoral College, the US Supreme Court and its rulings on the implementation of the 'Equal Protection' clause in the 14th Amendment to the Constitution; and of course numerous writings on voting-power considerations in the European Union. But, as far as we know, it took 36 years after Riker's paper was published before someone has followed Riker's lead in suggesting that the formation (or dissolution) of political coalitions should be examined from the viewpoint of a priori voting power, and using for this purpose the Shapley-Shubik (S-S) index.[2] That

[1] Indeed, Riker refers to it throughout – beginning with the paper's title – as '*the* power index'. As a matter of fact, L S Penrose (1946) had proposed another measure of a priori voting power; but it did not become widely known until it was reinvented by Banzhaf (1965) after whom it is generally named. Since then, other indices of a priori voting power have been proposed, but we need not dwell on any of them here.
For further details see Felsenthal and Machover (1998).

[2] Felsenthal and Machover (2000) criticized Riker's (1959) use of the S-S index as unsuitable because they argued that defections within the French National Assembly could be attributed to the defectors' wish to increase their I-power (for which the Penrose measure is suitable), not their P-power (for which the S-S index is more suitable). However, they agreed that for the purpose of testing Aumann's hypothesis the employment of the S-S index was probably more suitable than using the Penrose measure. These two types of voting power are further elaborated in footnote #4.

someone was the renowned game theorist (and the 2005 Nobel laureate in Economics), Robert J Aumann.

On 30 June 1995 Eric van Damme conducted an interview with Aumann on the state of the art in game theory. One of the practical applications of game theory mentioned by Aumann in this interview was the use of the Shapley value (Shapley, 1953; Shapley and Shubik, 1954) as an algorithm for predicting which governmental coalition is likely to form. Here are some relevant excerpts from this interview (cf. van Damme, 1997:11–13; van Damme 1998: 184–187):

> **Q:** From these examples, can one draw some lessons about the type of situations in which one can expect game theory to work in applications?
>
> **A:** What one needs for game theory to work, in the sense of making verifiable (and falsifiable!) predictions, is that the situation be structured. … For years I have been predicting the government that will form in Israel once the composition of the Israeli parliament is known after an election. That is a structured situation, with set rules. The parliament has 120 seats. Each party gets a number of seats in proportion to the votes it got in the election. To form a government a coalition of 61 members of parliament is required. The president starts by choosing someone to initiate the coalition formation process. (Usually, but not necessarily, this "leader" is the representative of the largest party in parliament.) The important point is that the process of government formation is a structured situation to which you can apply a theory. …
>
> For instance, in the governmental majority matter, one can set up a parliament as a simple game in the sense of Von Neumann and Morgenstern's cooperative game theory, where we model the party as a player; we get a coalitional worth function that attributes to a coalition the worth 1 when it has a majority and the worth 0 otherwise. And then one can work out what the Shapley values are;[3] the structure is there, it is clear, and one can make predictions. Now there are all kinds of things that are ignored by this kind of procedure, but one can go out and make predictions. Then, if the predictions turn out correct, you know that you were right to ignore what you ignored. …
>
> **Q:** In this example of coalition formation, you make predictions using an algorithm that involves the Shapley value. Suppose you show me the data and your prediction comes out correct. I might respond by saying that I don't understand what is going on. Why does it work? Why is the Shapley value related to coalition formation? Is it by accident or is it your intuition, or is it suggested by theory?
>
> **A:** There are two answers to that. First, certainly this is an intuition that arises from understanding the theory. The idea that the Shapley value does represent power comes from theory. Second, for good science it is not important that you understand it right off the bat. What is important, in the first instance is

[3] In the context of simple voting games the 'Shapley value' is usually referred to as the 'Shapley-Shubik (S-S) index'.

that it is correct, that it *works*. If it works, then that in itself tells us that the Shapley value is relevant.

Let me explain this a little more precisely. The theory that I am testing is very simple, almost naïve. It is that the leader – the one with the initiative – tries to maximize the influence of his party within the government. So, one takes each possible government that he can form and one looks at the Shapley value of his party within the government; the intuition is that this is a measure of the power of his party within the government.[4] This maximization is a nontrivial exercise. If you make the government too small, let's say you put together a coalition government that is just a bare majority with 61 members of [the Israeli] parliament – a minimal winning coalition – then it is clear that any party in the coalition can bring down the government by leaving. Therefore, all the parties in the government have the same Shapley value. So the hypothesis is that a wise leader won't do that. That is also quite intuitive, that such a government is unstable, and it finds its expression in a low Shapley value for the leader. On the other hand, too large a coalition is also not good, since then the leader doesn't have sufficient punch in the government; that also finds its expression in the Shapley value. Consequently, the hypothesis that the leader aims to maximize his Shapley value seems a reasonable hypothesis to test, and it works not badly. It works not badly, but by no means a hundred percent. For example, the current (June 1995) government of Israel is very far off from that, it is basically a minimal winning coalition. In fact, they don't even have a majority in parliament, but there are some parties outside the government that support it; though it is really very unstable, somehow it has managed to maintain itself over the past 3 years. But I have been looking at these things [in Israel] since 1977, and on the whole, the predictions based on the Shapley value have done quite well. I think there is something there that is significant. It is not something that works 100% of the time, but you should know that nothing in science works 100% of the time. In physics also not. In physics they are glad if things work more than half the time.

Q: Would you like to see more extensive empirical testing of this theory?

A: Absolutely. We have tried it in one election in the Netherlands where it seems to work not badly; but we haven't examined that situation too closely. The idea of using the Shapley value is not just an accident, the Shapley value has an intuitive content and this hypothesis matches that intuitive content.

Although more than a decade has elapsed since Aumann first phrased his above mentioned hypothesis, it has not been subjected, as far as we know, to an extensive empirical verification. The purpose of this paper is to conduct such an investigation.

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[4] Note that Aumann regards the Shapley value, interchangeably, as 'measure of influence' and as 'measure of power'. As argued by Felsenthal and Machover (1998:17–19, 35–36; 2005), one ought to distinguish between two types of a priori voting power: power as influence (I-power) – which is the a priori probability that a player's vote will be decisive, and power as prize (P-power) – which is the share a player can expect to receive in the prize distributed by a winning coalition among its members. Inasmuch as the Shapley value is a coherent notion, it measures the latter type of power; in the context of political coalitions the 'prize' is usually regarded as cabinet portfolios. See also in this context the similar distinction made by Laver and Schofield (1991: 91ff) between 'office-seeking' and 'policy-seeking' regarding what mainly motivates actors' bargaining in the process of forming governmental coalitions.

The way Aumann's hypothesis is phrased makes it easy to verify. This is so because it focuses only on the party charged with forming a (winning) governmental coalition,[5] hence one only needs to know the identity of this party and the distribution of seats of all the parties represented in parliament, or in Aumann's words: "So, one takes each possible government that he can form and one looks at the Shapley value of his party within the government".[6]

In investigating the predictive performance of Aumann's hypothesis we shall compare its performance with three modified versions of the hypothesis while benchmarking against two selected established theories of coalition formation. In carrying out this exercise, we make use of historical election data from eight European countries and Israel as tabulated in de Swaan's (1973) book. Motivated by Aumann's observation that his hypothesis appears to have worked quite well in the case of Israel between 1977 and 1995 as well as by Felsenthal and Machover's (1998: 205, fn 32) refutation of this observation,[7] we carried out a further check using detailed election results for Israel from 1949 until 2006. The statistical results do not support Aumann's hypothesis, but three variations of this hypothesis appear to perform somewhat better: restricting the maximization process to the set of closed majority coalitions, or likewise but with a further requirement that the coalition selected be of minimal size or of minimal range. However, none of these variations achieves a level of predictive performance comparable to Leiserson (1966) and Axelrod (1970:170 ff.) closed minimal range theory or with Riker's (1962) minimum size principle when confronted with the data.

The remainder of the paper is organized as follows. In the ensuing section, we discuss the hypothesis further and introduce a number of different variations to Aumann's hypothesis. In Section 3, we describe the data used in the analysis and explain the statistical tests that are used in our evaluation of the worth of each of the hypotheses considered. The results of our empirical analysis are summarized in Table 1 and discussed in Section 4; some concluding remarks follow.

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[5] Note that Aumann's hypothesis is not concerned with situations in which one party has an absolute majority of seats in parliament, nor with situations in which a minority governmental coalition is supported by one or more parties outside the coalition.

[6] The second-named author suggested to Aumann in an email message in November 2003 that in practice the party charged with forming a governmental coalition must gain the consent of each of the parties it wishes to incorporate in the coalition – and that some parties may refuse to join a proposed coalition for various political reasons. Hence he proposed that the phrase "each possible government that he can form" should perhaps be replaced with "each possible *politically feasible* government that he can form". Aumann responded (on 26 November 2003) that "[My] original hypothesis did not include the caveat 'politically feasible', which may be a little difficult to formulate for the purposes of empirical testing. However, in practice it may indeed be necessary to add such a caveat; or, perhaps, it may not be. It's a point worth thinking about." As is described in the sequel, given the positions of the various parties on a left-right ideological continuum, we defined operationally the concept 'politically feasible coalition' as 'ideologically closed coalition'.

[7] Felsenthal and Machover (1998: 205, fn 32) state that their calculations do not bear out Aumann's claim: 'We analyzed all governments formed in Israel during the period 1977–1996 and found that in all these cases the party charged with forming the government could have increased its S-S index within the government by either narrowing or widening the government that it actually formed.' Our own analysis (see Annex Table 2 and Annex Table 3) supports this conclusion.

2. Aumann's Hypothesis and Modifications

Aumann's hypothesis asserts that a party charged with forming a majority government will select that majority coalition that will maximize its Shapley-Shubik index.

However, before embarking on an empirical investigation of this hypothesis, one key point of clarification requires highlighting.

Let us return to the passage where Aumann says:

> For instance, in the governmental majority matter, one can set up a parliament as a simple game in the sense of Von Neumann and Morgenstern's cooperative game theory, where we model the party as a player; we get a coalitional worth function that attributes to a coalition the worth 1 when it has a majority and the worth 0 otherwise. And then one can work out what the Shapley values are;

From the loose way Aumann speaks in this passage – it is after all an interview, not a scholarly paper – it is at first unclear whether, for the purpose of calculating the S-S index, the term "majority" in this passage refers to (i) simple majority within the government (coalition), or (ii) simple majority in the parliament as a whole.

But from what Aumann says in a subsequent passage it becomes certain that he means simple majority in the parliament as a whole. In this passage he says:

> This maximization is a nontrivial exercise. If you make the government too small, let's say you put together a coalition government that is just a bare majority with 61 members of [the Israeli] parliament – a minimal winning coalition – then it is clear that any party in the coalition can bring down the government by leaving. Therefore, all the parties in the government have the same Shapley value.

From this passage two conclusions can be reached. First, a government with 61 seats in the Israeli parliament is a minimal winning coalition if the required majority is a simple majority of the parliament as a whole but not necessarily if the required majority is a simple majority of the coalition members. Second, the conclusion that "all the parties in the government have the same Shapley value" follows only if one assumes that the required majority is a simple majority of all members of parliament.

We therefore interpreted Aumann's description of the maximization process as implying the same threshold (quota) for each winning coalition considered, a threshold that is set equal to a simple majority of the sum of votes of all members of parliament. We shall henceforth refer to this interpretation of Aumann's hypothesis as the *maximal SSQ hypothesis.*

However, from the perspective of developing a predictive theory of coalition formation, it is important that one has a theory that provides a reasonably sharp or precise prediction as to the likely outcome. Parsimony – in terms of the size of the predicted set of likely coalitions – is an important attribute of a good predictive theory. Just like the von Neumann-Morgenstern *stable set*, however, there is a good chance that Aumann's hypothesis, too, will produce a number of predicted coalitions that is unreasonably large.[8]

An alternative route to avoiding the profusion of possibilities under the maximal SSQ hypothesis is to resort to *ad hoc* restrictions that will decrease the predicted set of coalitions by introducing social standards which, presumably, make some winning coalitions more reasonable than others.[9] To our minds, as long as these restrictions are sufficiently persuasive, due consideration should be given to them. In this spirit, we consider three variations of the maximal SSQ hypothesis.

2.1. *Variation I: Restriction to Closed Coalitions*

In this variation, we consider a restriction of the domain to the set of *closed* or *ideologically connected* coalitions.[10] The motivation for introducing this restriction is two-fold. First, as de Swaan (1973:148) has noted, out of 108 majority coalitions in nine countries which he investigated in his study, 85 (or 79 percent) of these were closed coalitions. Thus, empirically, the evidence appears to indicate a preference by political parties to form closed coalitions.[11] A second and related point is the observation that in a multi-party system, a coalition leader located at one extreme of the ideological continuum is highly unlikely to enter into a political union with a party located at the other extreme even though such a union may result in the highest S-S index for the coalition leader. The restriction to closed coalitions where the S-S index of the party charged with forming the coalition is maximized will help in making the predicted set more parsimonious. We will refer to Aumann's hypothesis under this domain restriction as the *Closed Maximal SSQ hypothesis*.

2.2**.** *Variation II: Restricting the predicted set of closed coalitions to the one with minimum size*

Even with the domain restriction introduced in variation I, the size of the predicted set may still be quite large. Since in the maximization process the stability issue arising from the defection of coalition members has already been factored into the calculation, it appears

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[8] As can be seen in the 5th column of Annex Tables 2 and 3, more than one coalition – and sometimes considerably more than one – is predicted to form according to Aumann's hypothesis in almost all investigated elections. Thus, for example, in the 1996 Israeli election (cf. Annex Table 3) fully one-third (246 out of 739) of the winning coalitions in which the leader is a member are predicted to form according to Aumann's hypothesis. There is a second point. When thinking of the coalition formation process along the line of an *n*-person non-transferable utility game, it is important to realize that it is reasonable to do so only to the extent that members of the same party in the assembly act in unison, that is, legislators operate strictly along party lines. Thus, Aumann's hypothesis, if it works at all, should work best in a context where party discipline is strict or tight. In political cultures where this is not the case, it would be unreasonable to expect the hypothesis to perform well.

[9] This seems to be the usual practice employed by coalition theorists as noted, for instance, by Luce and Raiffa (1957:213) and by Gamson (1961: 380).

[10] A coalition is 'closed' if all its members are adjacent to one another along the ideological continuum; otherwise the coalition is 'open'.

[11] It is tempting to attribute this to the notion that ideologically closed coalitions have lower levels of conflict of interest than those that are not ideologically closed and are thus preferred; but the evidence reported by Browne, Gleiber and Mashoba (1984) does not appear to support this position. Of the investigated coalitions in this study, France during the Fourth Republic is perhaps one exception to the case – and this, in turn, serves to highlight that it is probably not reasonable to expect one single theory to do well in all instances.

reasonable to present an argument along the line of the *minimum size* principle that only the *smallest* coalition in terms of the number of votes it controls in the closed maximal set will form. This may be so since it is not uncommon to expect that the actual distribution of cabinet positions among coalition members will be approximately proportional to the number of votes they control within the coalition.

Like the *minimum size* principle advocated by Gamson (1961) and Riker (1962),[12] this refinement of the predicted set will almost always yield a prediction that is a singleton or one that involves a relatively small number of coalitions. Such an approach is however distinct from the *minimum size* principle and it also overcomes the objection raised by Aumann concerning stability of the coalition when the *minimum size* principle (simpliciter) is invoked.[13]

We shall refer to this variation as the *Closed Maximal SSQ Minimum Size hypothesis*. Of course, under this variation, the predicted sets will be subsets, not necessarily proper subsets, of those obtained under Variation I, the closed maximal SSQ hypothesis.

2.3**.** *Variation III: Restricting the predicted set of closed coalitions to the one with minimal range*

As an alternative to the restriction of the predicted set to include only those closed maximal coalitions that are of minimal size or weight, we consider the restriction of the predicted set to the subset of closed maximal coalitions that are of *minimal range*.[14] Consideration of this variation is largely motivated by de Swaan's finding that the Axelrod-Leiserson closed minimal range hypothesis appears to fit the historical data rather well.[15] It also adds an additional dimension to the optimization process emphasizing, in addition to power, the desirability of homogeneity in ideological positions of coalition members, an idea that is rather intuitive.

Summarizing, in addition to considering the version of Aumann's hypothesis that we have referred to as the maximal SSQ hypothesis, we will also consider in our investigation the following three variations: (a) the variation of the SSQ version that restricts the maximization process to only those majority coalitions that are *ideologically closed*; (b) the variation that further restricts the prediction of the Closed Maximal SSQ version to the subset that is of *minimal size*; and (c) the variation that further restricts the prediction of the Closed Maximal SSQ version to the subset that is of *minimal range*.

While (b) is an analogue to the Riker-Gamson *minimum size principle*, (c) may be viewed as the analogue to the Axelrod-Leiserson *closed minimal range* theory. We note that the Riker-

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[12] Gamson calls the winning coalition controlling the smallest total number of seats (or votes) *cheapest winning coalition* while Riker calls it *coalition of minimal size*.

[13] This is so because a coalition of minimum size is often also a minimal winning coalition where the defection of any member renders it losing and where the Shapley value of all members is equal.

[14] Briefly, on a policy scale in which any two adjacent parties are regarded as one unit of distance apart, the range of a coalition is the distance between the two parties in the coalition whose positions on the policy scale are furthest apart.

[15] Of the 85 closed coalitions in De Swaan's study, 55 were of minimal range. Cf. de Swann (1973:148).

Gamson *minimum size principle* and the Axelrod-Leiserson *closed minimal range* theory are ideas that are already deeply entrenched in the political science literature.

3. Data and Analysis

3.1. *Data Sources*

In testing Aumann's hypothesis and its three different variants, the primary source of our data is the tabulation of election outcomes provided in de Swaan's (1973) book *Coalition Theories and Cabinet Formations*. This data tabulation covers nine countries and selected historical parliamentary elections. For some of these countries, elections from the end of the First World War up to the early 1970s were included.[16] de Swaan's data is particularly useful because, for each election, in addition to listing the number of seats controlled by parties which gained more than 2.5% of the seats in an assembly,[17] the parties have also been ordered along the left-right ideological continuum "according to the share of national income they wish to see redistributed by means of the government budget, military and police expenditures excepted. When this [did] not affect the ranking of the other actors, the criterion of nationalism [was] added to place the Fascist parties. The preference of the party's cadre [was] taken as indicative of the party's stand and, in the absence of such information, the judgment of parliamentary historians and other expert observers [were] accepted instead." (1973: 142). The rank-ordering of parties rendered the task of identifying closed majority coalitions in each election considerably less onerous.[18]

However, whereas the coalitions studied by de Swaan included many interim coalitions,[19] it seems to us that Aumann's hypothesis – and perhaps all other coalition-formation theories – is (are) concerned only with original coalitions because the formation of interim coalitions cannot be reasonably considered as independent of the formation of the original coalition when the leading party remains the same. Hence, except in one occasion, [20] we limited our investigation only to original coalitions.

[16] Specifically, the nine countries and the periods investigated by de Swaan were as follows: Denmark (1918–1971), Sweden (1917–1970), Norway (1933–1936; 1965–1969), Israel (1949–1969), Italy (1946–1972), The Netherlands (1918–1972), Finland (1919–1972), Germany's Weimar Republic (1919–1932), and France's Fourth Republic (1945–1957).

[17] de Swaan (1973:131) justifies the 2.5% restriction by arguing that parties controlling no more than 2.5% of the seats in an assembly were almost never included in a winning governmental coalition. Nevertheless he took into consideration the number of seats controlled by such small parties in calculating the needed majority to pass decisions by an assembly.

[18] Two other studies regarding coalition formations were conducted more or less simultaneously with (and independently of) de Swaan's study: the study by Taylor and Laver (1973) which dealt with western European coalitions during the period 1945-71, and the study by Dodd (1976) which dealt with coalitions in the early 1970s. Laver and Schofield (1991: 249–250) state that the ideological rankings of parties according to these two studies correspond closely to each other and to de Swaan's ranking.

[19] These are coalitions formed not immediately following a general election (which we call 'original coalitions') but as a result of change due to defections from or broadening of the original coalition.

[20] The exception is the 2001 Israeli direct elections of a prime minister (but not also the Knesset). Although the composition of the Knesset in 2001 remained the same as it was following the 1999 general elections, the new prime minister (Ariel Sharon) belonged to a different party than his predecessor (Ehud Barak), and hence decided to form a new governing coalition.

Our second departure from de Swaan's approach is in the definition of an admissible majority coalition. For each given election, de Swaan considers every possible winning coalition as a potential candidate for the formation of a government. However, in reality the party charged with forming a governing coalition is normally selected by the head of state, and is usually the party that, if successful in forming the governmental coalition, assumes the premiership. This is exactly the reason why Aumann's hypothesis is concerned only with the Shapley value of this party. Hence in our investigation of this hypothesis we consider only those winning coalitions that include the coalition leader which we identify *ex post* as the party of the prime minister.

The above considerations have meant that whereas de Swaan's conclusions are based on 108 original and interim coalitions formed in nine countries, ours are based, except for Israel, only on 65 original coalitions formed in these countries during the same period.

The case of Israel warrants special attention in this study. As we already mentioned, the findings by Felsenthal and Machover (1998) did not corroborate Aumann's statement that "predictions based on the Shapley value have done quite well" regarding Israeli elections between 1977 and 1995. de Swaan (1973:237) too noted that "Israel is a difficult country for the theories". However, as de Swaan's and Aumann's observations regarding Israel relate to different periods, we decided to investigate all 18 elections conducted in Israel during the period 1949–2006. [21] Moreover, because in Israel very small parties were sometime included in governmental coalitions, we included in our analysis all parties that gained representation in Israel's parliament and, consequently, our classification of some Israeli governmental coalitions diverges from de Swaan's. (As noted above, de Swaan, in contrast, considered as admissible members of winning coalitions only parties controlling more than 2.5% of the seats in the various parliaments that he investigated.)

3.2**.** *Evaluating the Worth of a Theory*

No theory is expected to correctly predict the outcome all of the time. In some contexts, a theory that performs marginally better than chance may be considered a reasonably good theory if other competing theories can do no better. Intuitively, when a restriction is placed on the set of admissible coalitions resulting in predicted sets that are sharper, it would appear that the frequency of obtaining a correct prediction is likely to be lower compared with the case when the domain is unrestricted. This is certainly the case when the restriction results in predicted sets that are proper subsets of the unrestricted predicted sets. This does not however automatically render the restricted theory a poorer predictive theory. Conversely, for the same election, different theories may give rise to predicted sets that differ considerably in terms of size, and the theory that gives rise to a larger predicted set will naturally have a better chance of correctly predicting the outcome. But such a theory is not necessarily a better predictive theory. Somehow, the tradeoff between the probability that the actual outcome is included in the predicted set and the parsimony of the predicted set will have to enter the calculus in determining which theory should be preferred.

In determining the worth of each theory in our analysis, we have kept in mind this tradeoff. Instead of attempting a direct comparison of the competing theories, our approach is to compare the different theories with their respective randomized counterparts so that the

[21] As mentioned in the previous footnote, of these 18 elections 17 were general elections to the Knesset and one (conducted in 2001) was a direct election of the prime minister.

evaluation of each theory is carried out on a level playing field. For the de Swaan data set, given a sample size of 65 which is reasonably large, we carry out our evaluation by invoking the Central Limit Theorem. For the more detailed investigation in the case of Israel, with a sample size of 18 elections, it appears grossly inappropriate to invoke the Central Limit Theorem. In this case, we computed the exact probability mass function under each theory/hypothesis to assist us in making our evaluation. A brief outline of our approach is provided below.

Let N_i denote the size of the set of winning coalitions given the distribution of seats secured by the various political parties in the i-th election, $i = 1, 2, \ldots, n$. Let S_{ij} denote the corresponding size of the predicted set under the j-th theory, $j = 1, 2, \cdots, m$. Then, for any theory, each election may be regarded as an independent Bernoulli trial with probability of success P_{ij} given by S_{ij}/N_i. The probability of success here refers to the probability that the prediction of the theory is consistent with the actual outcome. Since N_i and S_{ij} vary across elections, the probability of success P_{ij} also varies across elections. For each theory, therefore, the set of elections included in the analysis may be regarded as a sequence of independent Bernoulli trials with unequal probability of success across trials. Given these probabilities, the expected number of consistent predictions under each theory is given by $\sum_i P_{ij}$ with associated standard deviation given by $[\sum_i P_{ij}(1- P_{ij})]^{1/2}$. The expected number of consistent predictions $\sum_i P_{ij}$ is what one would expect if the theory in question is no better than a pure chance mechanism.

For *n sufficiently large,* the Central Limit Theorem for independent random variables[22] postulates that the expected number of consistent predictions will be *approximately normally distributed* with mean $\sum_i P_{ij}$ and standard deviation $[\sum_i P_{ij}(1- P_{ij})]^{1/2}$. In our analysis of de Swaan's data, we make use of this result to evaluate the worth of each theory. Specifically, a theory that produces a predictive outcome that departs significantly in the positive (negative) direction, measured in standard deviation units, from the mean value of its randomized counterpart would be regarded as one that is better (worse) than a pure chance mechanism and thus worthy (unworthy) of further consideration as a predictive theory of coalition formation.

In our supplemental analysis of the case of Israel, we have only 18 elections between 1949 and 2006. Because of the rather small sample it would be inappropriate to invoke the Central Limit Theorem here. This is particularly so when the probability of success, P_{ij}, in each of the independent Bernoulli trials, is close to either end of the unit interval as this will cause the probability mass function for the sum of the independent Bernoulli variables to be severely skewed. In this instance, while there are a number of alternative approaches that one may

[22] The Central Limit Theorem for independent variables states the following. Let $X_1, X_2, \cdots$ be a sequence of independent random variables having respective means and variances $\mu_i = E(X_i)$, $\sigma_i^2 = Var(X_i)$. If (i) the X_i are uniformly bounded; that is, for some M, $P\{|X_i|<M\}=1$ for all i, and (ii) $\sum_{i=1}^{\infty} \sigma_i^2 = \infty$, then

$$P\left\{ \frac{\sum_{i=1}^{n}(X_i - \mu_i)}{\sqrt{\sum_{i=1}^{n}\sigma_i^2}} \le a \right\} \to \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{a} e^{-x^2/2}\, dx \;\; as\; n \to \infty.$$ See Ross (2002).

employ to evaluate the worth of each theory, we have opted to compute the exact probability mass function for the randomized schemes under each of the theories considered, noting that the probability mass at k, the number of successes under the *j-th* theory, is given by:

$$P_j(x = k) = \sum_{a_k \in A_k} \left(\prod_{i \in a_k} P_{ij} \prod_{i \notin a_k} (1 - P_{ij}) \right)$$

In this expression, A_k denotes the set of situations that give rise to k successes and a_k is an element of this set. The cardinality of the set A_k is ${}^{n}C_k$ (i.e. the number of combinations of size k one can extract out of a population of size n) and the summation in the expression is over each of these ${}^{n}C_k$ different situations.

With the exact probability mass function for each theory in hand, and supposing the number of successes obtained under the *j-th* theory is k_j^*, we are able to calculate the exact probability that this number of successes or higher will be observed under its randomized counterpart scheme, i.e., we can calculate $P_j\left(x \geq k_j^*\right)$. This, in turn, will allow us to test the hypothesis that the theory in question is no better than the randomized scheme considered. A large value for $P_j\left(x \geq k_j^*\right)$ is indicative that this is indeed the case, whereas a very small value for $P_j\left(x \geq k_j^*\right)$ provides strong evidence against the null hypothesis, indicating that the predictive performance of the theory is unlikely to be the result of pure chance. In the latter situation, the theory in question would be regarded as one deserving further consideration.

We now turn to a discussion of our results based on the tests that we have described.

4. Main Results

As is evident from a cursory review of the summary statistics presented in Table 1, Aumann's SSQ hypothesis resulted in predicted sets that performed poorly both for the de Swaan data and the more detailed Israeli data.

For the 65 elections we considered in de Swaan's data set, the expected number of consistent predictions based on the randomized counterpart to the Maximal SSQ hypothesis is 13.2115, but under this hypothesis the number of consistent predictions obtained was only 8, a performance that is 1.8269 standard errors below its randomized mean. In fact, it is the only theory of the six theories considered that performed worse than its randomized counterpart. For the Israeli 1949–2006 data set, it is also one of five hypotheses that did not achieve a single success while at the same time having the highest randomized mean of the five, thus making it the least attractive theory. These considerations lead us to categorically rejecting the maximal SSQ hypothesis.

Table 1 about here

Detailed computational results appear in the Appendix: Annex Table 1 and Annex Table 2 for the de Swaan data set, and Annex Table 3 for the Israeli elections from 1949 through 2006.

In all these tables there are entries of the type *x*/*y* in the columns of the various tested theories. Thus, for example, in the first row of Annex Table 1 under the maximal SSQ column appears the entry 3/48. This entry should be interpreted thus: of the total 48 possible coalitions in which the leader's Shapley-Shubik index is maximized over all 17 elections considered in Denmark, only 3 coalitions were actually formed. The same interpretation applies, *mutatis mutandis*, to all other entries of this form.

In Annex Table 4, the exact probability mass functions that we employ in our evaluation of the detailed Israeli data set are tabulated.

For the interested reader we display in Annex Table 5a de Swaan's data regarding the distribution of parliamentary seats and ideological position of parties following 65 elections in nine countries, and in Annex Table 5b we present the detailed distribution of parliamentary seats and ideological position of all parties immediately following the 18 elections conducted in Israel during the period 1949–2006. In each of these two tables the parties represented in parliament following each of these elections are listed according to their position on the left-right ideological continuum: the party listed first is the most leftist party while the party listed last is the most rightist party. Following the abbreviated name of each party appears (in parentheses) its number of parliamentary seats. Parties typed in bold were members of the (original) governmental coalition, while the party typed in italic bold was the party charged with the formation of the governmental coalition. For each election in these two tables the quota is indicated between square brackets. Note that in Annex Table 5a the quota does not always constitute a simple majority of the listed parties' parliamentary seats; this is so because, as mentioned in footnote 17, de Swaan ignored parties who controlled no more than 2.5% of the seats.

All the three variations of the maximal SSQ version considered, namely the closed maximal SSQ variation, the closed maximal SSQ and minimal size variation, as well as the closed maximal SSQ and minimal range variation, performed better than the (pure) maximal SSQ variation both in terms of being more accurate in their predictions as well as in terms of their predictions being more parsimonious. However, these variations, too, are considerably inferior when compared to either the Gamson-Riker minimal size theory or to the Axelrod-Leiserson minimal range theory.

Like Aumann's hypothesis, Gamson-Riker's *cheapest coalition* or *minimum size* principle is simple and its informational requirements are similar. Unlike Aumann's hypothesis, however, it is extremely parsimonious and often produces predicted sets that are very small relative to the number of winning coalitions. As can be seen from Table 1, this principle works reasonably well for de Swaan's data set with 26 consistent predictions in 65 elections, or 10.484 standard errors above its randomized mean. As can also be seen from Table 1, this principle was more parsimonious and produced more accurate predictions than any of the three variations of Aumann's hypothesis that we examined. Also, its performance is inferior when compared with that achieved under the Leiserson-Axelrod *minimal range* theory. As pointed by Aumann, as well as by others before him, an apparent key weakness of forming a cheapest coalition (or a coalition of minimum size) is that the defection of even the smallest of its members turns it into a losing coalition, thereby apparently making the prospect of forming such a coalition very unattractive for a coalition leader. Nevertheless, there seem to be also considerable merits in forming coalitions of minimal size, as 40% of the examined coalitions in de Swaan's dataset were of this type.

As can be seen from Table 1, Annex Table 1 and Annex Table 3, both the predictive performance of the Leiserson-Axelrod *closed minimal range* theory as well as its parsimony were the best: of the 65 elections in de Swaan's data set, this theory predicted correctly the (original) governmental coalition formed immediately following 34 (52%) of these elections, and was the only theory of those investigated which scored two correct predictions in the detailed 18-election Israeli data set.

5. Concluding Remarks

According to the *Social Science Citation Index* the Shapley-Shubik (1954) paper is still, more than 50 years after it was first published, one of the most cited papers in social science research. Nevertheless, it is very doubtful whether many politicians have read this paper, or even heard about the Shapley-Shubik voting-power index, let alone use it as a tool for deciding which governing coalition they should form, join, or defect from. Aumann, in proposing his hypothesis, was of course aware of this when he responded to his interviewer that "…for good science it is not important that you understand it right off the bat. What is important, in the first instance is that it is correct, that it *works*. If it works, then that in itself tells us that the Shapley value is relevant."

So, in conclusion, does it work?

As can be seen from Table 1, as well as from Annex Table 1, Annex Table 2, and Annex Table 3, the answer is clearly 'no'. Aumann's hypothesis in its (pure) SSQ version achieved the smallest number of consistent predictions for both data sets considered, it lacks the parsimony required of a good predictive theory and hence should be rejected.

In conclusion, it appears that considerations of maximizing a priori voting power by the party charged with forming a governmental coalition – which seem quite reasonable from the point of view of game theory – do not account for which governing coalitions are formed in practice following general elections. This conclusion corroborates the (negative) findings of Felsenthal and Machover (1998) regarding Israeli governing coalitions during the period 1977–1996. It is also in line with the (negative) findings by Riker (1959) and by Felsenthal and Machover (2000) who discovered that defections from governing coalitions cannot be accounted for by considerations of a priori voting power.

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Table 1: Summary Test Statistics on the Predictive Accuracy of the Theories

	Maximal SSQ	Closed and Maximal SSQ	Closed Maximal SSQ and Minimal Size	Minimum Size	Closed Maximal SSQ and Minimal Range (Interval)	Closed Minimal Range (Interval)
De Swaan's Data						
Mean Number of Successes (Randomized Scheme)	**13.2115**	**7.2973**	**4.2307**	**4.6451**	**4.7588**	**5.5605**
Standard Deviation (Randomized Scheme)	**2.8527**	**2.3299**	**1.9454**	**2.0370**	**2.0411**	**2.1892**
Number of Consistent Predictions	**8**	**13**	**12**	**26**	**13**	**34**
Standardized Deviation from Randomized Mean	**-1.8269**	**2.4477**	**3.9937**	**10.4836**	**4.0376**	**12.9910**
Israel (1949-2006 Data)						
Mean Number of Successes (Randomized Scheme)	**0.3790**	**0.0133**	**0.0132**	**0.4013**	**0.0132**	**0.0195**
Number of Consistent Predictions	**0**	**0**	**0**	**1**	**0**	**2**
$P_j(x \geq k_j^*)$	**1.0000**	**1.0000**	**1.0000**	**0.3346**	**1.0000**	**0.000159**

APPENDIX

Annex Table 1: Summary of Predictive Accuracy (de Swaan's Data)

No	Country	Number of Elections	Maximal SSQ	Closed and Maximal SSQ	Closed Maximal SSQ and Minimum Size	Minimum Size	Closed Maximal SSQ and Minimal Range (Interval)	Closed Minimal Range (Interval)
1	Denmark	17	3/48	6/30	6/17	16/18	6/21	16/22
2	Finland	10	3/38	4/16	4/10	1/13	4/10	2/13
3	France	1	0/16	0/1	0/1	0/1	0/1	0/2
4	Israel	6	0/18	0/7	0/7	0/13	0/6	1/10
5	Italy	2	0/10	0/3	0/2	1/2	0/3	1/4
6	Norway	4	0/10	2/5	2/4	4/4	2/5	3/7
7	Sweden	7	1/29	0/13	0/7	3/7	0/8	5/9
8	The Netherlands	15	1/57	1/29	0/15	1/29	1/17	6/21
9	Weimar Republic	3	0/14	0/4	0/3	0/5	0/3	0/5
	Total	**65**	**8/243**	**13/108**	**12/66**	**26/92**	**13/74**	**34/93**
	Mean Number of Successes (Randomized Scheme)	*	**13.2115**	**7.2973**	**4.2307**	**4.6451**	**4.7588**	**5.5605**
	Standard Deviation (Randomized Scheme)	*	**2.8527**	**2.3299**	**1.9454**	**2.0370**	**2.0411**	**2.1892**
	Standardized Deviation from Randomized Mean	*	**-1.8269**	**2.4477**	**3.9937**	**10.4836**	**4.0376**	**12.9910**

Annex Table 2: Detailed Tabulation of Predictive Performance by Country

No	Election Year	# Winning Coalitions	# Closed Winning Coalitions	Maximal SSQ	Closed and Maximal SSQ	Closed Maximal SSQ and Minimum Size	Minimum Size	Closed Maximal SSQ and Minimal Range (Interval)	Closed Minimal Range (Interval)
a. Denmark									
1	1918	6	5	1/4	1/3	1/1	1/1	1/2	1/2
2	1920	6	4	0/1	1/3	1/1	1/1	1/1	1/1
3	1920A	6	4	0/1	1/3	1/1	1/1	1/1	1/1
4	1920B	6	4	1/4	1/3	1/1	1/1	1/1	1/1
5	1924	7	3	0/3	0/1	0/1	1/1	0/1	1/1
6	1926	6	4	1/4	1/3	1/1	1/1	1/1	1/1
7	1929	7	3	0/3	0/1	0/1	1/1	0/1	1/1
8	1932	7	3	0/3	0/1	0/1	1/1	0/1	1/1
9	1935	7	3	0/1	0/1	0/1	1/1	0/1	1/1
10	1939	7	3	0/1	0/1	0/1	1/1	0/1	1/1
11	1945	11	5	0/4	0/1	0/1	1/1	0/1	1/2
12	1953	14	6	0/1	0/1	0/1	1/1	0/1	1/1
13	1957	13	3	0/2	0/1	0/1	1/1	0/1	0/1
14	1960	15	7	0/4	0/2	0/1	1/2	0/2	1/2
15	1966	14	6	0/6	0/2	0/1	1/1	0/2	1/1
16	1968	10	5	0/2	1/1	1/1	0/1	1/1	1/2
17	1971	15	7	0/4	0/2	0/1	1/1	0/2	1/2
	Total	*	*	**3/48**	**6/30**	**6/17**	**16/18**	**6/21**	**16/22**

Annex Table 2: Detailed Tabulation of Predictive Performance by Country (cont'd)

No	Election Year	# Winning Coalitions	# Closed Winning Coalitions	Maximal SSQ	Closed and Maximal SSQ	Closed Maximal SSQ and Minimum Size	Minimum Size	Closed Maximal SSQ and Minimal Range (Interval)	Closed Minimal Range (Interval)
b. Finland									
1	1924	20	3	0/2	1/1	1/1	0/1	1/1	1/1
2	1930	12	2	1/8	1/1	1/1	0/1	1/1	1/1
3	1933	17	6	0/3	1/1	1/1	1/2	1/1	0/1
4	1939	16	6	0/1	0/1	0/1	0/1	0/1	0/1
5	1951	24	9	1/9	0/4	0/1	0/3	0/1	0/1
6	1954	18	5	0/2	0/1	0/1	0/1	0/1	0/2
7	1958	43	8	0/5	0/1	0/1	0/1	0/1	0/1
8	1962	23	8	0/3	0/2	0/1	0/1	0/1	0/2
9	1966	49	9	1/2	1/1	1/1	0/1	1/1	0/2
10	1970	42	9	0/3	0/3	0/1	0/1	0/1	0/1
	Total	*	*	**3/38**	**4/16**	**4/10**	**1/13**	**4/10**	**2/13**
c. France									
1	1947	44	9	0/16	0/1	0/1	0/1	0/1	0/2
	Total	*	*	**0/16**	**0/1**	**0/1**	**0/1**	**0/1**	**0/2**

Annex Table 2: Detailed Tabulation of Predictive Performance by Country (cont'd)

No	Election Year	# Winning Coalitions	# Closed Winning Coalitions	Maximal SSQ	Closed and Maximal SSQ	Closed Maximal SSQ and Minimum Size	Minimum Size	Closed Maximal SSQ and Minimal Range (Interval)	Closed Minimal Range (Interval)
d. Israel									
1	1949	115	16	0/5	0/1	0/1	0/1	0/1	0/1
2	1955	226	19	0/5	0/2	0/2	0/6	0/1	0/1
3	1959	118	15	0/3	0/1	0/1	0/2	0/1	0/3
4	1961	112	15	0/2	0/1	0/1	0/1	0/1	1/1
5	1965	54	9	0/2	0/1	0/1	0/2	0/1	0/2
6	1969	60	9	0/1	0/1	0/1	0/1	0/1	0/2
	Total	*	*	**0/18**	**0/7**	**0/7**	**0/13**	**0/6**	**1/10**
e. Italy									
1	1946	52	11	0/9	0/1	0/1	0/1	0/1	0/1
2	1972	29	9	0/1	0/2	0/1	1/1	0/2	1/3
	Total	*	*	**0/10**	**0/3**	**0/2**	**1/2**	**0/3**	**1/4**
f. Norway									
1	1936	7	3	0/3	0/1	0/1	1/1	0/1	0/1
2	1961	31	9	0/5	0/2	0/1	1/1	0/2	1/2
3	1965	9	3	0/1	1/1	1/1	1/1	1/1	1/2
4	1969	9	3	0/1	1/1	1/1	1/1	1/1	1/2
	Total	*	*	**0/10**	**2/5**	**2/4**	**4/4**	**2/5**	**3/7**

Annex Table 2: Detailed Tabulation of Predictive Performance by Country (cont'd)

No	Election Year	# Winning Coalitions	# Closed Winning Coalitions	Maximal SSQ	Closed and Maximal SSQ	Closed Maximal SSQ and Minimum Size	Minimum Size	Closed Maximal SSQ and Minimal Range (Interval)	Closed Minimal Range (Interval)
g. Sweden									
1	1917	12	6	1/8	0/4	0/1	0/1	0/1	0/1
2	1924	7	3	0/3	0/1	0/1	0/1	0/1	0/1
3	1932	14	6	0/6	0/2	0/1	0/1	0/1	1/1
4	1936	15	7	0/1	0/1	0/1	0/1	0/1	1/2
5	1952	7	3	0/1	0/1	0/1	1/1	0/1	1/1
6	1956	14	6	0/6	0/2	0/1	1/1	0/1	1/1
7	1970	15	7	0/4	0/2	0/1	1/1	0/2	1/2
	Total	*	*	**1/29**	**0/13**	**0/7**	**3/7**	**0/8**	**5/9**
h. The Netherlands									
1	1918	95	14	0/1	0/1	0/1	1/3	0/1	1/2
2	1922	25	9	0/5	0/1	0/1	0/1	0/1	1/3
3	1925	19	8	0/2	0/1	0/1	0/1	0/1	1/2
4	1929	48	11	0/2	0/3	0/1	0/1	0/1	1/2
5	1933	79	14	0/5	0/1	0/1	0/1	0/1	0/1
6	1937	83	15	0/2	0/2	0/1	0/3	0/2	1/2
7	1946	24	10	0/2	0/1	0/1	0/1	0/1	1/1
8	1948	21	8	1/8	0/3	0/1	0/1	0/1	0/1
9	1952	23	8	0/9	0/4	0/1	0/2	0/1	0/1
10	1956	24	8	0/1	0/4	0/1	0/1	0/1	0/1

Annex Table 2: Detailed Tabulation of Predictive Performance by Country (cont'd)

No	Election Year	# Winning Coalitions	# Closed Winning Coalitions	Maximal SSQ	Closed and Maximal SSQ	Closed Maximal SSQ and Minimum Size	Minimum Size	Closed Maximal SSQ and Minimal Range (Interval)	Closed Minimal Range (Interval)
h. The Netherlands (cont'd)									
11	1959	12	6	0/1	0/1	0/1	0/1	0/1	0/1
12	1963	48	14	0/5	0/2	0/1	0/1	0/1	0/1
13	1967	192	20	0/10	0/2	0/1	0/4	0/1	0/1
14	1971	64	11	0/1	1/2	0/1	0/2	1/2	0/1
15	1973	202	15	0/3	0/1	0/1	0/6	0/1	0/1
	Total	*	*	**1/57**	**1/29**	**0/15**	**1/29**	**1/17**	**6/21**
i. Weimar Republic									
1	1919	56	10	0/2	0/1	0/1	0/1	0/1	0/1
2	1925	150	12	0/11	0/2	0/1	0/2	0/1	0/2
3	1928	105	9	0/1	0/1	0/1	0/2	0/1	0/2
	Total	*	*	**0/14**	**0/4**	**0/3**	**0/5**	**0/3**	**0/5**

Annex Table 3: Summary of Predictive Accuracy (Israel 1949-2006)

No	Election Year	# Winning Coalitions	# Closed Winning Coalitions	Maximal SSQ	Closed and Maximal SSQ	Closed Maximal SSQ and Minimum Size	Minimum Size	Closed Maximal SSQ and Minimal Range (Interval)	Closed Minimal Range (Interval)
1	1949	1883	30	0/4	0/1	0/1	0/18	0/1	0/1
2	1951	15262	42	0/21	0/1	0/1	0/193	0/1	0/1
3	1955	1867	30	0/2	0/1	0/1	0/24	0/1	0/1
4	1959	1961	35	0/2	0/1	0/1	0/16	0/1	0/1
5	1961	929	25	0/1	0/1	0/1	0/17	0/1	0/4
6	1965	3733	41	0/3	0/1	0/1	0/60	0/1	0/1
7	1969	4069	44	0/3	0/1	0/1	0/18	0/1	0/2
8	1973	454	25	0/2	0/1	0/1	0/13	0/1	0/1
9	1977	3470	14	0/1	0/1	0/1	0/58	0/1	0/1
10	1981	384	13	0/4	0/1	0/1	0/20	0/1	0/2
11	1984	12875	27	0/3	0/1	0/1	0/587	0/2	0/1
12	1988	12288	30	0/1	0/3	0/1	0/492	0/1	0/1
13	1992	440	22	0/9	0/1	0/1	0/10	0/1	1/1
14	1996	739	12	0/246	0/1	0/1	0/20	0/1	0/1
15	1999	12703	39	0/6	0/1	0/1	0/301	0/1	0/3
16	2001	11059	23	0/1	0/1	0/1	0/297	0/1	0/2
17	2003	3618	16	0/3	0/1	0/1	0/57	0/1	0/1
18	2006	1641	33	0/1	0/1	0/1	1/33	0/1	1/1
Total		89375	501	0/313	0/20	0/18	1/2234	0/19	2/26
Mean Number of Successes (Randomized Scheme)		*	*	**0.3790**	**0.0133**	**0.0132**	**0.4013**	**0.0132**	**0.0195**
Number of Consistent Predictions				**0**	**0**	**0**	**1**	**0**	**2**
$P_j(x \geq k_j^*)$		*	*	**1.0000**	**1.0000**	**1.0000**	**0.3346**	**1.0000**	**0.000159**

Annex Table 4: Exact Probability Mass Function $P_j(x = k)$ for Randomized Scheme under the Minimum Size and Closed Minimal Range Theory

Number of Successes (k)	^nC_k	Minimum Size	Closed Minimal Range (Interval)
0	1	0.66543	0.98067
1	18	0.27515	0.019176
2	153	0.052672	0.000158
3	816	0.0062037	7.31E-07
4	3060	0.00050386	2.12E-09
5	8568	3.00E-05	4.15E-12
6	18564	1.35E-06	5.68E-15
7	31824	4.74E-08	5.60E-18
8	43758	1.31E-09	4.07E-21
9	48620	2.85E-11	2.21E-24
10	43758	4.95E-13	8.99E-28
11	31824	6.79E-15	2.74E-31
12	18564	7.32E-17	6.25E-35
13	8568	6.11E-19	1.05E-38
14	3060	3.85E-21	1.26E-42
15	816	1.77E-23	1.05E-46
16	153	5.56E-26	5.72E-51
17	18	1.06E-28	1.80E-55
18	1	9.31E-32	2.49E-60
Sum		1.00000	1.00000

Annex Table 5a: Seat Distribution of Parties in each Election by Country and Election Year ordered on the Left-Right Ideological Continuum (De Swaan's Data)

Denmark

1918	***SOCD***(39) **RAD**(33) LIB(45) CONS(22)	[Quota=71]
1920	SOCD(42) RAD(17) ***LIB***(49) **CONS**(28)	[Quota=71]
1920A	SOCD(42) RAD(16) ***LIB***(52) **CONS**(26)	[Quota=71]
1920B	SOCD(48) RAD(18) ***LIB***(52) **CONS**(27)	[Quota=75]
1924	***SOCD***(55) **RAD**(20) LIB(45) CONS(28)	[Quota=75]
1926	SOCD(53) RAD(16) ***LIB***(47) **CONS**(30)	[Quota=75]
1929	***SOCD***(61) **RAD**(16) LIB(44) CONS(24)	[Quota=75]
1932	***SOCD***(62) **RAD**(14) LIB(39) CONS(27)	[Quota=75]
1935	***SOCD***(68) **RAD**(14) LIB(29) CONS(26)	[Quota=75]
1939	***SOCD***(64) **RAD**(14) LIB(31) CONS(26)	[Quota=75]
1945	COMM(18) SOCD(48) **RAD**(11) ***LIB***(38) **CONS**(26)	[Quota=75]
1953	COMM(8) ***SOCD***(75) **RAD**(14) LIB(43) CONS(30)	[Quota=89]
1957	***SOCD***(70) **RAD**(14) LIB(46) CONS(30) **JUST**(9)	[Quota=88]
1960	LSOC(11) ***SOCD***(78) **RAD**(11) LIB(39) CONS(32)	[Quota=89]
1966	**LSOC**(20) ***SOCD***(70) RAD(13) LIB(35) CONS(34)	[Quota=89]
1968	LSOC(11) SOCD(63) **RAD**(27) ***LIB***(34) **CONS**(37)	[Quota=89]
1971	**LSOC**(17) ***SOCD***(73) RAD(27) LIB(30) CONS(31)	[Quota=90]

Party Abbreviation:

SOCD	Social Democrats (Socialdemokratiet)
CONS	Conservative People's Party (Det Konservative Folkeparti)
COMM	Communist Party (Danmarks Kommunistiske Parti)
JUST	Justice Party (Danmarks Retsforbund)
LIB	Liberals (Venstre)
LSOC	Socialist People's Party (Socialistisk Folkeparti)
RAD	Danish Social Liberal Party (Det Radikale Venstre)

Finland [Quota=101]

1924	SOCL(18) SOCD(60) ***AGR***(44) **PROG**(17) **SWPP**(23) **CONS**(38)
1930	SOCD(66) **AGR**(59) **PROG**(11) **SWPP**(21) ***CONS***(42)
1933	SOCD(78) **AGR**(53) ***PROG***(11) **SWPP**(21) **CONS**(18) NPF(14)
1939	**SOCD**(85) **AGR**(56) ***PROG***(6) **SWPP**(18) CONS(25) NPF(8)
1951	PDEM(43) **SOCD**(53) ***AGR***(51) PROG(10) **SWPP**(15) CONS(28)

1954	PDEM(43) **SOCD**(54) **AGR**(53) FPP(13) ***SWPP***(13) CONS(24)
1958	PDEM(50) ***SOCD***(38) SDWS(13) **AGR**(48) **FPP**(8) **SWPP**(14) **CONS**(29)
1962	PDEM(47) SOCD(39) ***AGR***(53) **FPP**(13) **SWPP**(14) **CONS**(32)
1966	**PDEM**(41) ***SOCD***(55) **SDWS**(7) **CENT**(49) LIB(9) SWPP(12) CONS(26)
1970	**PDEM**(36) **SOCD**(52) ***CENT***(37) **LIB**(8) **SWPP**(12) CONS(37) PAES(18)

<u>Party Abbreviation</u>:

AGR	Agrarian League
CENT	Centre Party of Finland or Finnish Centre (Suomen Keskusta, KESK)
CONS	National Coalition Party (Kansallinen Kokoomus or Samlingspartiet)
FPP	Finnish People's Party (Kansanpuolue)
LIB	Liberals (Liberaalit)
NPF	Patriotic People's Movement (Isänmaallinen Kansanliike)
PAES	Finnish Rural Party (Suomen Maaseudun Puolue)
PDEM	Finnish People's Democratic League (Suomen kansan demokraattinen liitto)
PROG	National Progressive Party (Kansallinen Edistyspuolue)
SDWS	Social Democratic Union of Workers and Small Farmers (Työvaen ja Pienviljelijain Sosialidemokraattinen Liitto)
SOCD	Finland Social Democratic Party (Suomen Sosialidemokraattinen Puolue)
SOCL	Communist Party of Finland (Suomen Kommunistinen Puolue)
SWPP	Swedish People's Party (Svenska Folkpartiet)

<u>France</u> [Quota=310]

1947	**PCF**(182) ***SFIO***(104) **MRP**(167) **RAD**(43) **UDSR**(26) **RI**(28) PRL(35)

<u>Party Abbreviation</u>:

MRP	National Republican Movement (Mouvement Répubicain Populaire)
PCF	French Communist Party (Parti Communiste Français)
PRL	Republican Party of Liberty (Parti Républicain de la Liberté)
RAD	Radical Party (Parti Républicain Radical et Radical-Socialiste)
RI	Independent Republicans (Républicains Indépendants)
SFIO	French Section of the Workers' International (Section Française de l'International Ouvrière)
UDSR	Democratic and Socialist Union of the Resistance (Union démocratique et socialiste de la résistance)

<u>Israel</u> [Quota=61]

1949	CPI(4) SOCL(19) ***SOCD***(46) **SEPH**(4) **NREL**(16) **PROG**(5) ZION (7)
1955	CPI(6) **SOCL**(9) **AGAV**(10) ***SOCD***(40) **MAFD**(11) **PROG**(5) AGDH(6) ZION(13) HRUT(15)
1959	**SOCL**(9) **AGAV**(7) ***SOCD***(47) **MAFD**(12) **PROG**(6) AGDH(6) ZION(8) HRUT(17)

1961	CPI(5) SOCL(9) **AGAV**(8) ***SOCD***(42) **MAFD**(12) AGDH(6) LIB(17) HRUT(17)
1965	**SOCL**(8) ***SOCD***(45) RAFI(10) **MAFD**(11) AGDH(6) **ILIB**(5) GACH(26)
1969	MAKI(4) ***ILAB***(56) STP(4) **MAFD**(12) AGDH(6) **ILIB**(4) GACH(26)

<u>Party Abbreviation</u>:

AGAV	Labor Unity (Achdut Ha'avodah)
AGDH	Union of Israel (Agudat Israel)
CPI	Communist Party of Israel (Maki or Miflaga HaKomunistit HaYisraelit)
GACH	Gahal (Gush Herut-Liberalim)
HRUT	National Jewish Movement or Freedom (Herut)
ILIB	Independent Liberal Party (Liberalim Atzmaiyim)
ILAB	Israeli Labor Party (Mifleget Ha-Avodah Ha-Yisraelit)
LIB	Liberal Party (Miflaga Liberalit)
MAFD	National Religious Party (Mafdal or Miflaga Datit Le'umit)
MAKI	Communist Party of Israel (Maki or Miflaga HaKomunistit HaYisraelit)
NREL	United Religious Front (Alliance comprising Mizrachi, HaPoel HaMizrahi, Poalei Agudat Israel and Agudat Israel)
PROG	Progressive Party (Miflaga Progresivit)
RAFI	Israel Labor List (Reshimat Po'aley Yisrael)
SEPH	Sephardim and the Edot of the Mizrah
SOCD	Worker's Party of Eretz Yisrael (Mapai or Mifleget Poalei Eretz Yisrael)
SOCL	United Workers' Party (Mapam or Mifleget Ha-Po'alim Ha-Me'ukhdet)
STP	State List (Hareshima Hamamlachtit)
ZION	General Zionists

<u>Italy</u>

1946	**PCI**(104) **PSI**(115) **PRI**(23) ***DCHR***(207) **PLI**(41) MON(16) UOMO(30)	[Quota=279]
1972	PCI(170) PSI(61) **PSDI**(29) ***DCHR***(267) **PLI**(20) MSI(56)	[Quota=316]

<u>Party Abbreviation</u>:

DCHR	Christian Democracy (Democrazia Cristiana)
MON	National Monarchist Party (Partito Nazionale Monarchico)
MSI	Italian Social Movement (Movimento Sociale Italiano)
PCI	Italian Communist Party (Partito Comunista Italiano)
PLI	Italian Liberal Party (Partito Liberale Italiano)
PRI	Italian Republican Party (Partito Repubblicano Italiano)
PSDI	Italian Democratic Socialist Party (Partito Socialista Democratico Italiano)
PSI	Italian Socialist Party (Partito Socialista Italiano)

UOMO	Common Man Party (Uomo Qualunque)

Norway [Quota = 76]

1936	***SOCD***(70) LIB(23) **AGR**(18) CONS(36)
1961	**SOCL**(2) ***SOCD***(74) LIB(14) CHPP(15) CENT(16) CONS(29)
1965	SOCD(68) **LIB**(18) **CHPP**(13) ***CENT***(18) **CONS**(31)
1969	SOCD(74) **LIB**(13) **CHPP**(14) ***CENT***(20) **CONS**(29)

Party Abbreviation:

AGR	Agrarian Party (Bondepartiet)
CENT	Centre Party (Senterpartiet)
CONS	Conservative Party (Høyre)
CHPP	Christian People's Party (Kristelig Folkeparti)
LIB	Liberal Left (Venstre)
SOCD	Labor Party (Arbeiderpartiet)

Sweden

1917	COMM(11) **SOCD**(86) AGR(12) ***LIB***(62) CONS(59)	[Quota=116]
1924	***SOCD***(105) AGR(23) **LIB**(33) CONS(65)	[Quota=116]
1932	COMM(8) ***SOCD***(104) **AGR**(36) LIB(24) CONS(58)	[Quota=116]
1936	COMM(6) ***SOCD***(112) **AGR**(36) LIB(27) CONS(44)	[Quota=116]
1952	***SOCD***(110) **AGR**(26) LIB(58) CONS(31)	[Quota=116]
1956	COMM(6) ***SOCD***(106) **CENT**(19) LIB(58) CONS(42)	[Quota=116]
1970	**COMM**(17) ***SOCD***(163) CENT(71) LIB(58) CONS(41)	[Quota=176]

Party Abbreviation:

AGR	Farmers' League (Bondeförbundet) Centre Party (Centerpartiet)
CENT	Centre Party (Centerpartiet)
COMM	Communist Party of Sweden (Sveriges Kommunistiska Parti)
CONS	Right-wing Party (Högerpartiet)
LIB	The Liberal (People's) Party ((Folkpartiet Liberalerna)
SOCD	Social Democrats (Socialdemokraterna)

The Netherlands

1918	SOCD(22) RAD(5) ***CATH***(30) **ARP**(13) **CHU**(7) ULIB(6) ECUN(3) FLIB(4)	[Quota=50]
1922	SOCD(20) RAD(5) ***CATH***(32) **ARP**(16) **CHU**(11) LIB(10)	[Quota=51]
1925	SOCD(24) RAD(7) **CATH**(31) ***ARP***(13) **CHU**(11) LIB(9)	[Quota=51]

1929	SOCD(24) RAD(7) ***CATH***(30) **ARP**(12) **CHU**(11) LIB(8) SQP(3)	[Quota=51]
1933	CPN(4) SOCD(24) **RAD**(7) **CATH**(28) ***ARP***(14) **CHU**(10) **LIB**(7) SGP(3)	[Quota=51]
1937	CPN(3) SOCD(23) RAD(6) **CATH**(31) ***ARP***(17) **CHU**(8) LIB(4) NSB(4)	[Quota=51]
1946	COMM(10) **SOCD**(29) ***CATH***(32) ARP(13) CHU(8) LIB(6)	[Quota=51]
1948	COMM(8) ***SOCD***(27) **CATH**(32) ARP(13) CHU(9) LIB(8)	[Quota=51]
1952	COMM(6) ***SOCD***(30) **CATH**(30) **ARP**(12) **CHU**(9) LIB(9)	[Quota=51]
1956	COMM(7) ***SOCD***(50) **CATH**(49) **ARP**(15) **CHU**(13) LIB(13)	[Quota=76]
1959	SOCD(48) ***CATH***(49) **ARP**(14) **CHU**(12) **LIB**(19)	[Quota=75]
1963	CPN(4) PSP(4) SOCD(43) ***CATH***(50) **ARP**(13) **CHU**(13) **LIB**(16)	[Quota=76]
1967	CPN(5) PSP(4) SOCD(37) D'66(7) ***CATH***(42) **ARP**(15) **CHU**(12) **LIB**(17) BP(7) [	Quota=76]
1971	CPN(6) SOCD(39) D'66(11) **CATH**(35) ***ARP***(13) **CHU**(10) **DS70**(8) **LIB**(16)	[Quota=76]
1973	COMM(7) **PPR**(7) ***SOCD***(43) **D'66**(6) **CATH**(27) **ARP**(14) CHU(7) DS70(6) LIB(22)	[Quota=76]

Party Abbreviation:

ARP	Anti-Revolutionary Party (Anti-Revolutionaire Partij)
BP	Farmers' Party (Boerenpartij)
CATH	Roman Catholic League of Electoral Associations (Roomsch-Katholieke Bond van kiesvereenigingen), Roman Catholic Political Party (Roomsch-Katholieke Staatspartij), Catholic Peoples Party (Katholieke Volkspartij)
CHU	Christian-Historical Union (Christelijke-Historische Unie)
COMM	Communist Party of Netherlands (Communistische Partij Nederland)
CPN	Communist Party of Holland (Communistische Partij Holland)
D'66	Democrats 66 (Democraten 66)
DS70	Democratic Socialists '70 (Democratisch-Socialisten 1970)
ECUN	Economical League (Economische Bond)
FLIB	Association of Free Liberals (Bond van Vrije Liberalen)
LIB	Liberal State Party (Liberale Staatspartij), People's Party for Freedom and Democracy (Volkspartij voor Vrijheid en Democratie), Freedom Party (Partij voor de Vrijheid)
NSB	National Socialist Movement (Nationaal-Socialistische Beweging)
PPR	Political Party Radicals (Politieke Partij Radicalen)
PSP	Pacifist-Socialist Party (Pacifistisch Socialistische Partij)
RAD	Freethinking Democratic League (Vrijzinnig Democratische Bond)
SOCD	Social Democratic Workers Party (Sociaal Democratische Arbeiders Partij, Labour Party (Partij van de Arbeid)
SGP	Political Reformed Party (Staatkundig Gereformeerde Partij)
ULIB	Liberal Union (Liberale Unie)

Weimar Republic

1919	USPD(22) *SPD*(165) **DEMP**(75) **ZENT**(73) BAYR(18) VOLK(19) NATP(44)	[Quota=212]

1925	KDP(45) SDAP(131) DEMP(32) **ZENT**(69) **BAYR**(19) ***VOLK***(51) WIRT(17) **NATP**(111) NAZI(14)	[Quota=247]
1928	KDP(54) ***SDAP***(153) **DEMP**(25) **ZENT**(62) **BAYR**(16) **VOLK**(45) WIRT(23) NATP(78)	[Quota=246]

Party Abbreviation:

BAYR	Bavarian People's Party (Bayerische Volkspartei)
DEMP	German Democratic Party (Deutsche Demokratische Partei)
KDP	Communist Party of Germany (Kommunistische Partei Deutschlands)
NATP	German National People's Party (Deutschnationale Volkspartei)
NAZI	National Socialist German Workers Party (Nationalsozialistische Deutsche Arbeiterpartei)
SDAP	Social Democratic Party of Germany (Sozialdemokratische Partei Deutschlands)
SPD	Socialist Party of Germany (Sozialistische Partei Deutschlands)
USPD	Independent Social Democratic Party of Germany (Unabhängige Sozialdemokratische Partei Deutschlands)
WIRT	Business Party (Wirtschaftspartei des Deutschen Mittelstandes)
VOLK	German People's Party (Deutsche Volkspartei)
ZENT	German Centre Party (Deutsche Zentrumspartei)

Annex Table 5b: Detailed Seat Distribution of Parties in each Israeli Election Year ordered on the Left-Right Ideological Continuum (Quota for all elections is 61)

1949	CPI(4) SOCL(19) **DPNZ** (2) ***SOCD***(46) **SEPH**(4) YEMA(1) **NREL**(16) **PROG**(5) WIZO(1) ZION(7) HRUT(14) FIGL(1)
1951	CPI(5) SOCL(15) **DLIA**(3) **KIVA**(1) **HAK** (1) ***SOCD***(45) SEPH(2) YA(1) **HAHA**(8) **HAMI**(2) PROG(4) **PAGI**(2)**AGDH**(3) ZION(20) HRUT(8)
1955	CPI(6) **SOCL**(9) **AGAV**(10) **DLIA**(2) **KIDVA**(2) **HAK** (1) ***SOCD***(40) **MAFD**(11) **PROG**(5) AGDH(6) ZION(13) HRUT(15)
1959	CPI(3) **SOCL**(9) **AGAV**(7) **DLIA** (2) **KIDVA**(2) **HAK** (1) ***SOCD***(47) **MAFD**(12) **PROG**(6) AGHD(6) ZION(8) HRUT(17)
1961	CPI(5) SOCL(9) **AGAV**(8) **SHIV** (2) **KIDMA** (2) ***SOCD***(42) **MAFD**(12) **PAGI**(2) AGDH(4) LIB(17) HRUT(17)
1965	CPI(1) RAKACH(3) HOH(1) **SOCL**(8) **SHIV** (2) **KIDMA**(2) ***SOCD***(45) RAFI(10) **MAFD**(11) **PAGI**(2) AGDH(4) **ILIB**(5) GACH(26)
1969	MAKI(1) RAKACH(3) HOH(2) **SHIV** (2) **KIDMA** (2) ***ILAB***(56) STP(4) **MAFD**(12) PAGI(2) AGDH(4) **ILIB**(4) HH(2) **GACH**(26)
1973	RAKACH(4) MOKED(1) RATZ(3) ALBV (1) KIDMA(2) ***ALIGN*** (51) **MAFD** (10) RELFRNT(5) **ILIB**(4) LIKUD(39)
1977	HADASH(5) SHELI(2) RATZ(1) UAL(1) ALIGN(32) **DASH** (15) **MAFD** (12) ILIB(1) PAGI(1) **AGDH** (4) PLATO(1) ***LIKUD***(43) **SHLMZION**(2)
1981	HADASH(4) RATZ(1) SHINUI(2) ALIGN(47) **TELEM**(2) **TAMI**(3) **AGDH** (4) **MAFD**(6) ***LIKUD***(48) TEHIYA(3)
1984	HADASH(4) RATZ(3) **SHINUI**(3) PROGPEACE(2) ***ALIGN*** (44) **OMETZ**(1) TAMI(1) **YAHAD**(3) **MORASHA**(2) **SHAS**(4) AGDH (2) **MAFD** (4) **LIKUD**(41) TEHIYA(5) KACH(1)
1988	HADASH(4) ARABDEM(1) PROGPEAC(1) SOCL(3) RATZ(5) SHINUI(2) **ALIGN** (39) **SHAS**(6) **AGDH** (5) **DEGEL**(2) TSOMET(2) ***LIKUD***(40) **MAFD**(5) TEHIYA(3) MOLEDET(2)
1992	HADASH(3) ARABDEM(2) **MERETZ**(12) ***ILAB***(44) **SHAS**(6) YDUTTORA(4) MAFD(6) TSOMET(8) LIKUD(32) MOLEDET(3)
1996	HADASH(5) UARAB(4) MERETZ(9) ILAB(34) **THIRDWAY**(4) **ISRALIYA**(7) **SHAS**(10) **YDUTTORA**(4) **MAFD**(9) ***LIKUD***(32) MOLEDET(2)
1999	HADASH(3) BALAD(2) UARAB(5) **MERETZ**(10) AMEHAD(2) ***ONEISR***(26) SHINUI(6) **CENTER**(6) **SHAS**(17) YDUTTORA(5) **MAFD**(5) **ISRALIYA**(6) LIKUD(19) IHUDLUMI(4) YISRBEIT(4)

2001[23]	HADASH(3) BALAD(2) UARAB(5) MERETZ(10) **AMEHA**D(2) **ONEISR**(26) SHINUI(6) CENTER(6) **SHAS**(17) **YDUTTORA**(5) **MAFD**(5) **ISRALIYA**(6) ***LIKUD*** (19) **IHUDLUMI**(4) **YISRBEIT**(4)
2003	HADASH(3) BALAD(3) UARAB(2) MERETZ(6) AMEHAD(3) ILAB(19) **SHINUI**(15) SHAS(11) YDUTTORA(5) **MAFD**(6) **ISRALIYA**(2) ***LIKUD***(38) **IHUDLUMI**(7)
2006	HADASH (3) BALAD(3) UARAB(4) MERETZ(5) **ILAB**(19) **GIL**(7) ***KADIMA***(29) **SHAS**(12**)** YDUTTORA(6**)** LIKUD(12) YISRBEIT(11) IHUDNRP(9)

Party Abbreviation

AGDH	Union of Israel (Agudat Israel)
AMEHAD	One People (Am Ehad)
ALAV	Arab List for Beduins and Villagers (Minority list associated with the Alignment)
ALIGN	Alignment (Maarach)
ARABDEM	Arab Democratic Part
BALAD	National Democratic Pact (Brit Le'Umit Democratit)
CENTER	Center Party (Mifleget Ha'Merkaz)
CPI	Communist Party of Israel (Maki or Miflaga HaKomunistit HaYisraelit)
DEGEL	Torah Flag (Degel Ha'Torah)
DASH	Democratic Movement for Peace
DLIA	Democratic List of Israeli Arabs (Arab list associated with SOCD)
DPNZ	Democratic Party of Nazareth (Arab list associated with SOCD)
FIGL	Fighters List
GACH	Gahal (Gush Herut-Liberalim)
GIL	Pensioners of Israel (Gimlaey Israel)
HADASH	Democratic Front for Peach and Equality (Hazit Democratit Le'Shalom Ve'Shivyon)
HAHA	Hapo'el Hamizrahi
HAMI	Hamizrahi
HAK	Agriculture and Development (Hakla'ut Uffituah – Arab list associated with SOCD)
HH	Free Centre (Ha'Merkaz Ha'Hofshi)
HOH	Ha'olam Hazeh–Koah Hadash
HRUT	National Jewish Movement or Freedom (Herut)
IHUDLUMI	National Unity (Ihud Le'Umi)

[23] As only the prime minister was directly elected in 2001, the distribution of seats among the parties in 2001 is the same as in 1999. However, the elected prime minister (Ariel Sharon, Likud) changed the composition of the governing coalition.

IHUDNRP	United Jewish National List (Ihud Le'Umi – Mafdal Reshima Yehoodit Leumit Meuhedet)
ILIB	Independent Liberal Party (Liberalim Atzmaiyim)
ILAB	Israeli Labor Party (Mifleget Ha-Avodah Ha-Yisraelit)
ISRALIYA	Israel Ba'Aliya
KACH	Kach (Rabbi Kahana)
KADIMA	Forward
KIDMA	Progress (Arab list associated with SOCD)
KIDVA	Progress and Labour (Kidma Va'avoda – Arab list associated with SOCD)
LIB	Liberal Party (Miflaga Liberalit)
LIKUD	Likud
MAFD	National Religious Party (Mafdal or Miflaga Datit Le'umit)
MAKI	Communist Party of Israel (Maki or Miflaga HaKomunistit HaYisraelit)
MERETZ	Meretz (Social Democratic Israel)
MOKED	Moked
MOLEDET	Fatherland
MORASHA	Morasha
NREL	United Religious Front (Alliance comprising Mizrachi, HaPoel HaMizrahi, Poalei Agudat Israel and Agudat Israel)
OMETZ	Courage
ONEISR	One Israel (Yisrael Ahat)
PAGI	Poalei Agudat Israel
PLATO	Plato-Sharon
PROG	Progressive Party (Miflaga Progresivit)
PROGPEACE	Progressive Movement for Peace
RAFI	Israel Labor List (Reshimat Po'aley Yisrael)
RAKACH	New Israel Communist Party (Reshima Komonistit Hadasha)
RATZ	Ratz
RELFRNT	United Torah Front
SEPH	Sephardim and the Edot of the Mizrah
SHAS	United Religious Sepharadic Jews
SHELI	Mahaneh Sheli
SHINUI	Shinui (Change)
SHIV	Cooperation and Fraternity (Shituf Ve'ahvah – Arab list associated with SOCD)
SHLMZION	Shlom Zion
SOCD	Worker's Party of Eretz Yisrael (Mapai or Mifleget Poalei Eretz Yisrael)
SOCL	United Workers' Party (Mapam or Mifleget Ha-Po'alim Ha-Me'ukhdet)
STP	State List (Hareshima Hamamlachtit)
TAMI	Movement for Renewal of Israel Tradition (Tenooa Le'Masoret Israel)

TEHIYA	Resurrection
TELEM	Movement for State Renewal (Tenooa Le'Hidhadshoot Mamlakhtit)
THIRDWAY	The Third Way
TSOMET	Crossroads
UARAB	United Arab List
WIZO	Women's International Zionist Organization
YAHAD	Together (Yahad)
YDUTTORA	Torah Jewry (Yahadut Ha'Torah)
YEMA	Yemenite Association
YISRBEIT	Israel Our Home (Yisrael Beitenu)
ZION	General Zionists