Extended Shapley-Owen Indices and Power Distribution in the III State Duma of the Russian Federation

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Abstract

New power indices extended well-known Shapley-Owen index are introduced. These indices are based on the consistency of groups' positions in spatial voting model in which each agent (group, faction) has preferences. These indices take into account the role of ideology in coalition formation, i.e., to what extent the preferences of coalescing factions are close.

The indices with and without taking into account the share of votes of each agent are introduced. An algorithm of their evaluation in the two-dimensional political space is given.

The analysis of power distribution among political factions in the III State Duma (2000-2003) of the Russian Federation is given.

1. Introduction

A well-known Shapley-Owen index takes into account agents' preferences to determine a possible coalition of voters according to their preferences closeness in political space. It is computed using the frequency of event, when the agent is pivotal. The pivotal (the formal definition is given below) agent is the agent who gives the last (decisive) vote to pass the law.

The suggested extensions of the Shapley-Owen index are those that we assign a weight to pivotal agent, computed by the consistency of agents' positions. The index of consistency of agents' political positions is based on the closeness of agents' preferences in political space, i.e., ideologically close agents behave identically while forming coalitions. A coalition will be formed rather by ideologically similar agents than by agents with different ideologies.

In Section 2 the Shapley-Owen scheme of the pivotal agent detecting and the power index computing is considered. In Section 3 an algorithm of power index evaluation in two-dimensional political space is examined. This algorithm was proposed in [6]. In Section 4 the different versions of extended Shapley-Owen index are introduced. The indices with and without taking into account the share of votes of each agent are studied. The first version of extended Shapley-Owen index is based on the distance between the agents' ideal points. Another version of extended Shapley-Owen index uses the agents' "center of mass". In Section 5 the analysis of power distribution among political factions in the III State Duma (2000-2003) of the Russian Federation is given.

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2. Shapley-Owen index

Simple games are widely used to describe coalition formation since L. Shapley and M. Shubik proposed to measure the power of an elected body's member by the frequency of event, when the agent is pivotal [8] (see also [5]). In their model it is assumed that all the coalitions are equally probable. They used for computing of agent's power index the approach in which only the information about the rule of decision making is important. The "non-symmetric" extension of Shapley-Shubik model was proposed by G. Owen and L. Shapley [7] (see also [4]). In this extension the power of an agent depends not only on the voting rule of decision making, but on the position of agents in political space, i.e., on ideology, as well. This index (the Shapley-Owen value, for short SOV) makes the special preference to the role of ideology in coalition formation, i.e., only the ideologically close agents will coalesce.

Let N be a finite set of n agents. Consider an n-person simple game in the form of characteristic function v, i.e., every coalition $S \subseteq N$ gets a payoff equal to 0 or 1. The coalition S with $v(S)=1$ is called winning coalition, otherwise it is called loosing. An agent i is called a pivot, if and only if the loosing coalition S becomes the winning one when the agent i joins the coalition, i.e., $v(S \cup \{i\})=1$.

Let each agent has its own ideal (bliss) point indicated as $P_i \in \Re^m$ in the m-dimensional Euclidean space. An ideal point reflects the preferred political outcome of each agent. Let $\Psi \subseteq \Re^m$ be the set of all voting outcomes. Each outcome is a vector $x \in \Psi$.

Let the function $u_i(x)$, $u_i : \Psi \to \Re^1$, measures the agent i's attitude to an outcome x. Using the values of this function, a strict order on the set N can be introduced, denoted as r. Thus, jri, if $u_j(x) - u_i(x) \geq 0$. This relation indicates that the agent j likes the outcome x more than the agent i.

Define $Y_{ij} = u_i(x) - u_j(x)$. Notice that if $Y_{ij} \leq 0$, then the agent j joins the coalition of agents supporting the outcome x more willingly than the agent i.

L. Shapley and G. Owen studied the power index in a spatial context. They considered the unit vectors $x \in \Re^m$ lying on the unit-sphere H^{m-1}, i.e., $\forall x \in \Psi \quad \langle x, x \rangle = 1$. Each vector defines a direction in

the space. They proposed that the values of function $u_i(x)$ is determined by the inner product, i.e., $u_i(x) = \langle x, P_i \rangle$.

Then each unit vector *x* randomly chosen from the uniform distribution induces an order relation r, such that, $i r_x j \Leftrightarrow \langle x, P_i \rangle \geq \langle x, P_j \rangle$.

The power index proposed by Shapley and Owen for the agent *i* can be written as ratio

$$SOV_i = \frac{q_i}{n!},$$

where q_i is the number of orderings for which the agent *i* is pivotal, and *n!* is the total number of all possible orderings.

3. SOV computational algorithm

This computational algorithm was proposed in [6]. The algorithm is the direct translation of the model discussed above. For this algorithm some changes have been made, namely,

1. instead of unit vector and its direction a line is taken, and fixing an origin, this line is rotated around this origin,
2. the ideal points of agents are projected onto this line for each increment of the rotation angles,
3. the pivotal agent is determined as the agent, who occupies the median position in the linear order of agents after projection of the agents' ideal points onto the line.

Table 1

Shapley-Owen model	Computational algorithm of [6]
Direction angles: $q_i, \quad i = 1, 2, ..., t$	Rotation angles: $q_i, \quad i = 1, 2, ..., t$
Direct unit vector: $x(q_i)$	Line: $L(q_i)$
$\langle x, P_i \rangle$	$\langle L, P_i \rangle$
Linear order of agents: $i r_x j \quad \Leftrightarrow \quad \langle x, P_i \rangle \geq \langle x, P_j \rangle$	Linear order of agents: $i r_L j \quad \Leftrightarrow \quad P_i \geq_L P_j$

Consider a finite set of *n* agents, *N*, in *m*-dimensional spatial voting model. Let the rule of decision making be the simple majority one.

Assume $P_i, \ i = 1, 2, ..., n$, determines an ideal point of agent *i* in the space under study. Introduce the strict order on *N*, $i r_L j \Leftrightarrow P_i \geq_L P_j$. The values P_i and P_j are compared after the ideal points are projected onto the line *L* which is rotated around the origin.

The implementation of the algorithm depends on the dimension of the space.

If the political space under study is one-dimensional, the pivotal agent is determined by a simple procedure. The ideal points of agents are located on the one axis, thereby, it is necessary to find an agent

with $\left[\frac{n}{2}\right]+1$ point in succession in this order. This agent will be pivotal, she can make winning both the coalition of agents located on the left side from her and the coalition of agents located on the right side from her. For an example of three agents (Fig. 1) with the ideal points P_1, P_2, P_3 the pivotal agent is the agent 2 with the ideal point P_2. This agent can form the coalition both with the agent 1 and with the agent 2.

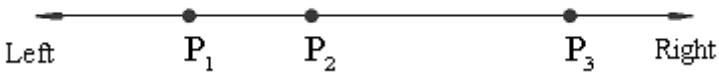

Fig. 1

For the two-dimensional space the problem how to find the pivotal agent can be solved by rotating the line *L* around the origin. These rotations can be randomly chosen, but it is easier to compute it, if the increments of angle will be uniform. Let Δq be an increment of angle q, t is the number of increments equal to *n!*, the total number of all the possible orderings of *n* agents. Assume $t \cdot \Delta q = p$, i.e., the first end of the line makes the half-turn, and another end of the line makes the half-turn as well, thereby, the line makes the full rotation around the origin.

The expression for Shapley-Owen index in the spatial context is

$$SOV_i(t) = \frac{q_i}{t},$$

q_i being the number of orderings for which the agent i is pivotal, t being the total number of angle increments for line rotation on p radian.

Consider the example of three agents in the two-dimensional space with the ideal points P_1, P_2, P_3. The values of Shapley-Owen index for these agents need to be found. In this case the number of angle increments is equal to *3!=6*, i.e., the total number of the algorithm steps is equal to 6. The increment of the angle is equal to $\Delta q = \frac{p}{6}$. Each step of the algorithm is shown on Fig. 2.

Step 1. The line *L* is rotated on the angle which is $q_1 = p/6$. The order of projections of agents' ideal points onto the line *L* is P_1, P_2, P_3.

Step 2. $q_2 = p/3$, the order of projections is P_1, P_2, P_3.

Step 3. $q_3 = p/2$, the order of projections is P_2, P_1, P_3.

Step 4. $q_4 = \frac{2p}{3}$, the order of projections is P_2, P_1, P_3.

Step 5. $q_5 = 5p/6$, the order of projections is P_2, P_3, P_1.

Step 6. $q_6 = p$, the order of projections is P_3, P_2, P_1.

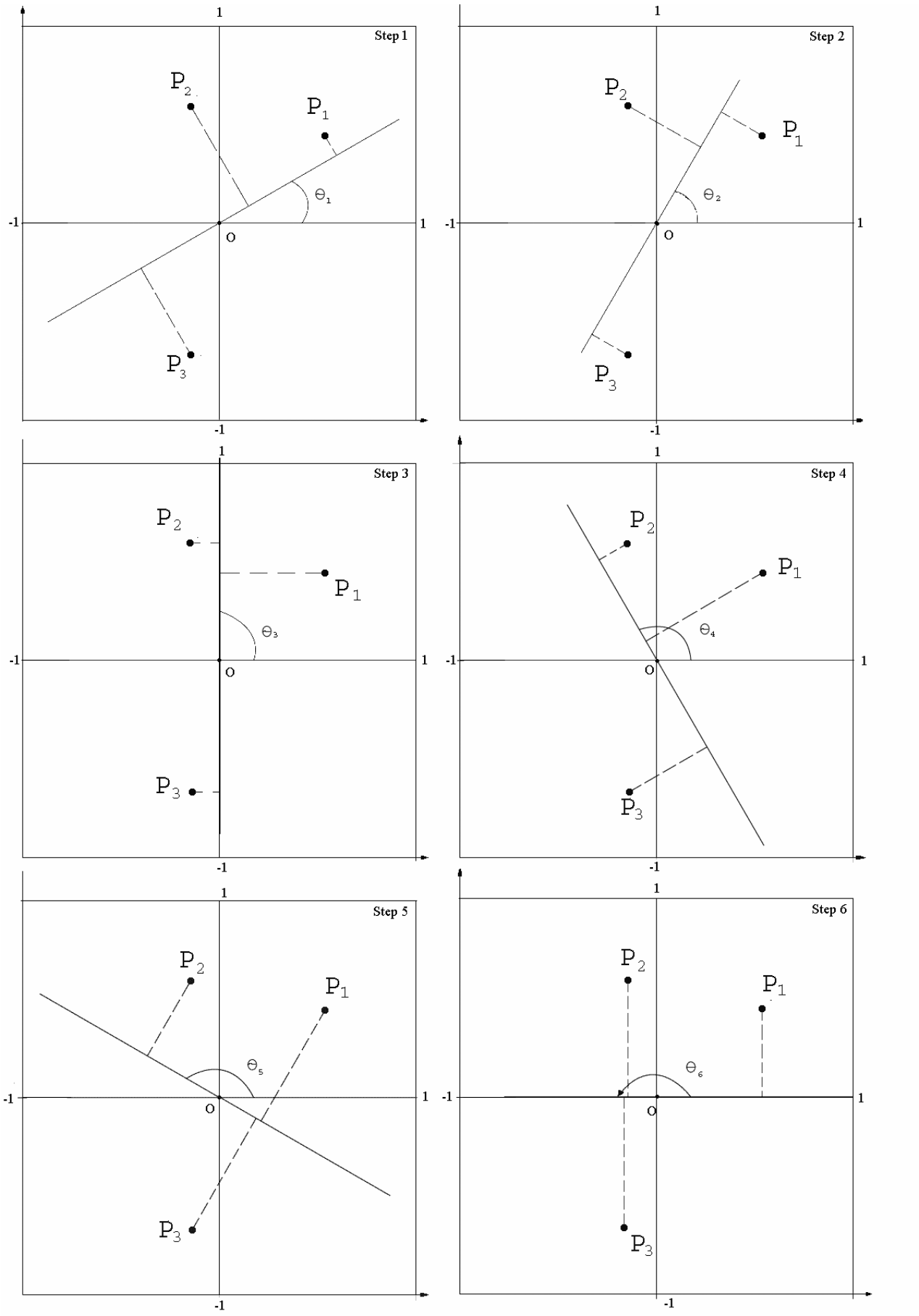
Step 1
Step 2
Step 3
Step 4
Step 5
Step 6
P1
P2
P3
θ1
θ2
θ3
θ4
θ5
θ6
o
1
-1

Fig. 2

The distribution of power is $SOV_1(6)=\frac{1}{3}$, $SOV_2(6)=\frac{1}{2}$, $SOV_3(6)=\frac{1}{6}$.

It is obvious that for the large number of *n*, the value *n!* is very large, thus, the number of steps equal to the number of angle increments *n!*, is inappropriate. It can be reduced to some sufficiently large number.

4. Extended Shapley-Owen indices

Introduce some modification of SOV computational algorithm. The suggested algorithm modification is that the steps with the repetitive orderings of agents are disregarded.

Recall the previous example of three agents. The modified algorithm disregards the steps 2 and 4, because at the step 2 the order of agents at the step 1 is repeated, and at the step 4 the order of agents at the step 3 is repeated. Thus, the steps 2 and 4 are disregarded, so the agent 1 becomes pivotal only once, the agent 2 becomes pivotal twice and the agent 3 becomes pivotal once.

Therefore, $SOV_1(4)=\frac{1}{4}$, $SOV_2(4)=\frac{1}{2}$, $SOV_3(4)=\frac{1}{4}$. As one can see the values of power of the first and third agents are equal, although in the previous case the power of the agent 1 is twice larger than the power value of the agent 3.

4.1. Extended Shapley-Owen index based on the consistency of agents' positions

Let d_{ij} be the Euclidean distance between the ideal points of agents *i* and *j* in the normalized two-dimensional political space, i.e., in the square [0, 1]×[0, 1]

Consider an index of consistency of two agents, introduced in [3],

$$k_{ij}=\frac{1}{\sqrt{2}}\left(\frac{1+\sqrt{2}}{1+d_{ij}}-1\right). \qquad (1)$$

In the Shapley-Owen model an agent is pivotal if she occupies the median position in the linear order obtained at every step, i.e., the pivotal agent splits the set of agents *N* to two disjoint subsets where one of them is the winning coalition.

The coalition located on the left side from the pivotal agent in the linear order obtained at any step is denoted as *S*. The coalition on the right side from the pivotal agent is denoted as *T* (Fig. 3). The pivotal agent can turn each of them into the winning coalition after joining one of them.

The weight of agent *i* which is pivotal is

$$w_{im}=\frac{1}{l}\sum_j k_{ij},\ i\neq j, \qquad (2)$$

It is computed as the sum of indices of consistency for each step of rotation *m=1, 2,..., t*. The summation in (2) includes the agents *j* joining the coalition which the agent *i* can turn into the winning, *l*

is the number of agents in this coalition.

Two values of the pivotal agent's weight are computed. The first value is defined by the sum of consistency indices of positions of pivotal agent and agents of coalition *S*. The second value is computed by the sum of consistency indices of positions of pivotal agent and agents of coalition *T*. For further computations the largest weight of pivotal agent is taken, so that the pivotal agent joins the coalition of agents, which are more consistent with him.

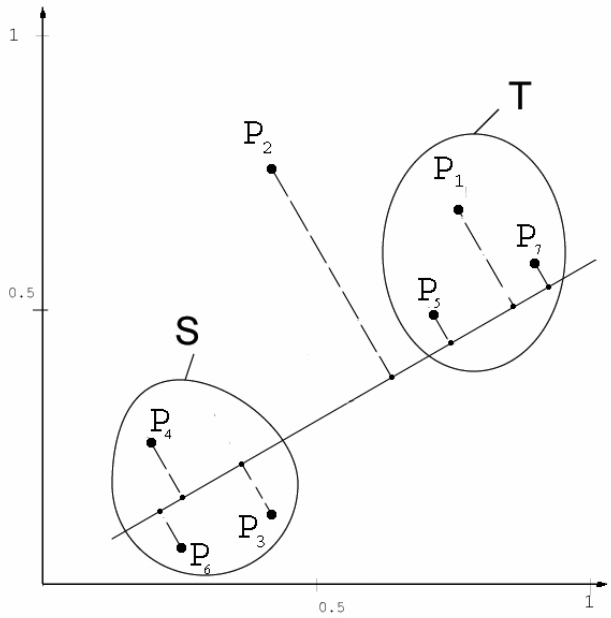

Fig. 3

Then an average value of agent i's weight is computed, *t* is the number of steps (rotations),

$$v_i(t) = \frac{\sum_{m=1}^{t} w_{im}}{t}. \qquad (3)$$

The agent i's power index is evaluated as

$$PI_1(i) = \frac{v_i(t) \cdot l_i}{\sum_{j=1}^{n} v_j(t) \cdot l_j}, \qquad (4)$$

where $l_i = n_i \Big/ \sum_j n_j$ is the vote share, and n_i is the number of votes of agent *i*.

4.2. Extended Shapley-Owen index based on the consistency of agents' positions with taking into account the share of agents' votes

This index is the same as the one introduced in Section 4.1, but it takes into account the share of agents' votes in the formula of distance between the ideal points of agents. Formally, the distance is defined by formula

$$\forall i, j = 1, 2, .., n \quad d_{ij} = \sqrt{(l_i x_i - l_j x_j)^2 + (l_i y_i - l_j y_j)^2},$$

where $l_i = n_i \Big/ \sum_j n_j$ is the vote share, n_i is the number of votes of agent *i*.

The index of consistency of agents *i* and *j*, the weight of agent as well as the power index are introduced as in Section 4.1, see (1)-(4). This index is indicated as $PI_2(i)$.

4.3. Extended Shapley-Owen index based on the consistency of agent's ideal point with the center of mass of agents' system

Coordinates of the center of mass are founded as

$$x_0 = \frac{\sum_i x_i l_i}{\sum_i l_i}, \quad y_0 = \frac{\sum_i y_i l_i}{\sum_i l_i},$$

$l_i = n_i \Big/ \sum_j n_j$ being the vote share, n_i being the number of the agent *i*'s votes. The vote share can be computed both for grand coalition of agents, then *j=1, 2,.., n*, and for some coalition of agents. In the second case only the agents joining the coalition are taken into account in this summation. We call the first case as the center of mass of the system, the second one – the center of mass of a coalition.

In the Shapley-Owen model an agent is pivotal who occupies the median position in the linear order, obtained at every step, i.e., the pivotal agent splits the set of agents *N* to two disjoint subsets, where one of them is the winning coalition.

Then $\forall i = 1, 2, .., n \quad d_{i0} = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2}$ is the distance between agent's ideal point and the coalition center of mass in the space under study.

The pivotal agent *i*'s weight is computed by formula (1) at every step *m=1, 2, .., t.*

The coalition located on the left side from the pivotal agent in the linear order obtained at any step is denoted as *S* (Fig. 4). The coalition located on the right side from the pivotal agent is denoted as *T* (Fig. 4). The pivotal agent can turn each of them into the winning one after joining it. The coordinates of the center of mass of *S* and *T* are defined. Two values of the weight of pivotal agent are specified. The first value is computed for the coalition *S* (the center of mass of the coalition is marked as ■ on Fig. 4). The second one is computed for the coalition *T* (the center of mass of the coalition is marked as • on Fig. 4).

The pivotal agent will join that coalition which center of mass is closer to the pivotal agent's ideal point. Thereby the pivotal agent makes the coalition winning. So the largest weight of pivotal agent is taken for further computations.

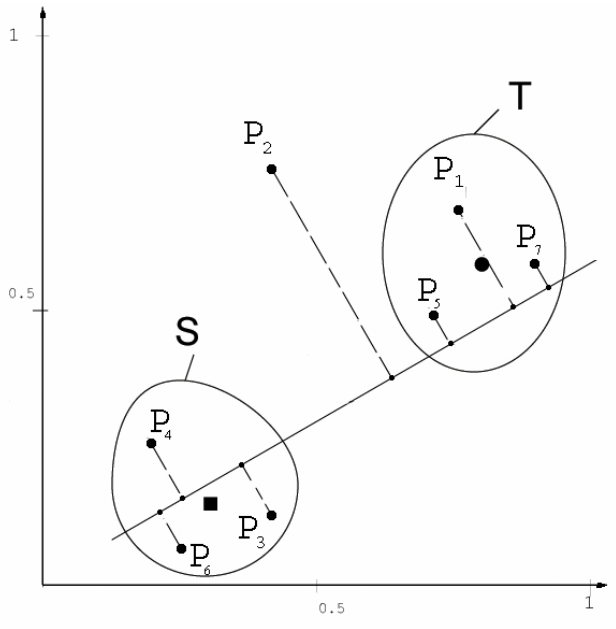

Fig. 4

Then the average value of the agent *i*'s weight is defined, *t* is the number of steps,

$$v_i(t) = \frac{\sum_{m=1}^{t} k_{im}}{t}.$$

The agent i's power index of the is computed as

$$PI_3(i) = \frac{v_i(t) \cdot l_i}{\sum_{j=1}^{n} v_j(t) \cdot l_j},$$

$l_i = n_i \Big/ \sum_j n_j$ being the vote share, n_i being the number of votes of agent *i*.

5. Power distribution in the III State Duma of the Russian Federation (2000-2003)

Let us estimate the distribution of power among political factions and groups in the III State Duma of the Russian Federation using the power indices introduced in Section 4. The data on the agents' preferences are represented by monthly[1] from 2000 to 2003.

The space under study consists of two dimensions defined as "Liberal – State oriented" (the horizontal axis) and "Antireform – Pro-reform oriented" (the vertical axis). Each dimension is measured by a floating-point scale ranged from 0 to 1. The preferences of agents are Euclidean. The decision making rule is the simple majority one.

The following factions and MPs groups were presented in the III Duma:

- APG (Agrarians);
- Edinstvo;
- CPRF (Communists);
- LDPR (Liberal-Democrats);
- Narodny Deputat;
- OVR;
- Regions of Russia;
- SPS;
- Yabloko.

The groups are classified by their size:

- large-size groups: Edinstvo, CPRF,
- middle-size groups: Narodny Deputat, OVR,
- small-size groups: Regions of Russia, APG,
- tiny-size groups: SPS, LDPR, Yabloko.

Fig. 5 represents all the factions' average power distribution measured as PI_1 for the period under study[2]. The computations of two other indices PI_2, PI_3 are represented in the Tables 3 and 4, respectively.

As one can see on Fig. 5, the faction Narodny Deputat had the greatest power values at the

[1] All computation are based on the data given [1,2]

[2] The extended Shapley-Owen indices in the classical computational version were analyzed as well. The classical version of the computational algorithm is that the steps with the repetitive orderings of agents are included. However the results obtained using this version and the results of the SOV computing are almost coincident.

beginning of 2000 ($\overline{PI}_1 \approx 0.29$). But to 2001 the value of the power becomes lower ($\overline{PI}_1 \approx 0.25$). In 2002 the average value of power index of Narodny Deputat falls down ($\overline{PI}_1 \approx 0.16$). It can be explained by the fact that in the beginning of the III Duma this faction "started" like centrists, the motion path of the Narodny Deputat's ideal points occupied a considerable area [1]. But to the 2002 it is noted that the ideal points of this faction moved from the center to the top left corner of a political map, and the area of motion path is decreased. It leads to the reduction of frequency of cases, when this faction was pivotal.

The faction Regions of Russia in 2000 has the average power value $\overline{PI}_1 \approx 0.16$, it is the second in the rating, and the average value of its power growths constantly during all the period under study. In 2003 $\overline{PI}_1 \approx 0.33$ for Regions of Russia, and as one can see this value is twice larger to the end of 2003. It means that the frequency of cases when this faction was pivotal is increased. The political map of Regions of Russia's motion paths shows that the faction ideal points' move is active, and the area covered by the move of the ideal points is considerable, moreover, this area is increased to the end of 2003.

Agrarians take the third place ($\overline{PI}_1 \approx 0.14$ in 2000), but the average value of index $\overline{PI}_1$ is strongly decreased to 2002 ($\overline{PI}_1 \approx 0.058$). It is increased to the end of the period under study ($\overline{PI}_1 \approx 0.078$). Communists and OVR are the next in our rating ($\overline{PI}_1 \approx 0.12$). Communists have the almost constant average value of power during all the period of 2000-2003. The average value of power $\overline{PI}_1$ of the group OVR changes strongly (the value is equal to 0.183 in 2001 and to 0.077 in 2002). The faction Edinstvo has strong changes in its power for all the period of 2000-2003. In 2000 the average value of its power is 0.08, but to 2002 the power of this faction increases strongly, more than 50 percent ($\overline{PI}_1 \approx 0.189$). All of these four factions had an exact faction discipline; the political maps of its motion paths demonstrate it. The motion paths of these factions have small changes, they occupy the small areas [1], so the average values of its power are small, and the changes of average values over time are small too. Till then the OVR group joined with Edinstvo in December, 2001, it had strong changes in its political opinion, the motion paths of OVR occupied the large area, these occurrences are reflected on the behavior of average power value.

The factions SPS, LDPR, Yabloko are tiny-size groups, they are in the end of rating. The average power values of these factions are less than 0.05, and there are not any important changes of power for all the period. The political map analysis shows that the ideal points' move actively in the space and the motion paths occupy large areas, but, however, the values of power are small. This phenomenon can be explained by the size of each group, firstly, and, secondly, by the displacement of the motion paths area from the center to some zone of political map. For example, the political positions of SPS locate in the liberal-reformist zone of political map (the bottom left area). LDPR in contrast supports reforms, that faction is state-oriented.

Thus, it may be concluded that both the greatest power values and the strong power changes over

time belong to that factions which change the political opinion constantly. It means that these groups have not a fixed political opinion; they can maneuver in order to receive the strategic advantages. Such a power is called a payoff-power (or P-power). Therefore, the power index measures the agent's ability to predict and adjust to the outcome. This hypothesis is confirmed by the political maps of ideal points' motion paths, presented by monthly for every year [1]. The ideal points of such group migrate throughout the political space under study.

Those factions whose power index is small have exact political views, a firm politics; they try to effect the outcome of voting. Such a power is called an influence-power (or I-power). This hypothesis is confirmed by the political maps of ideal points' motion paths, presented by monthly for every year [1].

The extended power index PI_1 curves for large-size (Edinstvo and CPRF), middle-size (Narodny Deputat and OVR), small-size (Regions of Russia and APG) and tiny-size groups (SPS, LDPR and Yabloko) are represented on Fig. 6-9 accordingly. They are marked by solid lines. On the same figures dashed lines present each faction SOV curves.

As one can see on Fig. 6, there is a peak of CPRF's power value in autumn of 2000. This peak is linked to the consideration of law on child's benefits supported by CPRF, APG (the power value of APG is also increased to 0.2), Narodny Deputat and Regions of Russia. The majority (263 votes) voted for this law passing, but it wasn't passed because of the Federation Council veto (300 votes were necessary to override the veto). Thus, this law was not voted through. The next power value peak observed in May, 2002 corresponds to the voting on the alternative military service federal law. The leftists assembled the majority which defeats this bill read twice for the ideological reasons. From May to September of 2003 a fall in CPRF's power is observed. It can be explained by some bills adoption, for example, the bill of local government reform or the federal budget bill, as well as the question about non-confidence vote to the Government was opened. In the voting on these bills CPRF was always in minority.

On Fig. 6 the power distribution curve of Edinstvo is represented. Examining the most important power value changes, it should be noted that there are some repetitive falls of power to zero during all the period under study. The first fall is observed in December, 2000 and January-February, 2001, when the alterations of pension federal law have been considered. The faction Edinstvo voted against the alterations, they were in the minority as well as the faction LDPR, which had power value falls at the same time (Fig. 9). The next Edinstvo's power value fall in November-December, 2001 is observed, when the questions of judicial authority reform have been discussed. The centrists had to reconcile the viewpoints with the leftists and the liberal factions for the adoption of this act. The left liberal factions in its turn took an advantage of the opportunity to the law liberalization. Thus, there are some observed increments of the power value of SPS, LDPR, APG (the power value increment to 0.107 took place for SPS, this value was the greatest of all SPS's power values, APG had the value $PI_1 = 0.138$, LDPR - $PI_1 = 0.05$). In spring of 2002 a power value peak of Edinstvo has been observed ($PI_1 = 0.4$). This peak

can be linked to the break of package agreement, accepted in the beginning of 2000. The package agreement break was initiated by the centrist factions and had been supported by SPS and Yabloko. Edinstvo was the pivotal agent in this voting. The next more important power value peak of Edinstvo is observed in September-November, 2002, it can be explained by the adoption of federal law of referendum of the Russian Federation. This draft law proposed by Edinstvo was supported by all the factions except Communists and Agrarians.

On Fig. 7 changes of middle-size factions' power distributions, namely, Narodny Deputat and OVR are represented. There are some strong changes of power values observed for these factions. In January-February, 2001 the value peak of Narodny Deputat's power is observed; it can be explained by the alteration in the federal law of pensions. Narodny Deputat was the pivotal agent in that voting when the veto was negotiated. There is power increasing observed for Narodny Deputat and OVR in September-November, 2001 (Narodny Deputat had $PI_1 = 0.45$ and OVR had $PI_1 = 0.266$, it is one of the most important peaks of this faction). It can be linked to the adoption of the most important bills of 2001 - the Russian Federation labor code. All the factions except CPRF and APG voted for this law, and Narodny Deputat was the pivotal agent in that voting, it had the highest power value at that time. The faction OVR had one another power peak observed in December, 2000. There is one of the highest values at that period, when the Russian Federation national symbol legislative package have been considered supported by all factions except SPS and Yabloko. The reason of this peak appearance is that the faction OVR votes were decisive in this voting.

On Fig. 8 the power distribution curves of small-size groups, namely, Regions of Russia and APG are represented. The faction Regions of Russia had the most interesting and important results in May-June, 2001, when the power value increased to 0.42. This is the highest value at this time interval. The power value peak can be explained by the law of political faction consideration. Edinstvo, OVR, LDPR, Narodny Deputat, Yabloko and Regions of Russia voted for the law adoption. Votes of the Regions of Russia were decisive in this voting, thereby, this is the reason of power value growth. There is one more peak of Regions of Russia power value observed in January-February, 2002 with $PI_1 \approx 0.52$. At that time some bills had been considered, namely, the act of nationalization, the termination of broadcasting of TV6 act, the law of nationality, the act of electric and heat energy rate management. The faction Regions of Russia was in majority voted for these laws, and the power value peak can be explained by the fact that Regions of Russia was the decisive agent in one of these votes. The next peak of Regions of Russia's power value is observed in March-April, 2003 with $PI_1 \approx 0.51$, when the questions of housing and communal services reform had been debated. The act of housing and communal services reform was accepted in third reading after some amendments to this act, and Regions of Russia had been the pivotal agent.

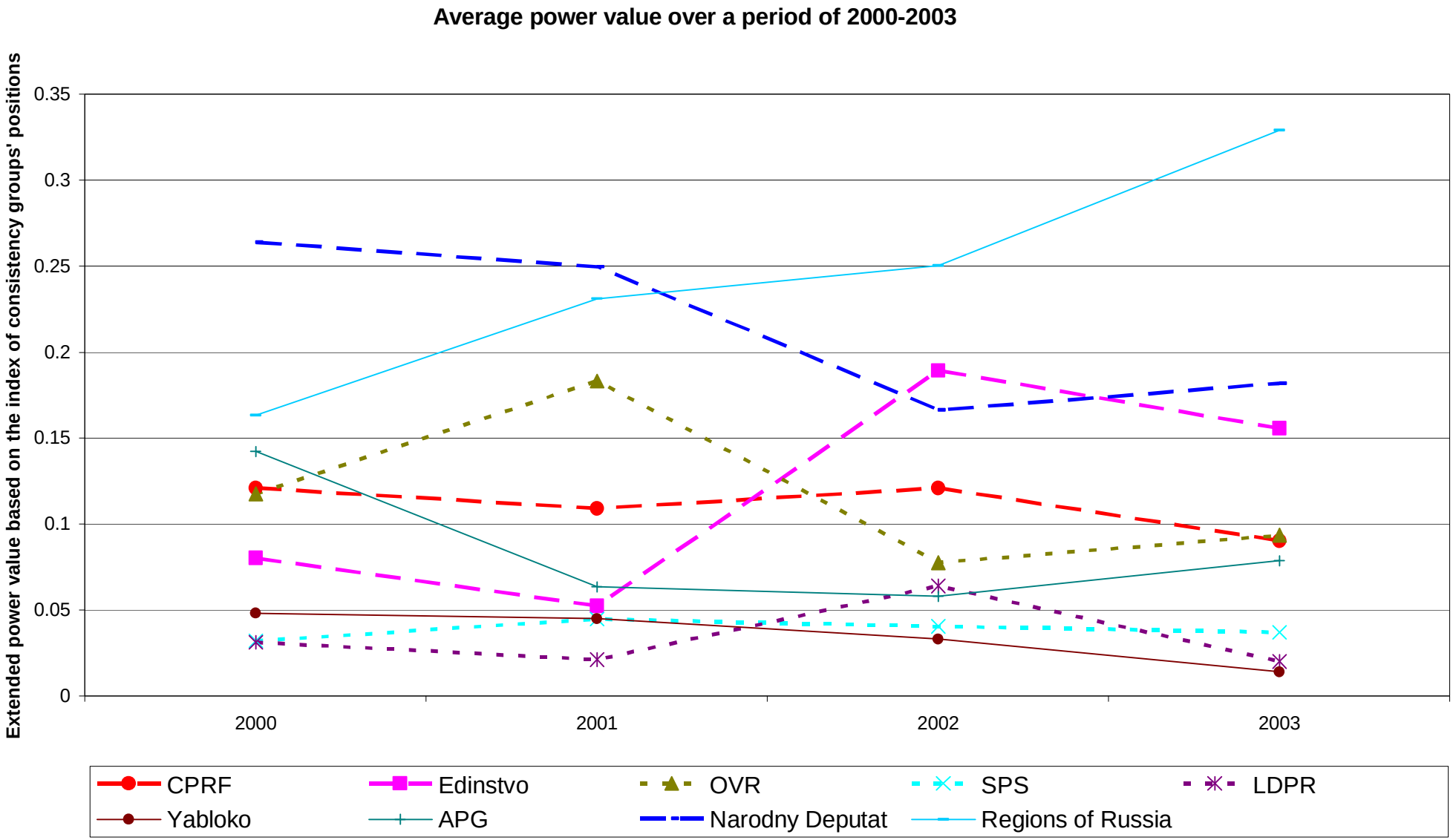
Average power value over a period of 2000-2003
Extended power value based on the index of consistency groups' positions
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
2000
2001
2002
2003
CPRF
Edinstvo
OVR
SPS
LDPR
Yabloko
APG
Narodny Deputat
Regions of Russia

Fig. 5

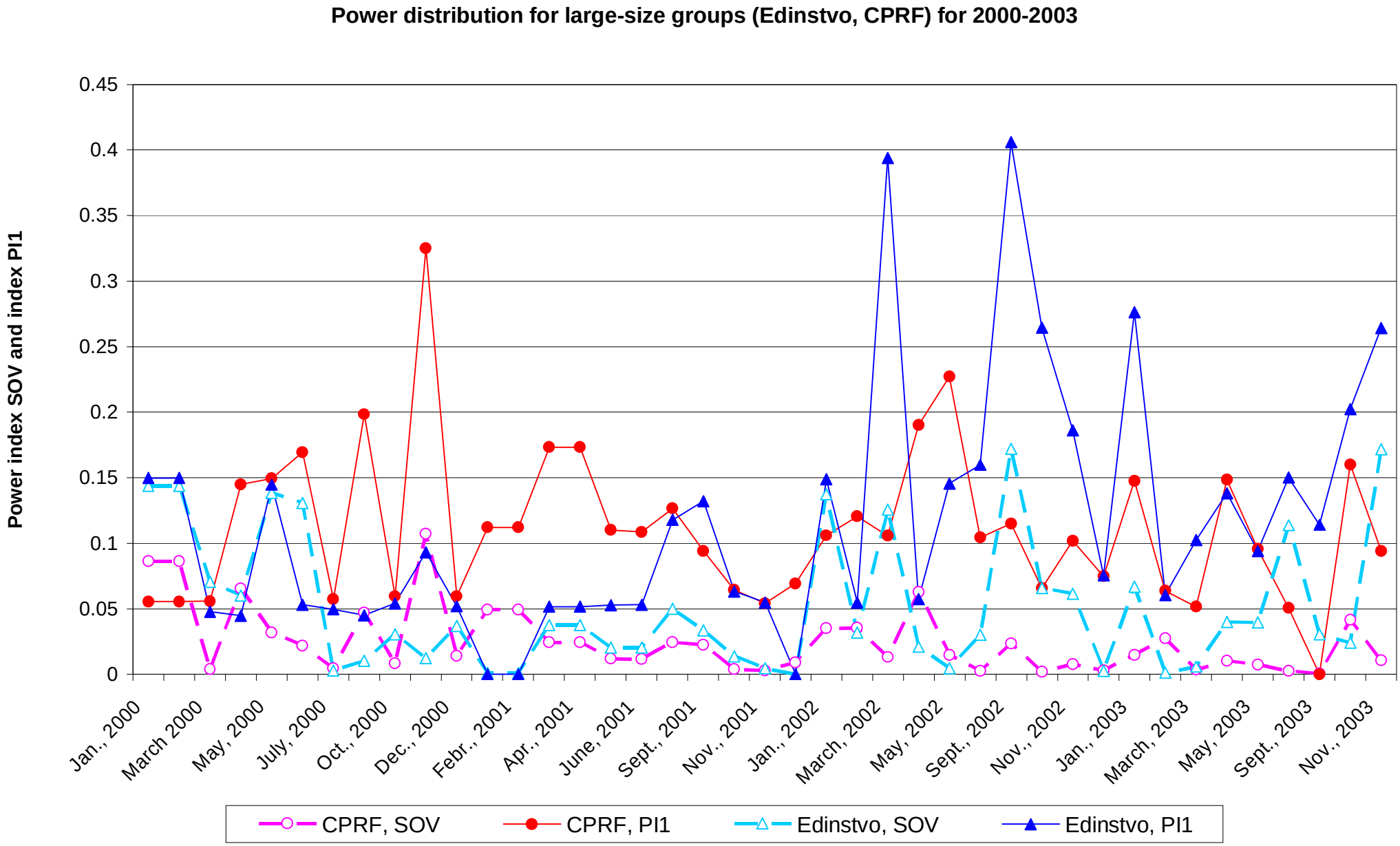
Power distribution for large-size groups (Edinstvo, CPRF) for 2000-2003
Power index SOV and index PI1
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Jan., 2000
March 2000
May, 2000
July, 2000
Oct., 2000
Dec., 2000
Febr., 2001
Apr., 2001
June, 2001
Sept., 2001
Nov., 2001
Jan., 2002
March, 2002
May, 2002
Sept., 2002
Nov., 2002
Jan., 2003
March, 2003
May, 2003
Sept., 2003
Nov., 2003
CPRF, SOV
CPRF, PI1
Edinstvo, SOV
Edinstvo, PI1

Fig. 6

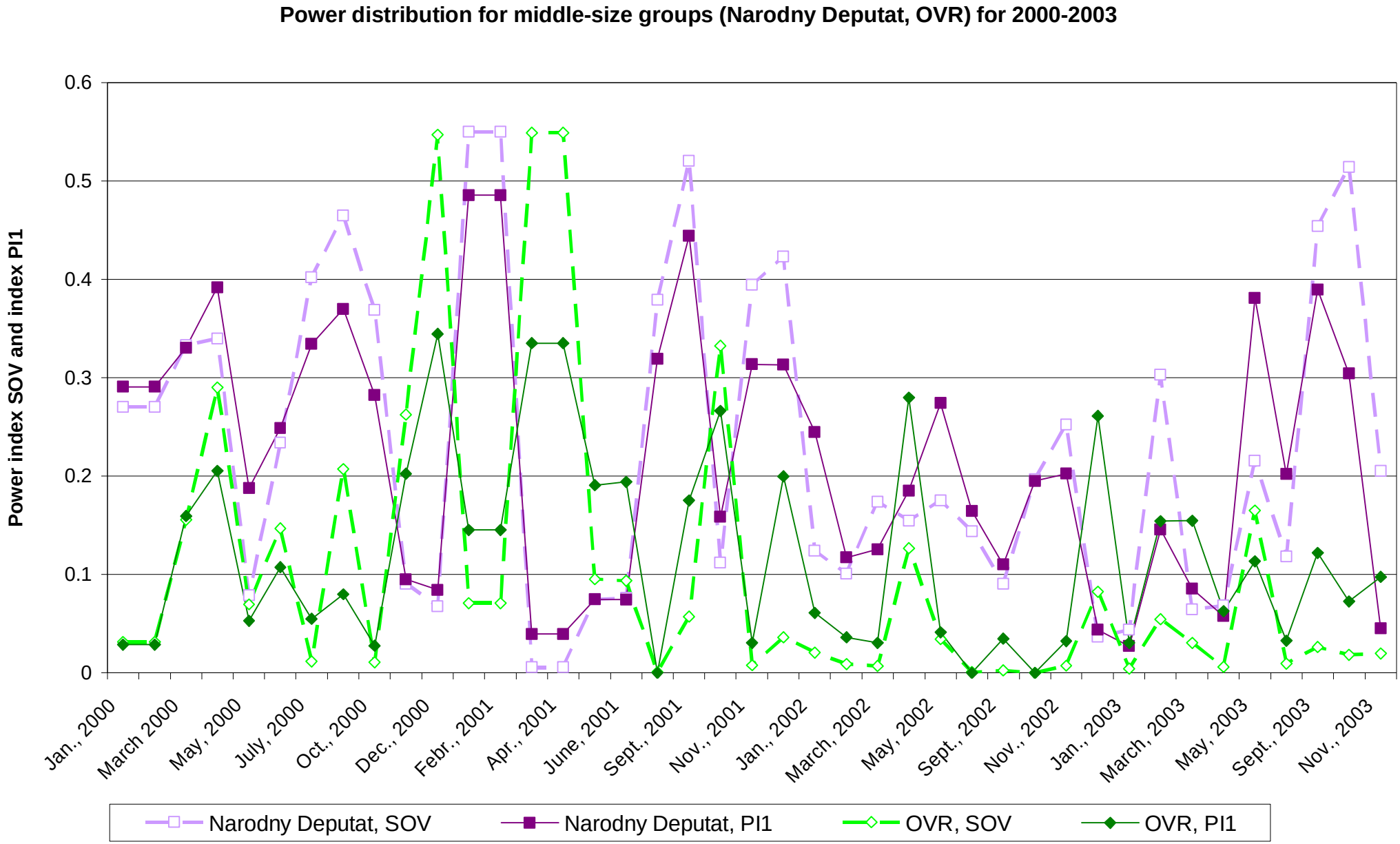
Power distribution for middle-size groups (Narodny Deputat, OVR) for 2000-2003
Power index SOV and index PI1
0.6
0.5
0.4
0.3
0.2
0.1
0
Jan., 2000
March 2000
May, 2000
July, 2000
Oct., 2000
Dec., 2000
Febr., 2001
Apr., 2001
June, 2001
Sept., 2001
Nov., 2001
Jan., 2002
March, 2002
May, 2002
Sept., 2002
Nov., 2002
Jan., 2003
March, 2003
May, 2003
Sept., 2003
Nov., 2003
Narodny Deputat, SOV
Narodny Deputat, PI1
OVR, SOV
OVR, PI1

Fig. 7

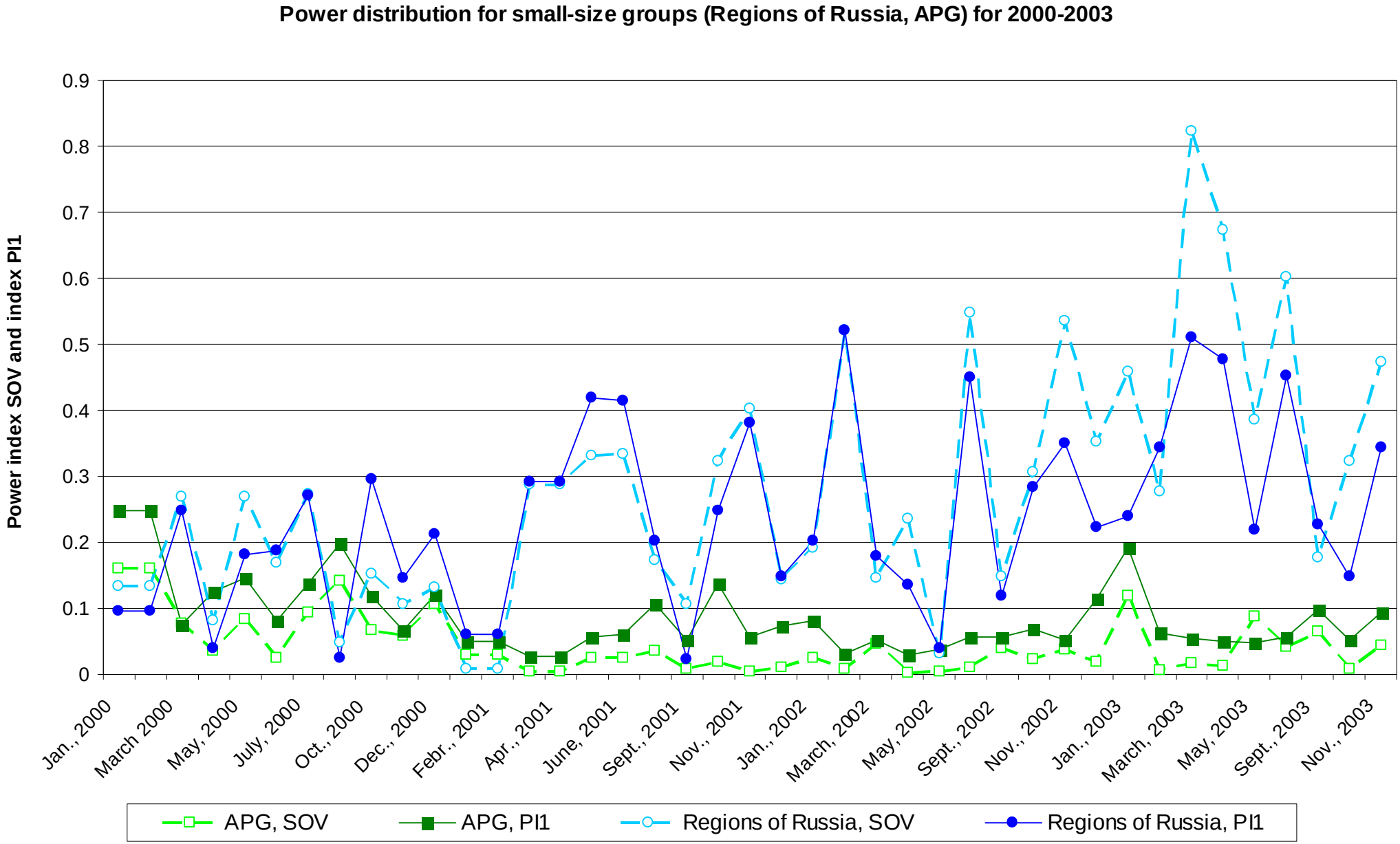
Power distribution for small-size groups (Regions of Russia, APG) for 2000-2003
Power index SOV and index PI1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Jan., 2000
March 2000
May, 2000
July, 2000
Oct., 2000
Dec., 2000
Febr., 2001
Apr., 2001
June, 2001
Sept., 2001
Nov., 2001
Jan., 2002
March, 2002
May, 2002
Sept., 2002
Nov., 2002
Jan., 2003
March, 2003
May, 2003
Sept., 2003
Nov., 2003
APG, SOV
APG, PI1
Regions of Russia, SOV
Regions of Russia, PI1

Fig. 8

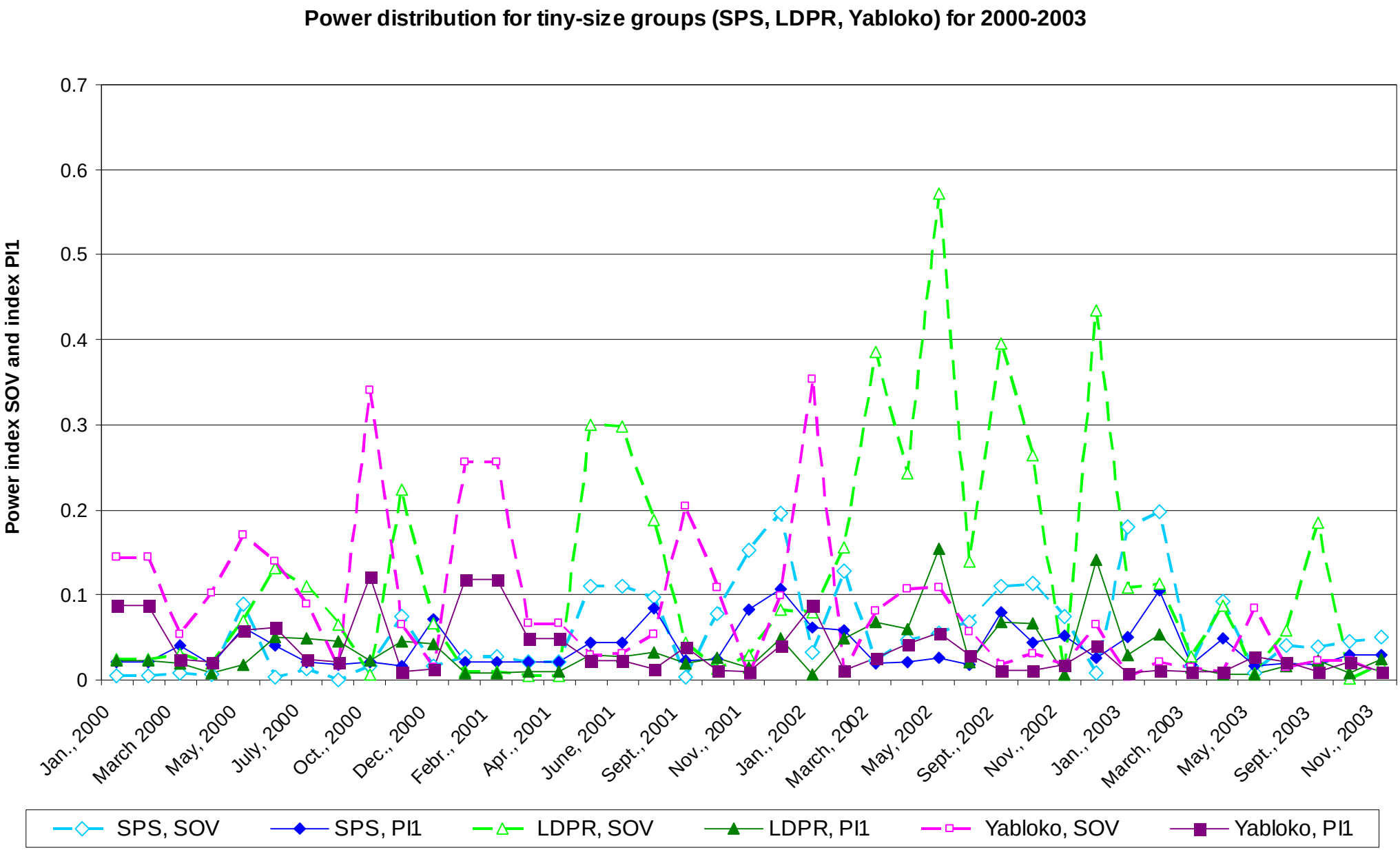
Power distribution for tiny-size groups (SPS, LDPR, Yabloko) for 2000-2003
Power index SOV and index PI1
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Jan., 2000
March 2000
May, 2000
July, 2000
Oct., 2000
Dec., 2000
Febr., 2001
Apr., 2001
June, 2001
Sept., 2001
Nov., 2001
Jan., 2002
March, 2002
May, 2002
Sept., 2002
Nov., 2002
Jan., 2003
March, 2003
May, 2003
Sept., 2003
Nov., 2003
SPS, SOV
SPS, PI1
LDPR, SOV
LDPR, PI1
Yabloko, SOV
Yabloko, PI1

Fig. 9

Table 2. Extended power index values PI_1 for the political factions represented in the III State Duma of the Russian Federation

	CPRF	*Edinstvo*	*OVR*	*SPS*	*LDPR*	*Yabloko*	*APG*	*Narodny Deputat*	*Regions of Russia*
Jan., 2000	0.055	0.150	0.029	0.022	0.022	0.088	0.247	0.291	0.097
Febr., 2000	0.055	0.150	0.029	0.022	0.022	0.088	0.247	0.291	0.097
March, 2000	0.056	0.048	0.160	0.041	0.019	0.025	0.076	0.330	0.247
Apr., 2000	0.145	0.045	0.205	0.018	0.008	0.022	0.124	0.392	0.041
May, 2000	0.150	0.145	0.053	0.061	0.017	0.059	0.147	0.188	0.182
June, 2000	0.169	0.053	0.107	0.040	0.051	0.061	0.082	0.249	0.187
July, 2000	0.057	0.050	0.055	0.022	0.049	0.024	0.138	0.334	0.270
Sept., 2000	0.198	0.045	0.080	0.019	0.045	0.021	0.198	0.370	0.024
Oct., 2000	0.060	0.054	0.028	0.020	0.022	0.121	0.118	0.282	0.295
Nov., 2000	0.325	0.093	0.202	0.017	0.045	0.009	0.067	0.095	0.146
Dec., 2000	0.060	0.052	0.345	0.071	0.043	0.013	0.120	0.084	0.212
Jan., 2001	0.112	0.000	0.145	0.021	0.009	0.119	0.050	0.485	0.060
Febr., 2001	0.112	0.000	0.145	0.021	0.009	0.119	0.050	0.485	0.060
March, 2001	0.173	0.051	0.335	0.022	0.010	0.049	0.028	0.040	0.292
Apr., 2001	0.173	0.051	0.335	0.022	0.010	0.049	0.028	0.040	0.292
May, 2001	0.110	0.053	0.191	0.044	0.028	0.023	0.056	0.075	0.420
June, 2001	0.108	0.053	0.194	0.044	0.028	0.023	0.060	0.074	0.416
July, 2001	0.127	0.118	0.000	0.084	0.032	0.013	0.107	0.319	0.201
Sept., 2001	0.094	0.132	0.175	0.022	0.020	0.038	0.051	0.444	0.023
Oct., 2001	0.064	0.063	0.266	0.024	0.026	0.011	0.138	0.159	0.248
Nov., 2001	0.054	0.055	0.030	0.083	0.015	0.010	0.057	0.314	0.382
Dec., 2001	0.069	0.000	0.200	0.107	0.049	0.041	0.073	0.313	0.148
Jan., 2002	0.106	0.149	0.061	0.062	0.007	0.088	0.082	0.245	0.201
Febr., 2002	0.120	0.055	0.036	0.059	0.048	0.011	0.032	0.117	0.521
March, 2002	0.106	0.394	0.030	0.019	0.068	0.026	0.051	0.125	0.180
Apr., 2002	0.190	0.057	0.279	0.021	0.059	0.042	0.030	0.185	0.136
May, 2002	0.227	0.146	0.041	0.025	0.153	0.055	0.038	0.274	0.040
June, 2002	0.104	0.160	0.000	0.018	0.020	0.029	0.056	0.164	0.449
Sept., 2002	0.115	0.406	0.035	0.079	0.069	0.011	0.056	0.110	0.120
Oct., 2002	0.066	0.264	0.000	0.044	0.067	0.011	0.070	0.195	0.283
Nov., 2002	0.102	0.186	0.032	0.051	0.007	0.018	0.051	0.202	0.350
Dec., 2002	0.075	0.075	0.261	0.026	0.141	0.040	0.115	0.044	0.223
Jan., 2003	0.147	0.276	0.031	0.050	0.029	0.008	0.193	0.027	0.239
Febr., 2003	0.064	0.060	0.154	0.105	0.053	0.011	0.063	0.145	0.345
March,2003	0.051	0.103	0.155	0.018	0.014	0.009	0.053	0.086	0.511
Apr., 2003	0.148	0.138	0.063	0.049	0.007	0.009	0.051	0.058	0.477
May, 2003	0.096	0.094	0.114	0.016	0.007	0.027	0.047	0.381	0.218
June, 2003	0.051	0.150	0.033	0.019	0.016	0.021	0.056	0.202	0.453
Sept., 2003	0.000	0.114	0.122	0.017	0.023	0.010	0.097	0.389	0.227
Oct., 2003	0.160	0.202	0.073	0.030	0.009	0.021	0.052	0.304	0.149
Nov., 2003	0.094	0.264	0.097	0.030	0.024	0.009	0.094	0.045	0.343

Table 3. Extended power index values PI_2 for the political factions represented in the III State Duma of the Russian Federation

	CPRF	Edinstvo	OVR	SPS	LDPR	Yabloko	APG	Narodny Deputat	Regions of Russia
Jan., 2000	0.058	0.157	0.031	0.022	0.023	0.087	0.212	0.298	0.111
Febr., 2000	0.058	0.157	0.031	0.022	0.023	0.087	0.212	0.298	0.111
March, 2000	0.054	0.047	0.166	0.039	0.010	0.025	0.076	0.335	0.248
Apr., 2000	0.147	0.044	0.181	0.018	0.009	0.024	0.120	0.411	0.046
May, 2000	0.150	0.142	0.053	0.056	0.019	0.059	0.148	0.177	0.197
June, 2000	0.168	0.051	0.114	0.020	0.051	0.063	0.080	0.265	0.189
July, 2000	0.054	0.050	0.056	0.020	0.050	0.025	0.133	0.341	0.270
Sept., 2000	0.166	0.050	0.083	0.000	0.051	0.024	0.184	0.415	0.027
Oct., 2000	0.058	0.056	0.029	0.022	0.022	0.116	0.113	0.289	0.296
Nov., 2000	0.282	0.096	0.225	0.019	0.037	0.011	0.073	0.105	0.152
Dec., 2000	0.058	0.056	0.356	0.069	0.041	0.013	0.112	0.085	0.210
Jan., 2001	0.102	0.000	0.140	0.021	0.008	0.118	0.051	0.503	0.056
Febr., 2001	0.102	0.000	0.140	0.021	0.008	0.118	0.051	0.503	0.056
March, 2001	0.162	0.052	0.346	0.022	0.009	0.050	0.026	0.041	0.291
Apr., 2001	0.162	0.052	0.346	0.022	0.009	0.050	0.026	0.041	0.291
May, 2001	0.107	0.050	0.191	0.047	0.030	0.025	0.052	0.075	0.424
June, 2001	0.107	0.050	0.191	0.047	0.030	0.025	0.052	0.075	0.424
July, 2001	0.123	0.118	0.000	0.083	0.035	0.013	0.096	0.334	0.197
Sept., 2001	0.095	0.131	0.170	0.021	0.020	0.039	0.049	0.451	0.025
Oct., 2001	0.060	0.059	0.257	0.028	0.026	0.012	0.126	0.171	0.260
Nov., 2001	0.049	0.050	0.028	0.091	0.015	0.010	0.053	0.323	0.381
Dec., 2001	0.063	0.000	0.194	0.114	0.047	0.039	0.069	0.325	0.148
Jan., 2002	0.102	0.149	0.060	0.060	0.007	0.095	0.081	0.243	0.203
Febr., 2002	0.108	0.053	0.032	0.065	0.048	0.012	0.028	0.114	0.540
March, 2002	0.095	0.386	0.030	0.019	0.072	0.032	0.052	0.139	0.175
Apr., 2002	0.170	0.057	0.272	0.023	0.059	0.049	0.030	0.201	0.137
May, 2002	0.211	0.138	0.041	0.027	0.163	0.058	0.036	0.285	0.040
June, 2002	0.098	0.148	0.000	0.020	0.022	0.032	0.052	0.172	0.457
Sept., 2002	0.099	0.415	0.034	0.084	0.070	0.011	0.055	0.110	0.122
Oct., 2002	0.060	0.244	0.000	0.051	0.069	0.013	0.066	0.208	0.288
Nov., 2002	0.088	0.177	0.029	0.056	0.007	0.019	0.047	0.212	0.365
Dec., 2002	0.066	0.067	0.259	0.028	0.143	0.044	0.108	0.046	0.239
Jan., 2003	0.134	0.268	0.029	0.054	0.028	0.009	0.189	0.029	0.259
Febr., 2003	0.056	0.057	0.150	0.116	0.056	0.012	0.061	0.156	0.336
March, 2003	0.045	0.091	0.148	0.018	0.015	0.010	0.049	0.090	0.535
Apr., 2003	0.136	0.131	0.059	0.054	0.007	0.010	0.048	0.059	0.497
May, 2003	0.091	0.086	0.115	0.017	0.007	0.028	0.047	0.378	0.231
June, 2003	0.051	0.147	0.033	0.020	0.016	0.021	0.053	0.194	0.465
Sept., 2003	0.000	0.110	0.118	0.021	0.027	0.011	0.088	0.402	0.223
Oct., 2003	0.140	0.188	0.070	0.037	0.008	0.020	0.052	0.316	0.167
Nov., 2003	0.084	0.256	0.095	0.034	0.023	0.009	0.093	0.049	0.357

Table 4. Extended power index values PI_3 for the political factions represented in the III State Duma of the Russian Federation

	CPRF	Edinstvo	OVR	SPS	LDPR	Yabloko	APG	Narodny Deputat	Regions of Russia
Jan., 2000	0.063	0.154	0.032	0.021	0.024	0.089	0.211	0.287	0.119
Febr., 2000	0.063	0.154	0.032	0.021	0.024	0.089	0.211	0.287	0.119
March, 2000	0.054	0.047	0.165	0.039	0.009	0.025	0.075	0.335	0.250
Apr., 2000	0.154	0.045	0.178	0.019	0.009	0.025	0.116	0.406	0.049
May, 2000	0.151	0.154	0.053	0.054	0.018	0.061	0.153	0.160	0.196
June, 2000	0.169	0.055	0.117	0.020	0.051	0.063	0.081	0.256	0.188
July, 2000	0.053	0.055	0.060	0.020	0.050	0.025	0.131	0.336	0.269
Sept., 2000	0.177	0.051	0.073	0.000	0.054	0.025	0.162	0.436	0.022
Oct., 2000	0.060	0.059	0.030	0.023	0.022	0.115	0.113	0.284	0.293
Nov., 2000	0.219	0.108	0.256	0.020	0.035	0.013	0.080	0.122	0.148
Dec., 2000	0.066	0.064	0.350	0.074	0.038	0.014	0.110	0.077	0.207
Jan., 2001	0.092	0.000	0.145	0.021	0.008	0.110	0.052	0.520	0.053
Febr., 2001	0.092	0.000	0.145	0.021	0.008	0.110	0.052	0.520	0.053
March, 2001	0.170	0.057	0.349	0.023	0.009	0.051	0.027	0.039	0.275
Apr., 2001	0.170	0.057	0.349	0.023	0.009	0.051	0.027	0.039	0.275
May, 2001	0.104	0.049	0.204	0.050	0.032	0.026	0.048	0.083	0.402
June, 2001	0.102	0.049	0.206	0.050	0.032	0.026	0.052	0.082	0.399
July, 2001	0.123	0.119	0.000	0.084	0.034	0.013	0.087	0.350	0.190
Sept., 2001	0.096	0.134	0.178	0.022	0.021	0.041	0.050	0.434	0.024
Oct., 2001	0.058	0.059	0.261	0.030	0.027	0.012	0.121	0.172	0.261
Nov., 2001	0.052	0.052	0.028	0.097	0.016	0.011	0.054	0.331	0.358
Dec., 2001	0.069	0.000	0.208	0.118	0.049	0.042	0.075	0.300	0.138
Jan., 2002	0.108	0.160	0.059	0.060	0.008	0.094	0.085	0.227	0.200
Febr., 2002	0.106	0.057	0.039	0.074	0.049	0.013	0.033	0.116	0.513
March, 2002	0.093	0.388	0.028	0.022	0.078	0.034	0.054	0.157	0.146
Apr., 2002	0.161	0.063	0.258	0.026	0.058	0.054	0.033	0.222	0.124
May, 2002	0.204	0.138	0.044	0.028	0.166	0.058	0.037	0.283	0.042
June, 2002	0.102	0.152	0.000	0.023	0.025	0.032	0.062	0.174	0.430
Sept., 2002	0.093	0.426	0.032	0.090	0.071	0.012	0.058	0.106	0.111
Oct., 2002	0.063	0.242	0.000	0.056	0.080	0.016	0.072	0.213	0.259
Nov., 2002	0.088	0.188	0.031	0.054	0.006	0.018	0.050	0.211	0.353
Dec., 2002	0.062	0.063	0.282	0.031	0.138	0.043	0.100	0.050	0.231
Jan., 2003	0.128	0.298	0.028	0.057	0.028	0.010	0.196	0.032	0.223
Febr., 2003	0.059	0.065	0.164	0.107	0.059	0.014	0.071	0.158	0.304
March, 2003	0.044	0.096	0.145	0.018	0.014	0.009	0.050	0.098	0.525
Apr., 2003	0.138	0.137	0.058	0.053	0.007	0.009	0.047	0.059	0.491
May, 2003	0.092	0.093	0.119	0.019	0.007	0.028	0.049	0.356	0.237
June, 2003	0.053	0.151	0.035	0.021	0.016	0.020	0.050	0.199	0.456
Sept., 2003	0.000	0.118	0.117	0.024	0.029	0.012	0.085	0.424	0.191
Oct., 2003	0.140	0.205	0.075	0.039	0.009	0.021	0.056	0.319	0.136
Nov., 2003	0.095	0.245	0.117	0.034	0.021	0.011	0.107	0.047	0.324

6. Conclusion

As one can see all the indices under study give almost the identical power distribution patterns. The results of computing of extended power indices differ from the SOV computing results. The most important changes are observed for the faction CPRF and the ally – the faction APG. The power value of these factions is higher than SOV. On the contrary, for the factions Narodny Deputat and LDPR these values are lower than SOV.

The results of power value computing for the political factions of the III State Duma badly correlate with the results obtained in [2]. The results for the political factions of the III Duma power analysis obtained in [2] on the basis of the standard Banzaf index show that the most powerful groups are the factions Edinstvo and CPRF; the factions Narodny Deputat and OVR take the third and forth places, accordingly, and the last are the factions Regions of Russia, APG and SPS. The results are the same for the tiny-size groups of the III Duma, namely, for the factions SPS, LDPR and Yabloko. The power analysis on the basis of both the standard Banzaf index and the extended Shapley-Owen indices demonstrates that the power of these groups is very low. The power values are the same for the factions OVR and APG, both analyses point out that these factions take the average positions in the rating of power.

The results of power analysis for the political factions of the III Duma based on the index of consistency obtained in [2] show that in the case of scenario 0.4, which is the closest to the real power distribution, the groups with the extreme policy (the factions CPRF, APG and Edinstvo) had the serious losses in power. But those factions who stand in the center (the factions Narodny Deputat, OVR and Regions of Russia) had large power values. The similar results were obtained using the extended Shapley-Owen indices for the power distribution analysis.

In this work the new approach of the problem of political groups' power estimation is introduced. This approach uses the information on agents' preferences. This information is based on the index of consistency of agents' positions defined by the closeness of the agents' preferences in political space. This index makes the special preference to the role of ideology in coalition formation, i.e., ideologically close agents will behave identically while forming coalitions. A coalition will be formed rather by ideologically similar agents than by agents with different ideologies.

7. References

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