Sozialökonomisches Seminar

der Universität Hamburg

Freedom, Power, and Success: A Game Theoretic Perspective

Matthew Braham

May 2004

Beiträge Zur Wirtschaftsforschung

Freedom, Power, and Success: A Game Theoretic Perspective

Matthew Braham*

May 2004

No. 135

ABSTRACT This paper is about the measurement specific freedoms – the freedom of an agent to undertake some particular action. In a recent paper, Dowding and van Hees (2003) discuss the need for, and general form of, a 'freedom function' that assigns a value between 0 and 1 to a right or freedom and that describes the expectation that a person may have about being in a position to exercise ('being free to perform') that freedom or legal right. An examination of the literature shows that such a measure has never been properly defined. Based on the framework of a game form, I develop a very simple and natural measure of specific freedom that turns out to the conditional variant of 'success', a measure that we know from the literature on voting power. Some properties and characteristics of the measure are discussed.

KEYWORDS coalitions; freedom, power, success

JEL classifications C71, D63, D71

* Institute of SocioEconomics (IAW), University of Hamburg, Von-Melle-Park 5, 20146 Hamburg, Germany. braham@.econ.uni-hamburg.de

ACKNOWLEDGEMENTS I would like to thank to thank Sebastiano Bavetta, Ian Carter, Ad van Deemen, Keith Dowding, Martin van Hees, Manfred Holler, Rudy Fara, Moshé Machover, Stefan Napel, Hannu Nurmi, Frank Steffen, Stefano Vannucci, and Peter Weikard for helpful and insightful comments and discussions at various stages in writing this paper. Earlier incarnations of this paper were presented at the Workshop on Freedom and Power, University of Bayreuth, 12 February 2004; University of Turku Political Science Worshop held in Tartu, Estonia, 15–17 August 2003; the Third Joint Hamburg–Siena Workshop in Applied Game Theory, Siena, 11–13 April 2003; the European Consortium of Political Research 31st Joint Sessions of Workshops ('New Approaches to Rights, Freedoms and Power'), Edinburgh, 28 March – 2 April 2003; the 2002 Annual Meeting of the European Public Choice Society (EPCS2002), Belgirate 4–7 April 2002; and the conference 'Analysis and Measurement of Freedom: Theoretical, Empirical, and Institutional Perspectives', Palermo, 27–29 September 2001. Financial support of the Deutsches Forschungsgemeinschaft (DFG) is gratefully acknowledged.

Freedom, Power, and Success: A Game Theoretic Perspective

Matthew Braham

1. *Introduction*

This paper is about the measurement specific freedoms – the freedom of an agent to undertake some particular action. In this regard, its general subject matter is not new. In a recent paper, Dowding and van Hees (2003) discuss, for example, the need for, and general form of, a 'freedom function' that assigns a value between 0 and 1 to a right or freedom and that describes the expectation that a person may have about being in a position to exercise ('being free to perform') that right or freedom. The usefulness of such a function is that in principle it could be used to define threshold values for indicating whether or not a person has a particular freedom or legal right and therefore for making non-welfaristic judgements about social states or to design the assignment of rights related to government policy, public regulation, or legal rules.

Much light, however, still needs to be shed on the actual nature of such a function. In their contribution, Dowding and van Hees leave the matter more or less open, claiming only that extent to which a person is free to perform a particular type of action or right depends only on the probabilities with which each of the relevant instances of the action or right will not be prevented. A straightforward example is that of determining our 'freedom of expression'. According to Dowding and van Hees, this is given by the probability that shouting 'Down with the Government' at Whitehall at a given time and date and doing the same thing at Piccadilly Circus, etc. will go unprevented.

Dowding and van Hees refer to the recent and burgeoning literature on measuring freedom for a hint as to how such a function could variously be defined (Arrow 1995, Carter 1999, Dowding 1992, Pattanaik and Xu 1990, Pattanaik and Xu 1998, Sugden 1998, Rosenbaum 2000). A perusal of this literature indicates, however, that as yet there is no agreed upon framework for defining this function as the 'probability of being unprevented'. The papers by Arrow, Dowding, Pattanaik and Xu, Rosenbaum and Sugden are all concerned with 'freedom of choice' rather than with the 'freedom to do *x*' *per se* (freedom *simpliciter*).[1] Even Carter's extensive analysis of measuring overall freedom as an aggregation of the probability of being unprevented to do *x*, *y*, and *z* does not suggest an explicit model for determining the 'input probabilities' into a freedom function. Instead, they enter into his measure as an exogenous variable.[2] It is, therefore, still an open question about (i) the source of the input probabilities and by implication (ii) how to aggregate these probabilities into an value as suggested by Dowding and van Hees. This paper provides a tentative answer to both issues.

In this paper it will be argued that the value describing *i*'s freedom to perform an action can be identified with the 'conditional probability of success'. This model makes an agent's freedom a function of the propensities of other agents to choose a strategy that does not oppose the agent performing an action *and* the 'decision rule', which is a function maps strategy choices into a unique outcome. The basic idea is that an agent is free (is unprevented) to perform a specific action if she belongs to a subset of agents (a coalition) that can guarantee the performance of the action. In a slogan, 'freedom is membership of powerful coalitions',[3] and a measure of specific freedom is the probability of being a member of a such a coalition. This clearly gives a twist to the meaning of 'success' which was independently introduced by Penrose (1946), Rae (1969), and Barry (1980a, 1980b) in the voting power literature.

In the process of constructing a freedom function I make four other contributions of general theoretical importance. First, I unearth an unrecognized link between the concepts of freedom and success.

Second, I provide an answer to the age-old question of the relationship between power and freedom. Starting from a basic opportunity concept of freedom I

---

[1] For a discussion of the importance of maintaining the distinction between freedom *simpliciter* and freedom of choice, see Carter (2004) and Kramer (2003a). However, one can – as van Hees (1998) has done – interpret the concept of an *opportunity set*, which underpins the freedom of choice literature, as expressing the extent of an person's specific freedom.

[2] In his review of Carter's measure of overall freedom, van Hees (2000) does not tackle this issue either. To the best of my knowledge, the only two papers that come anywhere close to hinting at a reasonable model for a freedom function are Sugden (1978) and Bavetta (1999). Both implicitly assume a game form. I will not discuss these contributions here because they are in fact only very suggestive; neither actually defines a freedom function in a precise way.

[3] This gives additional substance to Steiner's (1994: 39) slogan that 'Freedom is the possession of things'. That is, membership of powerful coalitions is the condition for 'possession of that action's physical components'. I thank Ian Carter for pointing out this extension of Steiner's claim.

am able to show that a specific freedom derives from a power structure and therefore power is the more basic of the two concepts. This conclusion itself hinges on a demonstration that an individual-agent based definition of negative freedom is logically untenable. Generically speaking, individual (negative) freedom is a collective property.

Third, I address the issue of how to measure freedom in a strategic rather than the parametric setting of social choice theory that developed since Sen's (1970) seminal contribution. Although a number of writers have, for some time, considered this to be a necessary step (Nozick 1974, Gärdenfors 1981, Sugden 1985, Gaertner et al. 1992, Pattanaik and Suzumura 1996, van Hees 2000), it is still a largely underdeveloped area (Deb 2004).

Fourth, I add to the nascent literature that seeks to develop formal models of freedom on an explicit philosophical framework (Steiner 1983, Carter 1999, Dowding and van Hees 2003, Bavetta 2004). In other words, I do not take for granted a particular notion of freedom, but rather base my measure on a philosophically grounded generic concept and syntax of freedom.

The remainder of this paper is organized as follows. In the next section I set out in more detail the type of freedom concept that I will work with. In the third section I discuss in some detail a formal definitional framework of specific freedom. In section 4 I present the game theoretic measure of specific freedom. Section 5 is a discussion of a number of conceptual issues relating to the measure that I have constructed. Section 6 concludes.

2. *The Concept of Specific Freedom*

2.1 *Opportunity and Exercise Concepts*

When constructing a measure of freedom, it is essential to be clear from the outset about the type of freedom that we want to deal with. Primarily, this means distinguishing between an 'opportunity' and an 'exercise' concept of freedom.[4] As Carter (2004), who employs this distinction in his dissection of Pattanaik and Xu's

[4] The opportunity–exercise distinction originates with a classic essay by Taylor (1979) in which he studies Berlin's (1969) distinction between *negative* and *positive* liberty. Taylor argues that the gamut of views of negative liberty fall into either the opportunity or exercise concepts but positive views are only ever exercise concepts. That is, opportunity and exercise concepts are generic categories into which any concept of freedom can be classified.

(1990) axioms of freedom of choice, puts it, 'where freedom is treated as an opportunity concept, it means the *possibility* for an agent of performing some action or actions' (Carter 2004: 64), where 'possibility' is understood as meaning a lack of constraints of various kinds. Taken in this sense, freedom is concerned with actions that *might* be performed, given the absence of constraints, at some moment subsequent (or identical) to that at which the agent possesses the freedom in question. On this view, freedom is a matter 'of what we can do, of what it is open to us to do, whether or not we do anything to exercise these options' (Taylor 1979: 177).

In contrast, freedom as an 'exercise concept' concerns the performance by an agent of some action or actions; it is 'to *do* certain things or to achieve certain outcomes in a certain way' (Carter 2004: 64). On this view, freedom usually involves exercising control over one's life, so that one is free to the 'extent that one has effectively determined oneself and the shape of one's life' (Taylor 1979: 177). Clearly the Hobbes–Bentham notion of negative freedom as simply the 'absence of external physical or legal obstacles' (Taylor 1979: 176) is an opportunity concept, while the Rousseau–Marx notions of positive freedom as 'self realization' or 'collective self-government' is an exercise concept.

Given that the aim of this paper is to define a freedom function that is applicable, although not solely restricted to, the language of legal rights, the type of freedom that we are interested in here is that of an opportunity concept. I make no apology for this restriction because the language of rights generally concerns the 'opportunity' to do things (voting, protesting, reading) and not 'exercising'. If I have a right to read a certain book, then I have that right whether or not I ever read it; or whether or not I read it as a Marxian 'species being'.

2.2 *MacCallum's Syntax*

Following MacCallum's (1967) now canonical analysis, we can define the opportunity conception of freedom as a triadic relation between agents, constraints (preventing conditions), and possible actions: '… freedom is thus always *of* something (an agent or agents), *from* something, *to* do, not do, become, or not become something …' (p. 314). MacCallum summarizes this relation in the format of: 'x is (is not) free from y to do (not do, become, not become) z,' where x ranges over agents, y ranges over 'preventing conditions' such as constraints, restrictions, interferences, and barriers, and z ranges over actions ('doings') or conditions of character (Marxian 'self-fulfillment' or 'realization of one's true nature') or circumstance ('becoming angry'). As discussed by MacCallum, disagreements about different conceptions of freedom boil down to different views about the content of the range variables, x, y, and z (e.g. whether the agent (x) is to be conceived as an individual or collectivity; whether the obstacles (y) are only external to the agent; and whether *any* action or condition are to be counted).[5]

For the purposes of defining a freedom function, it is clear that MacCallum's rather opened-ended syntax needs honing down. As Bavetta (2004: 34) has cogently noted, a conception of freedom requires a syntax *and* a set of arguments to fill in the content of the range variables in some specific way; 'it cannot coincide with the syntax itself.' Hence, a freedom function that can be used to analyse the extent of what Dowding and van Hees' (2003) call my *material* as against my *formal* rights or freedoms concerns not just *any* opportunity, but those that are *social*.

Obviously, socially determined opportunities restrict, without much ado, the domain of x to natural or juridical persons (or groups) and that of the preventing conditions (y) to those that are inflicted by the actions of other such agents or groups of agents. External conditions of natural origin in the 'wildest sense' are to be weeded out as are 'internal' psychological or neurobiological states of mind. This means – uncontroversially, I believe – that the form of the freedom function need not be applicable to determining the freedom of the mountaineer who has become physically stuck in a crevasse or the person who is hindered from performing an action because of a morbid fear or phobia, depression, or lack of awareness, etc. no matter how figuratively correct it may be to speak of their conditions in terms of freedom or lack of it. The language of freedom and rights, generally construed as a social relation, makes no sense in these circumstances.

While restricting the domain of x and y is a relatively straightforward affair, the z variable requires a little more philosophical consideration. The issue is whether or not we should, like Carter (1999: 16–17), narrow it down to only possible actions or 'doings' or should we, like Kramer (2003b: 156–169), be more expansive and include 'beings' and 'becomings'? The answer, it turns out, cuts both ways. However, because the main contribution of this paper hinges on the y

[5] For an application of MacCallum's triadic syntax to different measures of 'freedom of choice', see Bavetta (2004).

variable, I will ignore the issue and side with Carter on the grounds that by restricting ourselves to actions we obtain a concept of specific freedom 'on which all liberals, in a broad sense, can agree', by which he means, 'at least libertarians like Friedrich von Hayek and Robert Nozick and liberal egalitarians like John Rawls and Ronald Dworkin'. Kramer's position, while unquestionably valid for a fully fledged analysis of freedom, can be safely ignored in this context because it takes us into the Byzantine intricacies of the philosophy of action without adding anything to my own contribution.

3. *Formal Definitions*

3.1 *Basic Framework*

Having elaborated the concept of specific freedom and fleeted around the extensions to be assigned to the variables in MacCallum's syntax (and more or less settled on a broadly liberal concept of negative freedom), we can now state a pair of definitions for making ascriptions of specific freedom and specific unfreedom respectively, i.e. the freedom for an agent x to perform an action z. We will use these definitions in constructing a freedom function. (For the sake of clarity, the z variable in the triadic syntax will from now on be denoted by φ with the use of Latin lowercase reserved for agents).

Definition 3.1 (*Specific freedom*) 'i is free to φ' if no non-empty set of agents prevents i from φ-ing.

Definition 3.2 (*Specific unfreedom*) 'i is *un*free to φ' if some non-empty set of agents prevents i from φ-ing.

Before proceeding further, we need to train some careful scrutiny on a basic aspect of these pair of definitions, in absence of which misunderstandings can easily arise. Although the definitions 3.1 and 3.2 appear nearby indistinguishable from Carter's (1999: 27) Kramer's (2003b: 3), and in part Steiner's (1994: 8) definitions – 3.2 in particular – they do in fact differ in a small but highly significant way. And it is this difference upon which the main contribution of this paper pivots.

The usual method, which Carter, Kramer, and Steiner employ, is to define prevention in terms of the action(s) of natural or juridical *individual* agents. Indeed, a review of the literature on specific freedom indicates a preoccupation with dyadic relations: *j*'s preventing or not preventing *i* from φ-ing.[6] In the framework presented here, however, prevention arises not from the action of an individual agent, but from the combined actions of a non-empty *set* of individual agents, i.e. from individual agents who 'belong together' by dint of a common characteristic, that of the choice, coordinated or otherwise, of a specified action (or omission) that opposes ('is against'), but not necessarily one that can alone 'prevent', some other agent performing φ. In *n*-person game theory, these sets are referred to as *coalitions*.

It may be asked why we need to leap from agents to coalitions. The answer is that it is a solution to a logical conundrum that afflicts an individual agent-based definition. In order to convince the reader that we have the correct concept, let us consider the following example. Suppose we have a society made up of four members, denoted by the set $N = \{a,b,c,d\}$. Suppose further that there is some action φ that if *a* were to desire to perform it she would require the consent of at least two others. Now, if we apply an individual agent-based definition of specific freedom and unfreedom we will find that there is a configuration of agents who together prevent *a* from performing φ (by not giving their consent) but none of these agents can be said to be doing the preventing as such. This is the case when all other agents, i.e. the subset $\{b,c,d\}$, are against *a* doing φ.

To establish this, let us, without any loss of generality, take Carter's (1999: 27) definitions as our point of departure. (I do not take Steiner's because he only formally defines unfreedom and I do not take Kramer's because he includes a condition for an agent's personal ability to do something which in this context only complicates the issue without adding anything.) Carter says that an agent is free to φ 'if *every* other agent refrains from preventing her doing it' and she is unfree to φ 'if *some* other agent prevents her doing it'. Consider the configuration $\{b,c,d\}$, the members of which have chosen, either jointly or severally, not to consent to *a* performing φ. Obviously *a* cannot be free because, assuming that by 'refrains from

[6] Note that the literature that makes use of *n*-person game theory implicitly accounts for coalitions in our understanding of freedom, although this is not explicitly marked by the authors as being the *generic* formulation of a freedom ascription. See, among others, Gärdenfors (1981), Deb (1994), Peleg (1998), van Hees (1995, 2000). In a different context, Pettit (1996, 1997: 52) allows for 'collective agents' such as coalitions in his definition of freedom, but he neither discusses the necessity for doing so, the relationship between individual agents and the 'collectivity', nor the implications that follow.

preventing' means the same as 'not preventing', it is not the case that 'every other agent does not prevent *a* from φ-ing'.[7] But by Carter's account, *a* cannot be said to be unfree either because this would require that 'some other agent' (at least one) is preventing *a* from φ-ing, which is not the case. This can be established as follows. If *some* other agent is preventing *a*, it means, ceteris paribus, that there is *at least one* agent who, if she were to decide otherwise, would see to it that *a* was free to φ. If we hold the decisions of *c* and *d* constant, and ask whether *a* would be free to φ if *b* were to consent, the answer is 'no' (because at least two agents must do so). So it cannot be said that *b* is doing the preventing. A similar question can be asked of *c* and *d* and in both cases the answer is also 'no'. Thus, while the non-fulfilment of Carter's conditions for a specific unfreedom (it is *not* the case that some agent $j,k,\ldots,n$ is preventing i) logically entails the fulfilment of his conditions for a specific freedom (all other agents $j,k,\ldots,n$ are not preventing i) this entailment does not necessarily imply that an agent will possess a specific freedom, as our example unambiguously demonstrates.[8]

Hence, while there can be no dispute that the *logical* relationship between Carter's two conditions is correct (his use of the universal quantifier for freedom and existential quantifier for unfreedom assures this) what can be disputed is the *acceptability* of *both* his conditions. By defining prevention in terms of coalitions instead of individual agents we rid ourselves, in a very natural and simple way, of the pathology of logically ascribing *a* the freedom to φ when she is in fact unfree to do so: a 'preventing coalition' can *always* be identified.

In rounding up this section, a general point seems in order. The source of the difficulty with the individual agent-based definition appears to be its very *strong*

---

[7] The assumption that 'refrains from preventing' means the same as 'not preventing' is crucial, otherwise the fulfilment of Carter's condition for specific freedom is not straightforward. In a personal communication, Carter indicated that 'not preventing' is what he had in mind because he was not assuming anything about the opportunities or potential to prevent, i.e. it is not to be thought that this definition is referring to some action or strategy called 'refraining from prevention'. Note, therefore, that in the event of *b* and *c* consenting – i.e. the coalition $\{b,c\}$ is in favour – to *a* performing φ, this fulfils Carter's condition for the ascription of a specific freedom, even though not all other agents are consenting: *a* is free to φ because by consenting *b* and *c* are 'not preventing' and nor is *d*, who, despite being in opposition to *a* performing φ, cannot prevent it.

[8] Obviously if by 'agent' Carter – or for that matter, any other theorist working within a similar framework – would include 'collective agents' such as coalitions within the meaning of 'agent', then this criticism would not hold. But a close reading of Carter (and the work of others) suggests that by 'agent' he (and others) means an individual 'person'.

assumption about the nature of *power* relations: that they are individualistic. That is, every social state can be forced by *some* (at least one) individual agent (either i can see to it that she performs φ or there is some $j,k,\ldots,n$ who can see to it that she does not perform φ). As we have seen, this is neither logically nor empirically true. To belabour the point, if 'agental prevention' exists this is simply the special case of the singleton set. In the case of $\{b,c,d\}$ preventing a from φ-ing there is no such 'agental prevention', because none of $\{b\},\{c\},\{d\}$ can see to it that a is free to perform φ; but there is if $\{c,d\}$ is the preventing coalition, because $\{c\}$ and $\{d\}$ can see to it that she is free to do so. In a social context, then, power is a property to be ascribed to coalitions and not to individuals – tempting as it may be, i is not to be confused with $\{i\}$ (Holler and Widgrén 1999).[9] To confuse the two is to commit what is best called the 'individualistic fallacy'.

3.2 *An Modified Framework*

Although definitions 3.1 and 3.2 can, and will, be used in constructing a freedom function, some may be quick to point to a weakness which appears to be very counterintuitive. I want to discuss this in this section and show how the definitions can be tightened up with a natural modification, albeit one that is not without its own logical peculiarity. The issue is important because the modification leads to a different form for the freedom function.

The problem to be addressed is that because the definitional framework is purely behavioural, in the sense that it concerns only the presence or absence of prevention and not the hypothetical choices of the agent whose freedom we are interested in, definition 3.1 will ascribe i the freedom to φ even if she were compelled by the *force* (not coercion or threats) of another to do so (which she could not resist). In the now proverbial case of a's freedom to φ, this would be if $\{b,c,d\}$ or any of its two player subsets could not only see to it that a performs φ if a were to attempt to do so, but also see to it that she do so even if she were not to make such an attempt. That is, definition 3.1 ascribes a the freedom to φ even if $\{a\}$ cannot prevent the outcome in which a performs φ. This is obviously the favourite theme laboured by G. A. Cohen (1979: 9): 'that one is free to do what one is forced to do' by dint of the fact that one cannot, even by the force of another, do that

[9] This is not the place to elaborate, but it should be noted that this result poses serious problems for the 'responsibility view' of freedom (Miller 1983, Kristjánsson 1996).

which one is not free (unprevented) to do.

If we believe, and I am inclined to, that it is unnatural to ascribe as an instance of my 'freedom of expression' the case in which I am dragged by others to the gates of Downing Street and made to hold up a placard, then the natural solution is to impose a subjunctive restriction in definitions 3.1 and 3.2 that represents a minimal element of *personal agency* or *doing* (Cohen 1988: 245).[10] That is, freedom ascriptions require that we *do* something; when I am dragged by others to the gates of Downing street I am *not* doing anything. Hence, we arrive at:

Definition 3.1* (*Specific freedom*) 'i is free to φ': if i were to attempt to φ, then no non-empty set of agents prevents i from φ-ing.

Definition 3.2* (*Specific unfreedom*) 'i is *un*free to φ': if i were to attempt to φ, then some non-empty set of agents prevents i from φ-ing.

To observe this restriction at work, consider once more the case of a's freedom to φ. The restriction tells us to ascribe a the freedom to φ only in those instances that *if* she were to attempt it she would be unprevented from doing so, despite it being true that she can be forced to φ; and likewise the unfreedom to φ in those instances in which she is prevented *if* she were to attempt it. Thus, even if it were true that $\{b,c,d\}$ can force a to φ, the subjunctive restriction says we *ignore* this case as a component of assessing a's freedom to φ – but this is not to deny the truth of her being free to φ in this instance.[11]

Technically speaking, the subjunctive restriction says that the coalitions that we examine to determine a's freedom *must* include a, i.e. $\{a,b,c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a\}$. This is the basis of the slogan introduced at the beginning: 'freedom is membership of powerful coalitions' – because in the technical jargon we will use later, the coalitions which are sufficient for a to φ are denoted as 'powerful'.

The logical peculiarity of this step that has been alluded to is that while the

---

[10] Although not necessarily in the sense of 'acting freely' (Dworkin 1970), which is an exercise and not an opportunity concept of freedom.

[11] For those familiar with the literature on voting power, this heuristic step is akin to the one taken by Holler (1982) in the definition of his power index, the Public Good Index. Holler *ignores* the 'oversized' coalitions – he does not deny they may form – on the grounds that they are formed by 'luck' (in the sense of Barry (1980a, 1980b)) and therefore not relevant for making power ascriptions because they do not contribute to what a coalition can achieve.

subjunctive restriction is quite natural and undemanding, definitions 3.1* and 3.2* can imply a state of logical limbo for i's freedom because the antecedents may not be true (if i were *not* to attempt to φ) although the implications are. This is the state of affairs in which a does not attempt to φ but $\{b,c,d\}$ or any of its two member subsets nevertheless would not prevent her if she were to attempt it and cannot force her to do it either. Hence, i is logically neither free nor unfree to φ because the conditions in definitions 3.1* or 3.2* are not satisfied.[12] As I wish to shy away from discussing the 'bivalent' (one is either free or unfree) and 'trivalent' (one can be free, unfree, or neither free nor unfree) views of freedom,[13] I will simply side with the more intuitive bivalent position by *assuming* that an agent always attempts to φ, which means that the antecedent is always fulfilled.[14]

4. *A Game Theoretic Measure*

4.1 *Types and Tokens*

Having established (i) what we mean in a very weak sense by specific freedom (unfreedom) and (ii) the conditions for making a such an ascription, we can now turn our attention to the principle problem of constructing a freedom function that describes i's expectation that she is free (unfree) to φ. Following Dowding and van Hees (2003), we want this expectation to reflect the different instantiations $r_1,\ldots,r_n$, called *act-tokens*, of performing a particular type of action R, called an *act-type*, given by:

$$\Gamma_i(R) = \Gamma(p(r_1),\ldots,p(r_n)) \tag{4.1}$$

where $p(r_i)$ is the probability that an act-token r_i will not be prevented (the agent

[12] In a sense this is a specious problem (but I mention it in order to ward off anticipated criticism) because the subjunctive captures the counterfactuality of the case when i does not attempt to φ, by saing what happens *if i* were to. So the state of affairs *when i* does not is irrelevant to the freedom ascription. I am grateful to Kieth Dowding for pointing this out to me.

[13] On this see Steiner (2001) and Kramer (2003b: 41–60).

[14] I am aware that assuming that the antecedent is fulfilled (i always attempts) does not preclude the possibility of force being applied to i. My attempting to leave a room does not preclude that you will, and can, drag me out of the room at the same time. I ignore this possibility because it does not alter the fact that my attempt goes unprevented.

is free to perform an instance of the act-type or right, R).[15] The basic idea, is that while the formal existence of a class of acts (an *act-type*), R, is given by the possibility that at least one of its instantiations, the *tokens* $r_1,\ldots,r_n$, is *possible*, we want to determine how *probable* each of these tokens or instantiations are and from this derive a probabilistic judgement about the extent to which R can be said to *materially* and not just *formally* exist.

In the language that I have been using, an action φ can be taken as either an act-type, R, or an act-token, r_i, because a 'specific freedom' can be more or less 'specific' (Carter 1999, Steiner 1994, van Hees 2000). To use the example of 'freedom of expression' again, this is an act-type R that can be instantiated in the different ways we have said: r_1 is shouting 'Down with the government' at Whitehall at a particular time and date, r_2 is doing so at Piccadilly Circus, and so on. Each of these tokens can be specified further as act-types themselves: shouting 'Down with the government' at Whitehall alone or doing so with others, etc. An action to which there is unique corresponding event is an act-token; it is an action in which all spatiotemporal and physical components are specified. Thus in the example of a performing φ, each of the coalitions $\{a,b,c,d\}$, $\{a,b,c\}$, $\{a,b,d\}$, $\{a,c,d\}$ are the instantiations (tokens) $r_1,\ldots,r_n$ of a performing the act-type φ. Hence, given our definitional framework, we arrive at the central contribution of this paper: the natural way to define $\Gamma(\cdot)$ is on the domain of possible coalitions. From this we will demonstrate that specific freedom can be identified with the notion of success.

4.2 *Game Forms*

To define $\Gamma(\cdot)$ on the domain of coalitions in a systematic manner, we have to skip through some game theoretic preliminaries. The basic concept that we need is that of a *game form* (all of which has been implicit in our example of a's freedom to φ). A game form is a specification of a finite set of outcomes X, a finite set of individuals (or players) $N = \{1,\ldots,n\}$, a finite set of feasible actions or strategies A_i for each $i \in N$,[16] and an outcome function π (or decision rule) that yields some

[15] Note that most of the recent philosophical literature on the measurement of freedom discusses types and tokens in some detail. See Steiner (1994), Carter (1999:), van Hees (2000), and Kramer (2003b).

[16] If we would be interested in freedom under legal rules, then the set of feasible strategies should be restricted to those which are admissible. See Fleurbaey and Gaertner (1996) and

single outcome x for any given n-tuple $[a_i]$ if strategies, one strategy $a_i \in A_i$ for each i, i.e. $g = (A_i, i \in N; \pi)$. A game form can be said, therefore, to specify the 'rules of the game'.

For our purposes, we are interested in a particular game form in which the outcome set, X, has two elements, either i can perform φ (φ_i) or is cannot perform φ ($\neg\varphi_i$), i.e. $X = \{\varphi_i, \neg\varphi_i\}$ and in which each player (including i) has two possible strategies: to either agree that i should be free to φ or not, which we designate as $A_i = \{yes, no\}$. To be clear, by 'strategy' is not necessarily meant a particular action as such, but rather a 'bundle of actions'; they should be seen as courses of actions. Depending upon the context of the specific freedom, the act of agreeing to or hindering a's freedom to φ may involve different things. It could be as minor as a nod or a wink or providing a signature; or it could involve moving a heavy object; or it could even be an 'omission' in the sense of not doing something that is required, either consciously or unconsciously. In any case, what is involved are many actions (to provide a signature I must pick up a pen, put the pen to paper, hand over the signed form, etc), each of which I must be free to perform.[17] Note, then, that under this construction a specific freedom or unfreedom presupposes other prior specific freedoms and that the specific freedom or unfreedom in question is the outcome of a combination of such bundles of actions as determined by a 'decision rule', π.

Now, according to our definitional framework, π defines the subsets of agents, $S \subseteq N$, called coalitions, that can force an outcome in X. That is, we are looking at a game form with a very sharp distribution of power: a coalition S, which is a collection of members of N who have made the same strategy choice, has either *full power* (is 'winning') or *zero power* (is 'losing'). Thus, in our example, a has the support of b and c in $\{a,b,c\}$ and this coalition has the power to see to it that a can φ, while its complement, $\{d\}$, is powerless (cannot prevent a from φ-ing); while a only has the support of b in $\{a,b\}$, which because it is not enough is therefore powerless to see to it that a can φ, while its complement $\{c,d\}$ is powerful (can prevent a from φ-ing).

Such a game form is also called a *simple game* and can be represented by a

---

Fleurbaey and van Hees (2000).

[17] This point is discussed in detail in Braham and Holler (Forthcoming): an element of A_i by definition presupposes that a player is free to perform that strategy; otherwise it would not be in A_i and not part of the game form.

non-empty set $W \subseteq 2^N$ consisting of the winning coalitions. We assume, as is usual, that W satisfies three basic conditions: (i) $\varnothing \notin W$, otherwise all coalitions would be winning and no player could prevent anything; (ii) $N \in W$, i.e. the grand coalition is powerful; and (iii) if $S \in W$ and $S \subseteq T$, then $T \in W$, i.e. if a coalition is winning then additional support will not alter the outcome. Note that the first condition, $\varnothing \notin W$, guarantees the freedom game to be non-trivial because if the empty set is winning then every set would be one because every subset includes the empty set, which would imply that both φ_i and $\neg\varphi_i$ and would be the outcome. Note also that the non-emptiness of W implies that the specific freedom formally exists, i.e. it is possible.

To complete the preliminaries, there are four further definitions that we will need. The first concerns what we mean by saying that a player has *power* in a game form: we take it to simply be the ability of a player to bring about an outcome by changing a losing coalition to a winning coalition and vice versa. Formally, we say that i exerts power in S if $i \in S \in W$ but $S \setminus \{i\} \notin W$ (or power outside S if, and only if, $i \notin S \notin W$ but $S \cup \{i\} \in W$). Such instances are known as 'swings' or a player's 'decisiveness'. (Aggregating the swings gives us what are known as power indices – a frequently used one is the absolute Banzhaf score.)

The next two definitions concern types of players: a *dictator* is a member of every winning coalition, i.e. for all $S \subseteq N$, $S \in W$ if, and only if, $i \in S$ (or $\{i\} \in W$); while a *dummy* or *powerless* player is one that can never effect an outcome (never has a swing) – for all $S \subseteq N$, $S \setminus \{i\} \in W$ and $S \cup \{i\} \notin W$; and finally, a veto player (blocker) is a player that is a necessary member of $S \in W$, i.e. for all $S \subseteq N$, if $i \notin S$, then $S \notin W$ (or simply, $\{i\}$ is a blocking coalition).

4.3 *The Conditional Probability of Success*

Having identified the set of winning coalitions for a given specific freedom φ, it can be said that we have a freedom game form, $W(\varphi)$. We need make no other assumption as regards the decision rule; we take it to be 'natural' in the sense that no social law or convention need be contained in it.[18]

---

[18] Note: (i) $W(\varphi)$ can be a weighted game, i.e. a game in which there are non nonnegative weights $(w_1,\ldots,w_n)$ attached to the players and a *quota* $0 < q \leq \sum_{i \in N} w_i$ such that $S \in W$ iff $\sum_{i \in N} w_i > q$. The weights can be taken to represent resources such as money, social status, or authority. (ii) Unlike with most formal decision rules, $W(\varphi)$ need not be *proper*, i.e. N cannot be partitioned into two disjoint winning coalitions.

The essence of a 'freedom game' is that i's attempt to φ is, figuratively speaking, made in the form of a 'proposal' to the members of N who either accept or reject it by – again, figuratively speaking – choosing 'yes' or 'no' from their respective strategy sets; i's attempt to φ is registered by i choosing 'yes' from her strategy set.[19] That is, as a heuristic device we assume that S and $N \setminus S$ form. With this apparatus at hand, Definitions 3.1* and 3.2* reincarnate as:

Definition 4.1* (*Specific freedom*) 'i is free to φ' if $i \in S \in W(\varphi)$ ($N \setminus S$ *cannot* prevent i from performing φ, i.e. it is not a blocking coalition).

Definition 4.2* (*Specific unfreedom*) 'i is unfree to φ' if $i \in S \notin W(\varphi)$ ($N \setminus S$ *can* prevent i from performing φ, i.e. it is a blocking coalition).

Note: (i) In accord with definitions 3.1* and 3.2* $i \in S \in W(\varphi)$ implies that no set of agent prevents i from φ-ing because $S \setminus \{i\}$ is not preventing and neither is $N \setminus S$ because it is powerless; and if $i \in S \notin W(\varphi)$ then at least one set of agents is preventing because $N \setminus S$ has the power to do so. (ii) The subjunctive restriction in 3.1* and 3.2* is captured by the conditions $i \in S \in W(\varphi)$ and $i \in S \notin W(\varphi)$; without these restrictions we would have the much weaker $S \in W(\varphi)$ and $S \notin W(\varphi)$ respectively. (iii) Logically speaking, the conditions are sufficient but not necessary for freedom *per se*, because, as we discussed above, it may be true that i is free if $i \notin S \in W(\varphi)$ (this is discussed again below); if, on conceptual grounds we rule out 'being free while being forced', then the conditions can be taken to be necessary and sufficient. (iv) To avoid indeterminacy of the freedom ascription it is assumed that if $S \in W(\varphi)$ then i is free to φ, i.e. when looking at an $S \in W(\varphi)$ we exclude the possibility that the decisive members of S (those with a swing) will not in fact permit i to φ; if this would be the case then these members by definition belong to $N \setminus S$ (it is also assumed that those members of N not in S are in $N \setminus S$).

Thus to speak of i's freedom to φ in a freedom game form $W(\varphi)$ is to speak of membership of a powerful coalition. Following the idea that $\Gamma(\cdot)$ is to be an aggregation of the probabilities of act-tokens r_i of an act-type R as in (4.1), then $\Gamma(\cdot)$ is precisely the probability of such an $i \in S \in W(\varphi)$.[20] To define $\Gamma(\cdot)$ in a more precise

[19] This may not be so 'figurative' for a broad class of situations. My attempt to go from A to B may require that I ask my wife for her car keys; even the act of buying a bus ticket is a 'proposal'.

[20] Note that it may be more reasonable to restrict S to the set of *minimal winning coalitions* (MWC), W^m, where $S \in W^m$ if $S \in W$, but $T \subseteq S \notin W$. It is questionable if excess sized coalitions

fashion we need some additional structure and notation because calculating the probability of an $i \in S \in W(\varphi)$ requires a probability model for *S*. This means incorporating a minimal, but necessary, amount of behavioural information. That is, for any coalition *S* that may arise we may either know, are able to estimate, or make a reasonable *a priori* judgement as to the probability $p(S)$ that the players in *N* will choose an element of their strategy set such that *S* occurs.[21] In other words, $\Gamma(\cdot)$ is made up of two components, the 2^N elementary events denoted by each $S \subseteq N$ and a probability distribution $p : 2^N \to \mathbb{R}$ that associates each *S* with its probability of occurrence $p(S)$. That is, $p(S)$ gives the probability that players in *S* consent to *i* performing φ (by choosing 'yes' from their strategy set A_i) and those in $N \setminus S$ will not (by choosing 'no' from their strategy set A_i). (As is usual, $0 \le p(S) \le 1$ for any $S \subseteq N$, and $\sum_{S \subseteq N} p(S) = 1$). Our freedom function $\Gamma(R)$ is, then, specified by the pair $(W(\varphi), p)$.

With this basic set-up, the natural form for $\Gamma(\cdot)$ is given by a conditional probability. Note that we have said that the existence of an act-type *R* is given by $W(\varphi)$, so that for a given $W(\varphi)$ and *p*:

$$\Gamma_i(W(\varphi), p) =_{def} \text{Prob}\{\text{outcome is } \varphi_i \mid i \text{ chooses } \varphi_i\} = \frac{\sum_{S: i \in S \in W(\varphi)} p(S)}{\sum_{S: i \in S} p(S)} \tag{4.2}$$

To put it in another way, a player's specific freedom is simply a conditional variant of the notion of 'success' independently introduced by Penrose (1946) and Rae (1969) and more fully discussed by Barry (1980a, 1980b). It means, in a loose sense, getting the outcome you want, in this case φ-ing (recall the heuristic assumption accompanying 3.1* and 3.2*), without necessarily being able to force it (being powerful) – cashing out 'want' in the sense of choice and not in the sense of what you 'truly want'.

Before moving on to discuss some interpretive issues, I would like to briefly point out the significant effect that the subjunctive restriction in definitions 3.1*

---

add to the freedom of *i* to perform φ. In our example, if $\{a,b,c\}$ is sufficient for *a* to φ, in what way does $\{a,b,c,d\}$ contribute to *a*'s freedom? This is a question that can not be answered here.

[21] This does not include, however, information about intentions. The precise meaning of $p(S)$ is an open question. On the one hand it can be taken to reflect personal preferences; on the other it can be taken to reflect social structure and conventions. For a detailed discussion, see Braham and Steffen (2002).

and 3.2* has. We have already seen that in its absence we would use the weaker conditions $S \in W(\varphi)$ and $S \notin W(\varphi)$. We would therefore write definitions 3.1 and 3.2 as 'free if $S \in W(\varphi)$' and '*un*free if $S \notin W(\varphi)$'. This yields a different measure altogether: the probability of a winning coalition. For a given $W(\varphi)$ and p:

$$\Gamma_i(W(\varphi), p) =_{def} \text{Prob}\{\text{outcome is } \varphi_i\} = \sum_{S: S \in W(\varphi)} p(S) \tag{4.3}$$

Those familiar with the voting power literature will recognise (4.3) to be none other than Coleman's 'power of a collectivity to act' (Coleman 1971).

It should be obvious, however, that if we model the freedom game slightly differently and say that i is an 'outsider' and her attempt to φ is registered by her 'proposing' the action to the remaining $N \setminus \{i\}$ players in a game $W_{-\{i\}}(\varphi)$ (where the $-\{i\}$ in subscripts represents the reduced player set) and not by her strategy choice in that 'voting game', then (4.2) and (4.3) are formally equivalent. They are also conceptually equivalent because it is now as if we have two games $W_{\{i\}}(\varphi)$, with a player set $\{i\}$, and $W_{-\{i\}}(\varphi)$ forming a bicameral system $W_{\{i\}}(\varphi) \wedge W_{-\{i\}}(\varphi)$ and because $W_{-\{i\}}(\varphi)$ only has one player, the bicameral meet is equivalent to adding a single new veto player to $W_{-\{i\}}(\varphi)$. This has the result of saying that a winning coalition must contain i. Such 'veto power' was not assumed in (4.4) but it is implicit in the idea that we ignore all $i \notin S \in W(\varphi)$. Thus, under this alternative model we can still say that 'freedom is membership of powerful coalitions'.

4.4 *Three Interpretations*

Up to now I have not properly addressed the issue of what $\Gamma_i(W(\varphi), p)$ can really be said to accomplish.[22] Saying that it is a representation of $\Gamma(\cdot)$ which itself is a 'measure of specific freedom' does not say all that much because it is likely that any formal quantification of a concept such as freedom will not be without different shades of meaning. Those familiar with power indices will know this all too well. Any of the traditional power indices can be seen as having two primitive interpretations: as a direct quantification of an 'ability' called 'voting power' or as a 'reasonable expectation' of possessing this 'ability' (Holler 1998). It is not really any different here.

There are three immediate interpretations of $\Gamma_i(W(\varphi), p)$ that come to mind.

[22] I am indebted to Martin van Hees for drawing my attention to this issue.

The first is that it can be said to measure 'the degree to which a freedom-type exists' and is brought forward by the expression 'measurement of specific freedoms' with which this paper begins. This interpretation is problematic because as according to Steiner (1983), Carter (1999: 233ff), and Kramer (2003b: 169ff) it makes no sense to speak of degrees of existence; existence is binary, taking values from the set $\{0,1\}$, and *not* scalar, taking the values from the interval $[0,1]$. The existence of an act type is but the mere possibility of one of its tokens. If we submit to this the notion of existence then Steiner *et al.* are correct; and it finds its parallel in the non-emptiness of $W(\varphi)$. So for the moment, at least, we can eschew this interpretation.

What Steiner *et al.* would say is that as a probabilistic judgement $\Gamma_i(W(\varphi), p)$ captures 'being probably free' to perform a specific action, and not the extent of this freedom. And this is what is more properly meant by the locution 'material existence' that Dowding and van Hees use. This, however, does not solve the interpretive quibble because it yields two further options. The locution 'being probably free' can be cashed out as 'the probability that *i possesses* an act-type freedom' or 'the overall degree of freedom that the act-tokens represent', which is van Hees and Dowding's position. The first of these is a grander claim than the second, but in absence of a more extensive probability model I am not sure that this can be defended. To do so, it would have to be shown that $\Gamma_i(W(\varphi), p)$ gives a value for all possible probability distributions over this act-type. Hence we are left at this point with saying that $\Gamma_i(W(\varphi), p)$ is closer to the more modest Dowding and van Hees interpretation.

5. *Discussion*

No great claims are being made for $\Gamma_i(W(\varphi), p)$. From a theoretical point of view it is not an overly surprising result, even if it has not been obvious up to now; and from a practical point of view it is clearly of limited use – except in very specific institutional contexts where the game form is clearly defined or easily determined, it is unrealistic to believe that we can actually calculate an agent's specific freedom.[23] However, $\Gamma_i(W(\varphi), p)$ does have a fair amount of conceptual and program-

[23] Bureaucracies are a clear case of such an area of application. Here we generally find clear permission structures and decision rules. $\Gamma_i(W(\varphi), p)$ could also be used to give conceptual and empirical content to the management science literature on empowerment, which is often taken as

matic value.

First, the fine-grained process of constructing $\Gamma_i(W(\varphi), p)$ has churned up a major conceptual finding: an individual agent-based definition of specific freedom is logically unsustainable. Individual freedom is generically a collective property. Once this is accounted for we are able to bring the notions of freedom, success, and power into a single conceptual and formal framework. We have found that an agent's specific freedom, taken in the broadly negative sense, can be identified with her expected success of performing an action; and this is distinct from their social power, although dependant upon the structure and distribution of such power. By way of construction of $\Gamma_i(W(\varphi), p)$ we have obtained an answer to the age old and controversial problem of how power and freedom relate to each other in a conceptual hierarchy. Power is the more basic concept. Even the freedoms presupposed in the strategy sets A_i in a game form g presuppose a power structure: each element of the strategy set is itself an outcome of a prior (maybe trivial one-player) game form in which i is a 'dictator'. With respect to each element of A_i, a player can unequivocally see to it that she performs this strategy or not.

Second, $\Gamma_i(W(\varphi), p)$ provides a very natural and simple answer to an outstanding and significant question in the literature on the measurement of freedom and rights; and it is an answer that sits comfortably with the increasingly accepted game form approach to freedom and rights. It is also an answer that satisfies a number of intuitively appealing properties. This is not the place to go into the matter in detail, but it is not difficult to see that $\Gamma_i(W(\varphi), p)$ satisfies the following: (i) a *dictator* property (if i is a dictator with respect to φ, then i has maximal freedom to φ, i.e. $\Gamma_i = 1$); (ii) a *powerless player* (*dummy*) property (the addition of powerless players to N (or $N_{-\{i\}}$) does affect i's freedom to φ); and (iii) a *resource monotonicity* property (if in a game form g, i has more of the requisite resources needed for φ-ing than she has in a game form g', then i is at least as free to φ in g as in g'). The proof of (i) and (ii) are more or less trivial, following from the definitions of a 'dictator' and a 'powerless (dummy) player' respectively. Because a dictator is a member of every winning coalition the probability that i is free to φ is 1. In contrast, a dummy never effects an outcome so adding a dummy to set of players merely doubles the number of winning coalitions: for each old winning

meaning the 'freedom to do something' in an organisation. See, for example, Conger (1988), Gal-Or and Raphael (1998), Spreitzer (1995, 1996), Pfeffer (1992).

coalition S you now have two, S and $S \cup \{new\,dummy\}$, so the probability of obtaining a winning coalition is unchanged. The proof of (iii) is a little more involved. For my purposes here, we need only say that it is a form of 'global monotonicity' (Levínský and Silársky 2001), and can be derived from the proof of Proposition 3 in Laruelle and Valenciano (Forthcoming).

Third, inspection of $\Gamma_i(W(\varphi), p)$ indicates an interesting relationship between the distribution of power among the members of $N_{-\{i\}}$ and i's freedom to φ. Roughly speaking, as we move from a 'democratic' game in which each of the $N_{-\{i\}}$ players has an equal chance to determine the outcome (whether or not i performs φ) to games in which power is increasingly concentrated in the hands of fewer and fewer players, i's freedom will tend to decline.[24] While this is certainly unsurprising, what may be surprising is the fact that from a purely *a priori* perspective (i.e. when we make judgements behind a quasi-Rawlsian 'veil of ignorance' in which we apply the principle of insufficient reason and assume each player will say 'yes' or 'no' with equal probability), i's freedom to φ reaches its nadir not when all power is concentrated in the hands of a single player – a dictator – but when it is concentrated in the hands of a few – an oligarchy.[25] Formally, an oligarchic game is one in which there is a set of veto players that is also a winning coalition; while a dictatorial game is one in which there is a single individual that is also a winning coalition. This is not a mystery because an oligarchic game is in fact a unanimity game – a game in which the only winning coalition is N – on a reduced player set: the set of veto players minus all other players (who are dummies).[26] Needless to say, gaining the consent of one person (a dictator) is easier, ceteris paribus, than having to gain the consent from two or more players all of whom *must* consent (an oligarchy).[27] The basic point is that prob-

[24] A systematic examination of this relationship would plot $\Gamma_i(W(\varphi), p)$ on the vertical axis and an index of inequality of power (such as those axiomatized by Laruelle and Valenciano's (2004) on the horizontal axis. The idea would be for a fixed N and quota of weights q (see footnote 18) one would find different power distributions yielding 0.1 increments in the inequality index.

[25] The restrictiveness of an oligarchy was recognized by Oppenheim (1961: 198): 'The degree to which X is unfree to do x is a function of the number of actors Y who limit his freedom'; but he did not comment on the fact that, ceteris paribus, a dictator provides more freedom.

[26] For a review of the basic properties of oligarchic games, see van Deemen (1997: 126–129).

[27] If the $N_{-\{i\}}$ players act independently, and each have the same probability, p, to consent to i performing φ, but not necessarily 0.5, then this 'oligarchy result' holds for any $p < 1$ (assuming of course that in the dictator game the dictator has the same probability to consent as the members of the oligarchy). The robustness of this result under an asymmetric constellation of player propensi-

ability of a specific freedom does not *necessarily* decline monotonically with increasing inequality of power in $N_{-\{i\}}$. In fact, from a strictly *a priori* perspective, if *i* is interested in her pure negative freedom she would be indifferent between a 'democratic' and a 'dictatorial' game on $N_{-\{i\}}$ (or a game $W(\varphi)$ in which there is one other veto player): in both cases the odds are even that *i* will be free to φ, $\Gamma_i(W(\varphi), p) = 0.5$.

To many, the parity between a democratic and dictatorial game form might be considered an unacceptable pathology of $\Gamma_i(W(\varphi), p)$ as an index of specific freedom. But this would be mistaken for it is merely indicative of their insensitivity to procedural aspects of a game form: that the democratic one is an 'anonymous' decision rule (what matters is *how many* are in favour of *i* performing φ, not *who* is in favour), while a dictatorial one is not. Clearly, one could argue that a reasonable measure of freedom ought to account for such qualitative differences on the grounds that we may intuitively feel that 'procedures matter' in much the same way as we may feel that 'choices which matter' contribute more to our freedom than 'choices which do not matter', or 'matter less'. Or looking at it the other way, we may feel that the degree to which our freedom is curtailed depends somehow on the procedure by which it happens.[28]

To argue this way is, however, to posit a conception of freedom different to that which I have used here. Hence, it is not the measure that is procedure insensitive but the notion of negative freedom itself. As already alluded to in Section 2, from the standpoint of freedom as mere unpreventedness, it is entirely irrelevant whether the preventing conditions are inoperative because of a particular agent or because of a group of some agents. Notice that this is not something that mainstream liberals like Isaiah Berlin would object to. 'Liberty', remarked Berlin in his *Four Essays on Liberty* (1969), '… is principally concerned with the area of control, not with its source' (p. 129), and, he continues, 'The answer to the question "Who governs me?" is logically distinct from the question "How far does government interfere with me?" (p. 130). He goes on to tell us that it is conceptually possible

ties is a matter for future investigation.

[28] Obviously procedural aspects can be of relevance to how people *evaluate* and act in social circumstances. It is well known in experimental bargaining, for example, that the outcome of ultimatum games is sensitive to the presence of face-to-face communication (Roth 1995). This does not change the fact that there is no necessary connection between *i*'s freedom to φ and how that freedom has been obtained. See also Gaertner and Xu (2004).

for there to be more freedom in a dictatorship than in a democracy.[29] The question 'By whom am I ruled?' (p. 130), Berlin says, belongs to the domain of the 'positive' conception of freedom. The point is that the desire to be governed in a particular way (by myself in particular) may be 'as deep a wish as that of a free area of action … But it is not a desire for the same thing' (p. 130). To take issue, then, with this property of $\Gamma_i(W(\varphi), p)$ is tantamount to a rejection of the negative conception of freedom, not of the measure itself.

To complete our discussion, it would seem necessary to make a brief excursus on the matter of a republican conception of freedom *á la* Pettit (1997). Republicans of Pettit's mould would probably have difficulty endorsing $\Gamma_i(W(\varphi), p)$ on the grounds that its definitional framework is too weak in the sense that it only requires that potential constraints are inoperative for making a freedom ascription; not that they do not exist. For republicans, what definitions 3.1, 3.1*, and 4.1* lack is a reference to i's *power* to φ (i.e. the ability of i to φ despite potential opposition by others) because they ascribe i the freedom to φ even if constraints *could* have been operative had i chosen to φ, but *would not have been*, because the set of agents that *could* have made the constraints operative would not have done so (because the members of such a set had no common desire to do so). The unacceptable upshot is that these definitions allow us to say i is free to φ even if it is at the grace and favour of some set of agents who could, at will and with impunity, make i unfree to φ (this is what Pettit means by 'domination' or 'power'). For Pettit, such a situation is hardly deserving of the badge of freedom.

In Pettit's (1996, 1997) terminology, what the definitional framework is lacking is a criterion of 'non-domination' or 'antipower'. For i to be free to φ in a non-domination sense is to say that i cannot be prevented from φ-ing by *any* set of agents, viz. is immune from any interference. Only then, Pettit says, can we say that i is free to φ because there is no need for 'luck, cunning, or fawning' for unpreventedness. Freedom in this view is to enjoy 'noninterference resiliently' (Pettit 1996: 589); and it is this form of noninterference (in our terminology, 'unpreventedness') that Pettit calls 'republican freedom'.

The extent to which Pettit's conception of freedom is sustainable is something that we can probe here. On the one hand it seems to be plausible because we may find it counterintuitive to say that the slave is free to φ if it is only possible at the

---

[29] See, in particular, footnote 3 on p. 129 of *Four Essays on Liberty*.

behest of his master; on the other, it is equally counter-intuitive to say that he is unfree to φ given that his master has granted him permission. However, what is important is that even if it is a conceptually sustainable notion, it has no bearing upon the general form that we have given $\Gamma(\cdot)$. To see this, we need only note that Pettit's notion of non-domination ascribes *i* the freedom to φ *only* in those instances in which the set of preventing coalitions is empty. To be free in this sense is not only being free in the *actual* world where there is no effective resistance ($\{b,c,d\}$ do not oppose *a* from φ-ing), but also to be free even if there were resistance ($\{b,c,d\}$ or any of its subsets do oppose her doing it). Freedom as non-domination means freedom in *all nearby possible* worlds (List 2004), which in accord with our definitions happens to be the set of all possible coalitions on the set of agents. In Dowding's (1991: 48) terminology, this is having full 'outcome power' (in the technical jargon we have used, it is being a *dictator* with respect to φ); and this is completely covered by definitions 3.1, 3.1*, and 4.1*. It is trivially true that if $\{a\}$ has unqualified power – immune from any interference or encroachment – with respect to *a* φ-ing, then the conditions contained in these definitions are always satisfied because whatever the configuration of other agents, no coalition ever prevents *a* from performing φ. The point to emphasize, then, is that a person can have a specific freedom in an unpreventedness sense without having that freedom in a republican sense; but if a person has a freedom in republican sense then she also has it in an unpreventedness sense. We are do not need, therefore, an alternative framework to account for a republican conception of freedom; we only need to place a restriction on $W(\varphi)$.[30]

6. *Conclusion*

In this paper I have outlined and defended an interpretation of specific freedom as 'membership of powerful coalitions' (or 'freedom as conditional success') and from this constructed an elementary and natural function for its measurement. In wrapping up I would like to briefly point to two lines of further investigation that have been opened up by this idea.

[30] In the same way as we have suggested in footnote 20 that maybe we should only take into account MWCs. Note also that different conceptions of negative liberty can be contrasted on the domain of coalitions.

The first is to examine the possibility of using $\Gamma_i(W(\varphi), p)$ to construct a measure of *overall freedom*. This involves two tasks. The straightforward task is to determine the 'overall freedom of *i*' as the expectation that *i* is unprevented to perform *all* the specific freedoms that *i* can conceivably have; while the less straightforward one is to aggregate the overall freedom for each *i* into an measure for all members of *N*, i.e. a measure that would allow us to answer the question 'how free in an overall sense is society *A* compared to society *B*?' (Carter 1999: 28–29). While intuitively it would seem that the first task can be accomplished by simply summing over the value for each specific freedom and then dividing by the number of such freedoms in the spirit of Steiner and Carter, it is unclear how to proceed with the second task. For it is not obvious – to me at least – that one can add the probability that *i* will be unprevented to φ to the probability that $j, k, \ldots, n$ will be unprevented to φ in a conceptually meaningful way.

The second line of investigation concerns developing a game theoretic method for assessing the robustness an agent's specific freedom or the *sensitivity* of the freedom to φ to potential changes in the behaviour of others. This line of thought would take as its starting point a particular winning coalition and ask how sensitive this coalition is to a marginal behavioural change by its members (Napel and Widgrén Forthcoming). Such a measure would allow an agent to form a reasonable expectation of being left unhindered to φ once that freedom has been granted. It would seem natural to say that, ceteris paribus, that I have more freedom to φ in a game g than in a game g' if my freedom in g is less sensitive to changes in the behaviour of others than in g'. This would be the basis of developing a freedom function that can be said to determine the probability that an agent possesses an act-type freedom. From this we would then be able to speak in more precise terms about the degree to which a freedom is respected and can be said to materially exist in a *robust* sense.

References

Arrow, K.J. (1995) A Note on Freedom and Flexibility, in K. Basu, P.K. Pattanaik and K. Suzumura (eds) *Choice, Welfare and Development: A Festschrift in Honour of Amartya K. Sen*. Oxford: Oxford University Press.

Barry, B. (1980a) Is it Better to be Powerful or Lucky? Part 1, *Political Studies* 28: 183–94.

Barry, B. (1980b) Is it Better to be Powerful or Lucky? Part 2, *Political Studies* 28: 338–52.

Bavetta, S. (1999) The Measurement of Specific Freedom, *Notizie di Politeia* 56: 107–13.

Bavetta, S. (2004) Measuring Freedom of Choice: An Alternative View of a Recent Literature, *Social Choice and Welfare* 22: 29–48.

Berlin, I. (1969) *Four Essays on Liberty*. Oxford: Oxford University Press.

Braham, M. and M.J. Holler (Forthcoming) The Impossibility of a Preference-based Power Index, *Journal of Theoretical Politics*.

Braham, M. and F. Steffen (2002) Local Monotonicity of Voting Power: A Conceptual Analysis, Mimeo, University of Hamburg.

Carter, I. (1999) *A Measure of Freedom*. Oxford: Oxford University Press.

Carter, I. (2004) Choice, Freedom, and Freedom of Choice, *Social Choice and Welfare* 22: 61—81.

Cohen, G.A. (1979) Capitalism, Freedom and the Proletariat, in A. Ryan (ed.) *The Idea of Freedom: Essays in Honour of Isaiah Berlin*. Oxford: Oxford University Press.

Cohen, G.A. (1988) *History, Labour, and Freedom: Themes from Marx*. Oxford: Oxford University Press.

Coleman, J.S. (1971) Control of Collectivities and the Power of a Collectivity to Act, in B. Lieberman (ed.) *Social Choice*. New York: Gordon and Breach.

Conger, J.A. and R.N. Kanungo (1988) The Empowerment Process: Integrating Theory and Practice, *Academy of Management Review* 13: 471–82.

Deb, R. (1994) Waiver, Effectivity and Rights as Game Forms, *Economica* 61: 167–78.

Deb, R. (2004) Rights as Alternative Game Forms, *Social Choice and Welfare* 22: 83–111.

Dowding, K. (1991) *Rational Choice and Political Power*. Aldershot: Edward Elgar.

Dowding, K. (1992) Choice: Its Increase and its Value, *British Journal of Political Science* 22: 301–14.

Dowding, K. and M. van Hees (2003) The Construction of Rights, *American Political Science Review* 97: 281–93.

Dworkin, G. (1970) Acting Freely, *Nous* 4: 367–83.

Fleurbaey, M. and W. Gaertner (1996) Admissibility and Feasibility in Game Forms, *Analyse & Kritik* 18: 54–66.

Fleurbaey, M. and M. van Hees (2000) On Rights in Game Forms, *Synthese* 123: 295–326.

Gaertner, W., P.K. Pattanaik and K. Suzumura (1992) Individual Rights Revisted, *Economica* 59: 161–77.

Gaertner, W. and Y. Xu (2004) Procedural Choice, *Economic Theory* 24: 335–249.

Gal-Or, E. and R. Amit (1998) Does Empowerment Lead to Higher Quality and Profitability?, *Journal of Economic Behavior and Organization* 36: 411–31.

Gärdenfors, P. (1981) Rights, Games and Social Choice, *Noûs* 15: 341–56.

Holler, M.J. (1982) Forming Coalitions and Measuring Voting Power, *Political Studies* 30: 262–71.

Holler, M.J. (1998) Two Stories, One Power Index, *Journal of Theoretical Politics* 10: 179–90.

Holler, M.J. and M. Widgrén (1999) The Value of a Coalition is Power, *Homo*

Oeconomicus 15: 497–511.

Kramer, M.H. (2003a) On Being Free Without Having Any Choices, *Journal of Theoretical Politics* 15: 445–71.

Kramer, M.H. (2003b) *The Quality of Freedom*. Oxford: Oxford University Press.

Kristjánsson, K. (1996) *Social Freedom*. Cambridge: Cambridge University Press.

Laruelle, A. and F. Valenciano (2004) Inequality in Voting Power, *Social Choice and Welfare* 22: 413–31.

Laruelle, A. and F. Valenciano (Forthcoming) Some Voting Power Postulates and paradoxes Revisited, *Social Choice and Welfare*.

Levínský, R. and P. Silársky (2001) Global Monotonicity of Values of Cooperative Games: An Argument Supporting the Explanatory Power of Shapley's Approach, in M.J. Holler and G. Owen (eds) *Power Indices and Coalition Formation*. Dordrecht: Kluwer.

List, C. (2004) The Impossibility of a Paretian Republican, *Economics and Philosophy*.

MacCallum, G.C. (1967) Negative and Positive Freedom, *Philosophical Review* 76: 312–34.

Miller, D. (1983) Constraints on Freedom, *Ethics* 94: 66–86.

Napel, S. and M. Widgrén (Forthcoming) Power Measurement as Sensitivity Analysis – A Unified Approach, *Journal of Theoretical Politics*.

Nozick, R. (1974) *Anarchy, State, and Utopia*. New York: Basic Books.

Oppenheim, F.E. (1961) *Dimensions of Freedom*. New York: St. Martins Press.

Pattanaik, P. and Y. Xu (1990) On Ranking Opportunity Sets in Terms of Freedom of Choice, *Recherches Economiques de Louvain* 56: 383–90.

Pattanaik, P.K. and K. Suzumura (1996) Individual Rights and Social Evaluation: A Conceptual Framework, *Oxford Economic Papers* 48: 194–212.

Pattanaik, P.K. and Y. Xu (1998) On Preference and Freedom, *Theory and Decision* 44: 173–98.

Peleg, B. (1998) Effectivity Functions, Game Forms, Games and Rights, *Social Choice and Welfare* 15: 67–80.

Penrose, L.S. (1946) The Elementary Statistics of Majority Voting, *Journal of the Royal Statistical Society* 109: 53–7.

Pettit, P. (1996) Freedom as Antipower, *Ethics* 106: 576–604.

Pettit, P. (1997) *Republicanism: A Theory of Freedom and Government*. Oxford: Oxford University Press.

Pfeffer, J. (1992) *Managing with Power: Politics and Influence in Organizations*. Boston MA: Harvard Business School Press.

Rae, D.W. (1969) Decision-Rules and Individual Values in Constitutional Choice, *American Political Science Review* 63: 40–63.

Rosenbaum, E. (2000) On Measuring Freedom, *Journal of Theoretical Politics* 12: 205–27.

Roth, A.E. (1995) Bargaining Experiments, in J.H. Kagel and A.E. Roth (eds) *The Handbook of Experimental Economics*. Princeton: Princeton University Press.

Sen, A.K. (1970) The Impossibility of a Paretian Liberal, *Journal of Political Economy* 78: 152–7.

Spreitzer, G.M. (1995) Psychological Empowerment in the Workplace: Dimensions, Measurment, and Validation, *Academy of Management Journal* 38: 1442–65.

Spreitzer, G.M. (1996) Social Structural Characteristics of Psychological Empowerment, *Academy of Management Journal* 39: 483–504.

Steiner, H. (1983) How Free: Computing Personal Liberty, in A.P. Griffiths (ed.) *Of Liberty*. Cambridge: Cambridge University Press.

Steiner, H. (1994) *An Essay on Rights*. Oxford: Blackwell.

Steiner, H. (2001) Freedom and Bivalence, in I. Carter and M. Ricciardi (eds) *Freedom, Power and Political Morality*. Basingstoke: Palgrave.

Sugden, R. (1978) Social Choice and Individual Liberty, in M.J. Artis and A.R. Nobay (eds) *Contemporary Economic Analysis*. London: Croom Helm.

Sugden, R. (1985) Liberty, Preference, and Choice, *Economics and Philosophy* 1: 213–29.

Sugden, R. (1998) The Metric of Opportunity, *Economics and Philosophy* 14: 307–37.

Taylor, C. (1979) What's Wrong with Negative Liberty, in A. Ryan (ed.) *The Idea of Freedom: Essays in Honour of Isaiah Berlin*. Oxford: Oxford University Press.

van Deemen, A.M.A. (1997) *Coalition Formation and Social Choice*. Dordrecht: Kluwer.

van Hees, M. (1995) *Rights and Decisions*. Dordrecht: Kluwer.

van Hees, M. (1998) On the Analysis of Negative Freedom, *Theory and Decision* 45: 175–97.

van Hees, M. (2000) *Legal Reductionism and Freedom*. Dordrecht: Kluwer.

Seit 1997 sind erschienen:

NR. 94: R. PETERS, Evolutionary Stability and Cooperation: The Infinitely Repeated Prisoner's Dilemma, Juni 1997.

NR. 95: R. PETERS, Evolutionary Stability in Social Environments: The Ultimatum Game, Juni 1997.

NR. 96: V. TIMMERMANN, Zur Geschichte der Geldlehre, Juli 1997

NR. 97: L. GOERKE UND M. J. HOLLER, Strategic Standardization in Europe: A Public Choice Perspective, November 1997.

NR. 98: M. J. HOLLER, G. KNIEPS UND E. NISKANEN, Standardization in Transportation Markets: A European Perspective, Dezember 1997.

NR. 99: M. J. HOLLER, Two Stories, One Power Index, Dezember 1997.

NR. 100: E. SCHOLING UND V. TIMMERMANN, Der Zusammenhang zwischen politischer und ökonomischer Freiheit: Eine empirische Untersuchung, November 1998.

NR. 101: M. J. HOLLER AND M. WIDGREN, The Value of a Coalition is Power, Dezember 1998.

NR. 102: M. J. HOLLER AND M. WIDGREN, Why Power Indices for Assessing EU Decision-making, Dezember 1998.

NR. 103: M. J. HOLLER, On the Regulation of Telecommunications Markets, März 1999.

NR. 104: Y. ALTUNBAS, S. P. CHAKRAVATY AND F. STEFFEN, Proportional Representation in the National Assembly for Wales, März 1999.

NR. 105: M. J. HOLLER AND B.-A. WICKSTRÖM, The Use of Scandals in the Progress of Society, May 1999.

NR. 106: R. ONO, Limit Property of a Multi-Choice Value: Comparison with the Fuzzy Value, June 1999.

NR. 107: M. J. HOLLER, R. ONO, AND F. STEFFEN, Constrained Monotonicity and the Measurement of Power, October 1999.

NR. 108: M. J. HOLLER, Fairness, Power and Their Relationship, November 1999.

NR. 109: F. STEFFEN, When is A Priori Voting Power really A Priori?, November 1999.

NR. 110: A. POMPERMAIER, Downsizing, X-efficiency and Social Efficiency, February 2000.

NR. 111: I. LINDNER AND H. STRULIK, Property Rights and Growth, February 2000.

NR. 112: A. POMPERMAIER, Imperfect Competition and Firm Efficiency: A General Equilibrium Approach, April 2000.

NR. 113: M. J. HOLLER, Telecommunications Regulation and Economic Development, May 2000.

NR. 114: M. J. HOLLER AND V. KANNIAINEN, Compatibility Effects of Set-up Costs in the Organization of Service Systems, May 2000.

NR. 115: E. SCHOLING, Messung der Bevölkerungsgesundheit in Entwicklungsländern auf der Grundlage eines MIMIC-Modells, Mai 2000.

NR. 116: C. MENZEL, Collusion in Auctions: A Rotating-Bid Mechanism, July 2000.

NR. 117: C. HILLINGER, Consistent Aggregation and Chaining of Price and Quantity Measures, August 2000.

NR. 118: M. J. HOLLER AND S. NAPEL, On Interpersonnel Comparison of Value, October 2000.

NR. 119: M. BRAHAM AND F. STEFFEN, Voting Power in Games with Abstentions, July 2001.

NR. 120: M. BRAHAM AND F. STEFFEN, Notes on Power in Hierarchical Structures, July 2001.

NR. 121: I. LINDNER, Probabilistic Characterization of Power Indices with Abstentions, July 2001.

NR. 122: M. BRAHAM AND F. STEFFEN, Voting Rules in Insolvency Law: A Simple Game Theoretic Approach, December 2001.

NR. 123: M.J. HOLLER, Classical, Modern, and New Game Theory, December 2001.

NR. 124: I. LINDNER AND M. MACHOVER, L.S. Penrose's Limit Theorem: Proof of a Special Case, April 2002.

NR. 125: I. LINDNER AND H. STRULIK, Distributive Politics and Economic Growth: The Markovian Stackelberg Solution, April 2002.

NR. 126: M.J. HOLLER AND S. NAPEL, A Pragmatic Interpretation of Ken Binmore's Theory of Justice, October 2002.

NR. 127: M.J. HOLLER AND G. OWEN, Present and future of Power Measures, December 2002.

NR. 128: M.J. HOLLER AND P. SKOTT, The Importance of Setting the Agenda, June 2003.

NR. 129: M.J. HOLLER AND I.LINDNER, Mediation as signal, June 2003.

NR. 130: M.BRAHAM AND M.J. HOLLER, The Impossibility of a Preference-based Power Index, July 2003.

NR. 131: F. BOLLE AND M. BRAHAM, A Difficulty with Oaths: On trust, trustwortiness, and signalling, July 2003.

NR. 132: M.J. HOLLER, The Artist as a Secret Agent: Liberalism Against Populism, August 2003

NR. 133: W.GÜTH AND S.NAPEL, Inequality Aversion in a Variety of Games: An Indirect Evolutionary Analysis, November 2003

NR. 134: E. SCHOLING, Zur ökonomischen Analyse der Kinderarbeit, April 2004

NR. 135: M. BRAHAM, Freedom, power, and success: A game-theoretic perspective, May 2004