Success versus decisiveness: conceptual discussion and case study

July 3, 2004

Abstract

In this paper, we vindicate the relevance of the notion of success or satisfaction for the normative assessment of voting rules. We provide arguments in support of this view and emphasize the conceptual and analytical differences between this notion and that of decisiveness. The conclusions are illustrated in the case study provided by three different voting rules that have been proposed for the Council of Ministers of the European Union.

1 Introduction

When a set of individuals makes decisions by means of a voting rule that specifies for which configurations of votes a proposal is accepted, the question of the "power" or "voting power" that the voting rule confers to each voter arises. This issue is at the basis of a considerable piece of literature, both theoretical and applied. In particular a variety of "power indices" intended to assess different variations of the notion of "power" under different conditions have been proposed[1].

Since Shapley-Shubik's (1954) interpretation of their index[2] as the probability of being "pivotal" or decisive in the make of a decision, the notion of decisiveness has *de facto* been widely accepted by many scholars as the right formalization of the notion of "voting power". Banzhaf (1965) and Coleman (1971, 1986), in spite of their right criticism of Shapley-Shubik's index, also assume the notion of decisiveness at the basis of their indices[3]. These indices, as well as others, have been often applied with normative purposes to different situations from the real world, and have been the basis of different normative recommendations about the "right" voting rule in a variety of actual committees of representatives.

In spite of this dominant view, some authors have raised doubts as to the relevance of this interpretation of "power" as decisiveness, especially for normative purposes, suggesting as more relevant to this effect the notion of "satisfaction" or "success". That is, focussing on the likelihood of having the result one voted for irrespective of whether one's vote was crucial for it or not. Rae (1969) is the first to propose a measure of success for symmetric voting rules, and Dubey and Shapley (1979) extend this "Rae index" to arbitrary voting rules. Later, a few authors have paid attention to the notion of success, as Brams and Lake (1978), Barry (1980) (from whom we take the term of "success" that we use here), Straffin, Davis, and Brams (1981), and more recently König and Bräuninger (1998)[4].

Nevertheless, as Benoît and Kornhauser (2002) remark, "although a voter's satisfaction is arguably more important than a voter's power, the former concept has received comparatively little attention from game theorists." Moreover, it must be said that the

[1] See, e.g., Felsenthal and Machover's (1998) monography, or a recent synthesis in Laruelle and Valenciano (2004a).

[2] Their index is the result of applying the Shapley (1953) value to the simple game that results from the voting rule by assigning 'worth' 1 to the 'winning coalitions' and 0 to the losing ones.

[3] This is also the case with Penrose's (1946) pioneer measure.

[4] In a recent paper Barberà and Jackson (2004), in a different framework, address the issue of the "efficient" voting rule, for which, under certain assumptions, the aggregated expected utility of the voters is maximized.

two concepts have been often confused or at least insufficiently separated. Possibly, this is partly due to the relationship pointed out by Dubey and Shapley (1979). They showed that there exists an affine-linear relation between the "Rae index" and the Banzhaf index[5]. This no doubt has contributed to overlooking the question of success in the literature, where success is often either ignored or considered a sort of appendix or secondary ingredient of decisiveness, commonly considered as the substantial notion.

The purpose of this paper is to vindicate the relevance of the notion of success or satisfaction for the assessment of voting rules with normative purposes, and emphasize the conceptual difference between this notion and that of decisiveness. To this end we examine some analytical relations and their generality, and some differences of consequence in practical quantifications within the setting introduced in Laruelle and Valenciano (2004).

The conclusions are illustrated in a case study: the three voting rules that have been recently considered for EU's Council of Ministers[6]: Nice's voting rule, the one proposed by the Convention and the one proposed by the Spanish government[7]. This application is a small step towards filling a gap that seems to exist in the literature, where most studies apply indices of decisiveness and hardly pay any attention to the question of success. We show that the conclusions of a comparison between these three voting rules based on the point of view of success may differ from the conclusions based on the decisiveness point of view. We pay a special attention to what seems to be an important concern of the Member states: the probability of being imposed a proposal to which they oppose.

2 Success versus decisiveness: analytical discussion

2.1 Background and notation

We consider voting rules to make dichotomous choices (acceptance and rejection) by a voting body. Let $N = \{1, 2, .., n\}$ denote the set of seats. If any vote different from 'yes' is assimilated into 'no', there are 2^n possible *vote configurations*. Each vote configuration can be represented by the set $S \subseteq N$ of 'yes' voters. The cardinal of S will be denoted by s. An N-*voting rule* is fully specified by the set W of *winning vote configurations*, that

[5]More precisely, they show this to be so for the "raw" (i.e., Banzhaf index as defined originally) and a natural extension of Rae measure of success for arbitray voting rules.

[6]Many studies have been devoted to the European Union Council of Ministers and in particular to the Nice rule (see, for instance, Laruelle and Widgrén (1998), Felsenthal and Machover (2001), Lane and Maeland (2002), Leech (2002)).

[7]When the first draft of this paper was written president Aznar's Spanish government endorsed adamantly what we call here "Spanish rule". Whatever the future policy of the new Spanish government, the comparison with the other rules provides an interesting example to illustrate the claims presented in section 2.

is, those which lead to the acceptance of a proposal. We assume the set W satisfies the following conditions: (i): The unanimous 'yes' leads to the acceptance of the proposal: $N \in W$; (ii): The unanimous 'no' leads to the rejection of the proposal: $\emptyset \notin W$; (iii): If a vote configuration is winning, then any other configuration containing it is also winning: If $S \in W$, then $T \in W$ for any T containing S; (iv): If one vote configuration leads to the acceptance of a proposal, the opposite configuration will not: If $S \in W$ then $N \backslash S \notin W$.

We assume that a set of (N-labelled) voters uses a voting rule W as a 'take-it-or-leave-it' committee. That is, a committee that can only accept or reject proposals submitted to it by some external agency[8]. As in Laruelle and Valenciano (2004) we assume that a second input describes the voting situation: a probability distribution over the set of all possible vote configurations. The most natural choice for normative purposes seems to be assuming all vote configurations being equally probable. Nevertheless, we introduce the basic notions in terms of an arbitrary probability distribution, which can be interpreted as a "common prior" about the voters voting behavior. As we will see this provides a wider perspective that allows for a better understanding of the concepts involved and their relationships.

Let p denote a probability distribution over the set of vote configurations, and let $p(S)$ denote, for each $S \subseteq N$, the probability of S being the vote configuration. For a given p and a given W, several features can be evaluated *ex ante*. First, the ease or difficulty to pass proposals, can be evaluated by the probability of a proposal being accepted or rejected, given by

$$\alpha(W,p) := Prob \text{ (the proposal is accepted)} = \sum_{S:i \in S} p(S), \tag{1}$$

and $\bar{\alpha}(W,p) :=$*Prob* (the proposal is rejected) $= 1 - \alpha(W,p)$.

A voter's probability of being decisive (i.e., having the result one voted for and being crucial for it) is given by

$$\Phi_i(W,p) := Prob \text{ (i is decisive)} = \sum_{\substack{S:i \in S \in W \\ S \backslash i \notin W}} p(S) + \sum_{\substack{S:i \notin S \notin W \\ S \cup i \in W}} p(S), \tag{2}$$

while the likelihood of being successful (having the result one voted for) for a voter i is given by

$$\Omega_i(W,p) := Prob \text{ (i is successful)} = \sum_{S:i \in S \in W} p(S) + \sum_{S:i \notin S \notin W} p(S). \tag{3}$$

[8]This limitation implicit in the traditional approach to the assessment of voting situations based on the sole voting rule is systematically minimized (if not completely ignored) in the power index literature.

In spite of the obvious difference of meaning between the notions of success and that of decisiveness, in view of the dominant confusion alluded to in the introduction it seems convenient providing additional arguments for a clear distinction. To this end we examine in the next subsections some relationships and their very particular character, as well as some relevant differences.

We will deal also with the following 'interim' evaluations (i.e., conditional expectations[9] updated with the private information of each voter's own vote) for which we use the following notation:

$$\begin{aligned}
\Phi_i^{i+}(W,p) &:= Prob\ \ (i \text{ is decisive} \mid i \text{ votes 'yes'})\,, \\
\Phi_i^{i-}(W,p) &:= Prob\ \ (i \text{ is decisive} \mid i \text{ votes 'no'})\,, \\
\Omega_i^{i+}(W,p) &:= Prob\ \ (i \text{ is successful} \mid i \text{ votes 'yes'})\,, \\
\Omega_i^{i-}(W,p) &:= Prob\ \ (i \text{ is successful} \mid i \text{ votes 'no'})\,.
\end{aligned}$$

We will also denote

$$\gamma_i(p) := Prob\ \ (i \text{ votes 'yes'}) = \sum_{S: i \in S} p(S). \tag{4}$$

2.2 Some especial relationships

As commented in the introduction, the notion of decisiveness has been widely accepted as the right formalization of the notion of "voting power" or "voting influence". Whatever the relevance of this interpretation of power, it can be argued that it seems more relevant from the voters' point of view the likelihood of having the result they voted for irrespective of their being crucial for it or not.

Despite of the clear conceptual difference, the notions of success and decisiveness are still largely conflated and seen as the two faces of a same coin. The confusion arises from certain particular relations that hold exclusively for the special distribution of probability that assigns the same probability to all vote configurations. Namely,

$$p^*(S) := \frac{1}{2^n} \quad \text{for all configuration } S \subseteq N.$$

This distribution of probability is the underlying assumption of different power indices proposed in the literature (see Laruelle and Valenciano, 2004). This is the case of the Banzhaf (1965) index ($Bz_i(W)$), the Rae (1969) index ($Rae_i(W)$), Coleman's (1971) indices to prevent action ($Col_i^P(W)$) and to initiate action ($Col_i^I(W)$), and König-Brauninger's (1998)

[9] The conditional probability $Prob(A \mid B) = \frac{Prob(A \cap B)}{Prob(B)}$ only makes sense if $Prob(B) \neq 0$.

inclusiveness index ($KB_i(W)$), respectively given by:

$$\begin{aligned} Bz_i(W) &= Prob_{p^*}(i \text{ is decisive}) = \Phi_i(W, p^*) \\ Rae_i(W) &= Prob_{p^*}(i \text{ is successful}) = \Omega_i(W, p^*) \\ Col_i^P(W) &= Prob_{p^*}(i \text{ is decisive} \mid \text{the proposal is accepted}) \\ Col_i^I(W) &= Prob_{p^*}(i \text{ is decisive} \mid \text{the proposal is rejected}) \\ KB_i(W) &= Prob_{p^*}(i \text{ is successful} \mid \text{the proposal is accepted}). \end{aligned}$$

Dubey and Shapley (1979) established the well-known relation between the Banzhaf index and their extension of Rae' index[10]

$$2Rae_i(W) = 1 + Bz_i(W),$$

which[11] in the current notation can be restated like this

$$2\Omega_i(W, p^*) = 1 + \Phi_i(W, p^*). \tag{5}$$

It is also well-known that the Banzhaf index can also be expressed as the probability of being decisive conditional to voter i voting 'yes' or voting 'no':

$$\begin{aligned} Bz_i(W) &= Prob_{p^*}(i \text{ is decisive} \mid i \text{ votes 'yes'}) \\ &= Prob_{p^*}(i \text{ is decisive} \mid i \text{ votes 'no'}). \end{aligned}$$

In other terms

$$\Phi_i(W, p^*) = \Phi_i^{i+}(W, p^*) = \Phi_i^{i-}(W, p^*). \tag{6}$$

Relation (5), though it does not justify the confusion, has no doubt contributed to overlooking the notion of success or satisfaction, often considered as just a sort of appendix or secondary ingredient of decisiveness because of it. According to Dubey and Shapley (1979, p. 124): "It was not noticed for several years that this 'Rae index' is nothing but the Banzhaf index in disguise." More recently, Hosli and Machover (2004) commenting about the "likelihood of a member's vote being critical and the likelihood of that member being successful in securing desired outcomes", claim that "as a matter of fact these two concepts of voting power, far from being opposed to each other, are virtually identical, and

[10]Note that we have divided Dubey and Shapley's equation (53) by 2^n.

[11]Much more recently, Lane and Maeland (2000) also show a similar relation between König and Bräuninger's (1998) "inclusiveness" index and Coleman's (1971) "power to prevent action". Namely,

$$2KB_i(W) = 1 + Col_i^P(W).$$

This is equation (28) in Lane and Maeland (2000), though they did not seem to be aware that what they define as the individual probability of blocking is Coleman's index to prevent action.

differ only in using a different scale of measurement." But as we argue in 2.3, equation (5), though correct, has been considerably misleading.

On the other hand, relation (6) conveys the idea that there is no difference between the "approval power" and the "blocking power". As Straffin (1982 p. 267-268) puts it: "(..) we mentioned 'blocking coalitions' which could prevent a proposal from passing, even though they might not be able themselves to pass a proposal. This would suggest that in addition to studying 'approval power' as we have here, we should also study 'blocking power'." But he concludes that the Banzhaf (or Shapley-Shubik's) index can serve to "effectively measure both kinds of power."[12] Again in this respect, as will be discussed in 2.3, equation (6) has also been a source of misunderstanding. Moreover, along with (5), relation (6) may induce a somewhat unconscious but definitely wrong conclusion: that a relation similar to (6) holds for success. But as we will presently see this is false.

2.3 Some relevant differences

In order to achieve a deeper understanding of the meaning of the relations mentioned in 2.2, we examine the possibility of generalizing them within the setting given in 2.1. As we will see, these mathematical relations do not hold in general, but arise only due to the extreme symmetry of the particular probability distribution p^*. This provides additional arguments in support of a clear differentiation between the notions of success and decisiveness.

To this purpose it will be of use the only relation that, apart from the evident $\Phi_i(W,p) \leq \Omega_i(W,p)$, is really a genuine and general relationship between these two notions. This is Barry's (1980) equation: 'Success' = 'Decisiveness' + 'Luck,' which remains valid in a much more precise and general version. Namely, for any rule W *and any arbitrary* p, we have

$$\Omega_i(W,p) = \Phi_i(W,p) + \Lambda_i(W,p), \tag{7}$$

where $\Lambda_i(W,p)$ denotes Barry's "luck" (though perhaps a more suitable term should be "irrelevance", or strictly speaking "success without decisiveness"), given by

$$\Lambda_i(W,p) := Prob\ (i \text{ is successful and not decisive})\,.$$

If there existed a linear relation extending (5), success and decisiveness would just be two faces of a same coin. But in general,

$$2\Omega_i(W,p) \neq 1 + \Phi_i(W,p).$$

[12] Unlike $\Phi_i^{i+}(W,p^*)$ and $\Phi_i^{i-}(W,p^*)$, Coleman's "power to initiate action" and "power to prevent action", though based on the notion of decisiveness, differ. For this reason Coleman's indices have been sometimes used to distinguish what is indistinguishable from the interim a priori decisiveness point of view.

Moreover relation (5) is *exceptional* in the following sense: it *holds only when all vote configurations are equally probable*, as the following proposition shows.

Proposition 1 *Relation* $2\Omega_i(W,p) = 1+\Phi_i(W,p)$ *holds for every* W *if and only if* $p = p^*$.

Proof. *Sufficiency*[13]*:* For all W, all p and all i, we have from the generalization of Barry's equation (7),

$$\begin{aligned}
\Omega_i(W,p) &= \Phi_i(W,p) + \Lambda_i(W,p) \\
&= \Phi_i(W,p) + \sum_{\substack{S:i\in S\in W \\ S\setminus i\in W}} p(S) + \sum_{\substack{S:i\notin S\notin W \\ S\cup i\notin W}} p(S) \\
&= \Phi_i(W,p) + \sum_{\substack{T:i\notin T \\ T\in W}} p(T\cup i) + \sum_{\substack{T:i\in T \\ T\notin W}} p(T\setminus i)
\end{aligned}$$

$$= \Phi_i(W,p) + \sum_{T\in W} p(T\cup i)) - \sum_{\substack{T:i\in T \\ T\in W}} p(T\cup i) + \sum_{T\notin W} p(T\setminus i) - \sum_{\substack{T:i\notin T \\ T\notin W}} p(T\setminus i).$$

For the distribution $p = p^*$, as $p^*(T\cup i) = p^*(T) = p^*(T\setminus i) = 1/2^n$, the last equation can be re-written as:

$$\begin{aligned}
\Omega_i(W,p^*) &= \Phi_i(W,p^*) + \sum_{T\in W} 1/2^n - \sum_{\substack{T:i\in T \\ T\in W}} 1/2^n + \sum_{T\notin W} 1/2^n - \sum_{\substack{T:i\notin T \\ T\notin W}} 1/2^n \\
&= \Phi_i(W,p^*) + \alpha(W,p^*) - \Omega_i(W,p^*) + 1 - \alpha(W,p^*),
\end{aligned}$$

which yields (5).

Necessity: Assume $2\Omega_i(W,p) = 1 + \Phi_i(W,p)$ holds for every W. We will prove that for any T of any size t ($1 \le t \le n$) it holds $p(T) = p(T\setminus i)$. We will proceed by inverse induction on the size t. Let $W = \{N\}$. In this case

$$\Omega_i(W,p) = p(N) + \sum_{S:i\notin S} p(S) \quad \text{and} \quad \Phi_i(W,p) = p(N) + p(N\setminus i).$$

Thus in this case the assumed relation implies

$$2p(N) + 2\sum_{S:i\notin S} p(S) = 1 + p(N) + p(N\setminus i). \tag{8}$$

Now consider $W = \{N, N\setminus i\}$. In this case we have

$$\Omega_i(W,p) = p(N) + \sum_{S:i\notin S} p(S) - p(N\setminus i) \quad \text{and} \quad \Phi_i(W,p) = 0,$$

[13]The sufficiency of this condition is well known, but we prove it in order to see explicitly what role plays the assumption $p = p^*$.

and substituting in the assumed equality yields

$$2p(N) + 2 \sum_{S:i \notin S} p(S) = 1 + 2p(N \setminus i). \tag{9}$$

From (8) and (9) we obtain $p(N) = p(N \setminus i)$ and

$$\sum_{S:i \notin S} p(S) = \frac{1}{2}. \tag{10}$$

Thus the claim is proved for $t = n$. Now assume that $p(S) = p$ for any S of size $s \geq t$, for some t ($t \geq 1$). We show that for any T of size t it holds $p(T \setminus i) = p$. Fix any such T and consider $W = \{S : T \subseteq S\}$. For all $i \in T$, we have

$$\Omega_i(W,p) = \sum_{S:T \subseteq S} p(S) + \sum_{S:i \notin S} p(S) \quad \text{and} \quad \Phi_i(W,p) = \sum_{S:T \subseteq S} p(S) + \sum_{\substack{S:T \setminus i \subseteq S \\ i \notin S}} p(S).$$

Then the assumed relation along with (10) yields

$$\sum_{S:T \subseteq S} p(S) = \sum_{\substack{S:T \setminus i \subseteq S \\ i \notin S}} p(S).$$

Which, using the induction assumption ($p(S) = p$ for all S such that $s \geq t$), becomes

$$\left(\binom{n-t}{0} + ... + \binom{n-t}{n-t} \right) p = p(T \setminus i) + \left(\binom{n-t}{1} + ... + \binom{n-t}{n-t} \right) p.$$

Therefore $p = p(T \setminus i)$. Thus the claim is proved and consequently $p(S) = \frac{1}{2^n}$, for all $S \subseteq N$. ■

A similar result holds for the following special relationship between the interim variants of success and decisiveness[14]. But note that in this case, the relation is not linear affine any more, but the probability to pass a proposal enters the relation.

Proposition 2 *The following relations*

$$\Omega_i^{i+}(W,p) = \alpha(W,p) + \frac{1}{2}\Phi_i^{i+}(W,p)$$

and

$$\Omega_i^{i-}(W,p) = \bar{\alpha}(W,p) + \frac{1}{2}\Phi_i^{i-}(W,p),$$

hold for every W if and only if $p = p^*$.

[14]The same can be said about the relation between König-Brauninger index and the Coleman' power to prevent action alluded to in footnote (9).

Proof. *Sufficiency:* For all W, all p and all i, we have

$$\Omega_i^{i+}(W,p) = \frac{1}{\gamma_i(p)} \sum_{S:i\in S\in W} p(S) = \frac{1}{\gamma_i(p)} (\sum_{\substack{S:i\in S\in W \\ S\setminus i\notin W}} p(S) + \sum_{\substack{S:i\in S\in W \\ S\setminus i\in W}} p(S))$$

$$= \Phi_i^{i+}(W,p) + \frac{1}{\gamma_i(p)} \sum_{T:i\notin T\in W} p(T\cup i)$$

$$= \Phi_i^{i+}(W,p) + \frac{1}{\gamma_i(p)} (\sum_{T:T\in W} p(T\cup i) - \sum_{T:i\in T\in W} p(T\cup i)).$$

Now for $p = p^*$, as $p^*(S) = 1/2^n$ for all S, and $\gamma_i(p^*) = 1/2$ for all i

$$\Omega_i^{i+}(W,p^*) = \Phi_i^{i+}(W,p^*) + \frac{1}{1/2} (\sum_{T:T\in W} p^*(T) - \sum_{T:i\in T\in W} p^*(T)$$

$$= \Phi_i^{i+}(W,p^*) + 2\alpha(W,p^*) - \Omega_i^{i+}(W,p^*).$$

That is,

$$\Omega_i^{i+}(W,p^*) = \alpha(W,p^*) + \frac{1}{2}\Phi_i^{i+}(W,p^*).$$

The other equality is obtained similarly.

Necessity: Assume that for a given p both relations hold for any rule W. Then, taking into account that both $\Omega_i(W,p)$ and $\Phi_i(W,p)$, are the averages of their respective interim variants, by multiplying both equations by $1/2$ and adding them up, equation $2\Omega_i(W,p) = 1 + \Phi_i(W,p)$ is obtained. But, in view of Proposition 1, this implies $p = p^*$. ■

In sum: relation (5) between Rae's and Banzhaf's indices do not extend to their natural extensions for arbitrary priors different from p^*. In other terms, it has more to do with the very especial character of this probability distribution than with any general relationship between these two notions.

Now we turn our attention to relation (6), that provides three alternative interpretations of the Banzhaf index, either as the unconditional probability of being decisive or as the interim evaluations (conditional to a voter voting 'no' or voting 'yes', respectively) of this probability. A first natural question that arises is whether relation (6) can be generalized to any p. The answer is negative, as in general,

$$\Phi_i(W,p) \neq \Phi_i^{i+}(W,p) \neq \Phi_i^{i-}(W,p).$$

In fact, as proved in Laruelle and Valenciano (2004), relation (6) holds only if every voter votes 'yes' or 'no' with a certain probability *independently from the others*[15].

[15]Therefore, in particular the relation does not hold for the p for which $\Phi(W,p)$ becomes the Shapley-Shubik index. Or the other way round, the Shapley-Shubik index can be interpreted also as interim evaluation of the probability of being decisive (either as $\Phi_i^{i+}(W,p)$ or $\Phi_i^{i-}(W,p)$), but *for different probability distributions.*

In view of (5) the question of whether a similar relation to (6) holds for success, at least for p^* or some other special distribution arises. The answer again is negative. In general,

$$\Omega_i(W,p^*) \neq \Omega_i^{i+}(W,p^*) \neq \Omega_i^{i-}(W,p^*).$$

A voter's probability of getting a proposal accepted when the voter favors the proposal may differ from the probability of getting the proposal rejected when the voter votes against it, even under the assumption that all voting configurations are equiprobable.

Example: For the unanimity rule $W = \{N\}$, it holds

$$\Omega_i^{i+}(W,p^*) = \frac{1}{2^{n-1}}, \quad \text{while} \quad \Omega_i^{i-}(W,p^*) = 1.$$

This perfectly reflects the intuition that under the majority rule, the likelihood of getting a proposal accepted when a given voter favors the proposal will be quite small (the larger the number of seats, the smaller the probability), while the proposal will surely be rejected whenever a voter votes against it, no matter the number of seats (any voter has a veto right).

More generally, the three evaluations of success coincide *only* for the trivial case in which all voters vote always unanimously, as the following proposition shows.

Proposition 3 *The relation $\Omega_i(W,p) = \Omega_i^{i+}(W,p) = \Omega_i^{i-}(W,p)$ holds for any voting rule if and only if all voters vote always unanimously. That is, with a certain probability γ $(0 < \gamma < 1)$ all voters vote 'yes', and with probability $1-\gamma$ all vote 'no'.*

Proof. First note that the two conditional probabilities make sense only if the case where any voter i votes 'yes' (or 'no') with probability zero is excluded. Thus, we assume $0 < \gamma_i(p) < 1$, for all i. Assume that $\Omega_i(W,p) = \Omega_i^{i+}(W,p)$ holds for every W. Let $W = \{N\}$. In this case

$$\Omega_i(W,p) = p(N) + \sum_{S:i\notin S} p(S) \quad \text{and} \quad \Omega_i^{i+}(W,p) = \frac{p(N)}{\gamma_i(p)}.$$

This yields that necessarily $p(N) = \gamma_i(p)$ or $\gamma_i(p) = 1$ But as $\gamma_i(p) < 1$, it must be $p(N) = \gamma$, and $p(\emptyset) = 1-\gamma$, for some γ $(0 < \gamma < 1)$. On the other hand, it is straightforward that for this trivial voting behavior the equality of the three evaluations holds. ■

This is a relevant difference between success and decisiveness, as it is often the case that voters (or the analyst) are differently concerned with the prospect of getting the result they want depending on which is the sense of the decision: acceptance or rejection. Especially when rejection means keeping the *status quo* there may exist a bias in either sense, giving priority to one or another form of success. Thus the assessments based on either interim measures of success may differ not only at the quantitative level. They may rank differently voting rules, which may be relevant for the comparison of voting rules, as will be illustrated in the EU case study in section 3. This nuance is completely lost by Banzhaf's decisiveness index.

3 Case study: Three voting rules for the EU Council

A relevant case study that has been once and again approached from the power indices point of view is the European Council of Ministers. Most decisiveness indices have been applied (see, for instance, Hosli (1993), Widgrén (1994), or Brückner and Peters (1996)), but to the best of our knowledge the only application of an index of success is by König and Bräuninger (1998). Practitioners have often raised objections about the power indices approach. Part of this criticism is right (e.g., some applications of some power indices lack any clear sense), other times is based on a misinterpretation of the meaning of this approach, and concerns the assumption that all vote configurations have the same probability ($p = p^*$)[16], which is a natural assumption from a normative point of view when the goal is to assess the voting rule itself. But practitioners have also pointed out a more serious criticism that, it has to be admitted, has been usually ignored by scholars in spite of its serious motives: why to pay so much attention to decisiveness, where success seems a more important issue for the involved voters? As Moberg, in a comment on the Banzhaf index and the Intergovernmental Conference of Nice, puts it: "(...) it is very doubtful that this concept of power is relevant in EU politics. There is hardly any indication that Member States were actually seeking power in that sense in IGC 2000. Instead they were trying to make sure that they could safeguard their essential national interests, together with other like-minded countries, whether they had a pivotal position or not." (Moberg, 2002, p. 261). He also distinguishes between "blocking power" ("it means a country contribution to a blocking minority") and "partner in qualified majorities". Also, in a recent paper, Hosli and Machover (2004) recognize that too little attention has been paid to blocking power in its own right.

This line of thought seems to suggest that attention should shift from decisiveness to other issues involved in a voting situation. Namely, the probability of a proposal being accepted (α), the probability of a proposal being accepted given that voter i votes in its favor (Ω_i^{i+}), and the probability of a proposal being rejected given that voter i votes against it (Ω_i^{i-}). Member States that are worried for their sovereignty will surely be interested in minimizing the probability of being imposed a proposal that they reject, that is to say,

[16]For instance, Moberg (2002, p. 261): "the vast majority of the millions of theoretically conceivable coalitions are highly unlikely."

$1 - \Omega_i^{i-}$, or equivalently, to maximizing Ω_i^{i-}. More pro-integration members may value more the probability of a proposal being accepted α, and in particular having the proposal accepted once they vote in its favor, that is, maximizing Ω_i^{i+} may also matter.

Here we compare three rules that have been proposed for the enlarged Council of 25 Member States, which are, along with their populations[17] (pop_i, in thousands): Germany (82165), United Kingdom (59623), France (58747), Italy (57680), Spain (39442), Poland (38654), Netherlands (15864), Greece (10546), Czech Republic (10278), Belgium (10239), Hungary (10043), Portugal (9998), Sweden (8861), Austria(8092), Slovakia (5399), Denmark (5330), Finland (5171), Ireland (3775), Lithuania (3699), Latvia (2424), Slovenia (1988), Estonia (1439), Cyprus (755), Luxemburg (436), and Malta (380).

The first rule that we consider is the Nice rule (W^{Ni}), a rule based on the re-weighting proposed in the Intergovernmental Conference that concluded in Nice, December 2000. The second rule (the Convention rule (W^{Co})) was suggested by the Convention set up after the summit of Laeken in 2001. The third rule (the "Spanish rule" (W^{Sp})) was proposed as an alternative to the Convention by the Spanish government in 2003 with the support of Poland.

All the three voting rules require the support of two majorities in order to pass a decision. In the three cases the first majority is a simple majority of Member States. The second is a weighted majority. In the Nice rule, the weights are not proportional to the populations as it is the case with the Convention rule and the alternative rule proposed by the Spanish government. The difference between the two latter rules is the level of the quota. The Convention rule states the quota at 60% of the total population while the Spanish government proposed to increase the quota to 66% of the total population. Formally, the three rules are respectively:

1. The Nice rule:[18]

$$W^{Ni} = \left\{ S \subseteq N : \sum_{i \in S} w_i \geq 232 \text{ and } s \geq 13 \right\},$$

where the vector of weight is:

$$w = (29, 29, 29, 29, 27, 27, 13, 12, 12, 12, 12, 12, 10, 10, 7, 7, 7, 7, 7, 4, 4, 4, 4, 4, 3).$$

[17]Source EUROSTAT (2000), quoted from Galloway (2001).

[18]Here we ignore the "population safety net" that stipulates: "When a decision is to be adopted by the Council by a qualified majority, a member of the Council may request verification that the Member States constituting the qualified majority represent at least 62% of the total population of the Union. If that condition is shown not to have been met, the decision in question shall not be adopted." In fact the results that we obtain with or without this clause differ very little.

2. The Convention rule:

$$W^{Co} = \left\{ S \subseteq N : \sum_{i \in S} pop_i \geq 0.60 \sum_{i \in N} pop_i \text{ and } s \geq 13 \right\}.$$

3. The Spanish rule:

$$W^{Sp} = \left\{ S \subseteq N : \sum_{i \in S} pop_i \geq 0.66 \sum_{i \in N} pop_i \text{ and } s \geq 13 \right\}.$$

The comparison of the three rules yields the following conclusions. First, note that $W^{Sp} \subseteq W^{Co}$: if a voting configuration is winning under the Spanish rule, it is also be winning under the Convention. As a consequence, whatever the prior p for the assessment, it will always be more complicated to pass a proposal under the Spanish rule that under the Convention. That is, for any p, we always have

$$\alpha(W^{Co}, p) \geq \alpha(W^{Sp}, p).$$

Similarly, whatever the p, it is easy to see that any voter's interim probability of being successful in the passage of a proposal is larger with the Convention rule, while the interim probability of being successful in the rejection of a proposal is larger under the Spanish alternative rule. That is, for any state i:

$$\begin{aligned} \Omega_i^{i+}(W^{Co}, p) &\geq \Omega_i^{i+}(W^{Sp}, p), \\ \Omega_i^{i-}(W^{Co}, p) &\leq \Omega_i^{i-}(W^{Sp}, p). \end{aligned}$$

Unlike success, decisiveness' behavior is rather erratic: no inequality concerning the decisiveness can be derived in general from an inclusion between rules.

Although there are more winning vote configurations in W^{Co} than in W^{Ni}, it can be checked that no inclusion exists between W^{Ni} on the one hand, and W^{Co} or W^{Sp}, on the other. As a consequence no general conclusions about success can be derived for arbitrary the distributions of probability. For normative purposes, deliberately ignoring any information beyond the rule itself, the natural choice of p to compare the three rules is p^*. This particular distribution of probability will allow us to make quantitative comparisons.

First, let us consider the ease (respectively, difficulty) of accepting proposal. Computing the probability of a proposal being accepted (respectively, rejected). That is, computing $\alpha(W, p^*)$ (respectively, $\bar{\alpha}(W, p^*) := 1 - \alpha(W, p^*)$) for the three rules, we obtain:

$$\begin{aligned} \alpha(W^{Co}, p^*) = 0.225, \quad \bar{\alpha}(W^{Co}, p^*) = 0.775, \\ \alpha(W^{Sp}, p^*) = 0.148, \quad \bar{\alpha}(W^{Sp}, p^*) = 0.852, \\ \alpha(W^{Ni}, p^*) = 0.036, \quad \bar{\alpha}(W^{Ni}, p^*) = 0.964. \end{aligned}$$

It means that with the rule proposed in Nice it would be *a priori* extremely difficult to pass proposals. The rule proposed by the Convention substantially increases the probability of passing a proposal (or reduces the probability of a proposal being rejected). The probability of a proposal being rejected under the Spanish rule is obviously larger than under the Convention rule (recall this was so *for any* p) but smaller than under the Nice rule.

The probabilities of success for every country, unconditional and interim in both senses, are given in Table 1. The following comments can be made on this table. Starting with the Nice rule, a first comment concerns the probability of a proposal being rejected given that a certain country votes against it (Ω_i^{i-}): this probability is strikingly high for France, Germany, Italy, Poland, Spain and UK (more than 0.99). It means that the rule gives a great veto power to these big states. Even small states have a very high probability of having a proposal rejected if they vote against it (nearly 0.97 for the smallest state, Malta). The rule proposed by the Convention reduces this probability for all states (the range of probabilities is between 0.93 -for Germany- and 0.79 -for Malta). The probabilities that are obtained with the Spanish rule are intermediate between the two above mentioned rules. We have that for any state i:

$$\Omega_i^{i-}(W^{Co}, p^*) < \Omega_i^{i-}(W^{Sp}, p^*) < \Omega_i^{i-}(W^{Ni}, p^*).$$

The counterpart is that with the Nice rule, the probability of a proposal being accepted given that a state votes for it (Ω_i^{i+}) is very small for all states (for the largest state, Germany, this probability is 0.06). The probability of having a proposal accepted given that a state votes for it increases substantially (for instance, for Germany this probability passes from 0.06 to 0.26). The probabilities that are obtained with the Spanish proposal are again intermediate between the two above mentioned rules. We have for any state i:

$$\Omega_i^{i+}(W^{Co}, p^*) > \Omega_i^{i+}(W^{Sp}, p^*) > \Omega_i^{i+}(W^{Ni}, p^*).$$

In absolute terms, these probabilities are closer to the ones obtained with the rule proposed by the Convention than with the Nice rule.

The probability of getting the outcome (acceptance or rejection) one votes for with the Nice rule is around 0.50 (between 0.55 for Germany and 0.50 for Malta). This probability is larger for the Spanish rule than with the Nice rule, and the largest under the Convention rule. That is, for any state i:

$$\Omega_i(W^{Co}, p^*) > \Omega_i(W^{Sp}, p^*) > \Omega_i(W^{Ni}, p^*).$$

In sum, the results are the following. Among the three rules, the Convention rule is the one that confers all the states the largest probability of getting the outcome one votes for.

The Convention rule is also the rule that yields the largest probability of a proposal being accepted. This is however done at the expense of being more often imposed proposals one does not favor. In other terms, if the criterion that prevails is "keeping national sovereignty" and not being imposed a proposal one does not want, then the best rule of the three is the Nice rule. In any case, contrary to what is sometimes claimed, the choice of the rule is not a zero-sum game between large and small states. It is more a problem of the choice of the criterion. Those who are more in favor of further integration will surely prefer the Convention rule (for its larger probability of passing proposals in the different senses considered), while those who are mainly worried about keeping the national sovereignty will prefer the Nice rule.

Finally, let us briefly compare these results with the ones obtained from the point of view of a priori decisiveness, as evaluated by the Banzhaf index[19]. First, as implied by (5), the ranking between the rules is the same as the one obtained for the probability of success. So for any state i we have:

$$\Phi_i(W^{Co}, p^*) > \Phi_i(W^{Sp}, p^*) > \Phi_i(W^{Ni}, p^*).$$

That is, the probability of being decisive would be the largest under the Convention rule, and the smallest under the Nice rule. But note that for any voting rule, as pointed out in section 2, we have equality (6) for any state. In other words the assessment based on decisiveness is insensitive to the differences between the interim evaluations which are conspicuous from the point of view of success and no doubt matter all State Members.

4 Conclusion

A first conclusion is the clear conceptual and analytical distinction between the notions of success and that of decisiveness. It has been shown that, in addition to the obvious difference of meaning, these two concepts are independent in the sense that neither of them can be derived from the other in general, and their "interim" conditional variations behave differently. While the unconditional and interims variants coincide for decisiveness under certain conditions on the prior p (met in particular for the usual normative probability distribution p^*), this is not so for the corresponding evaluations of success. The

[19]It may be worth recalling that the normalization of the Banzhaf index (still common in the literature) completely distorts the conclusions. By normalizing the Banzhaf index by dividing each component by the sum of the components, the probabilistic interpretation is lost. In addition to that it makes the comparison of the rules meaningless. It may be that a state's probability of being decisive increases while its percentage of the sum of probabilities decreases. The normalization forces a comparison in terms of a zero-sum game, in the sense that comparing percentages it is impossible for all percentages to increase or to decrease.

relationship between Banzhaf's and Rae's measures established by Dubey and Shapley, whose very especial character has been shown, may partly explain but never justify the overlooking of success in the literature. Even if one only cares about the normative point of view provided by p^*, for with an affine relation holds, it remains the question of which is the most relevant notion.

Perhaps the fascination raised by the notion of "power" has caused a distortion of focus in the field. It can be argued that decisiveness seems intuitively closer to the notion of "power" than that of success, but this does not grant greater credit to recommendations based on this interpretation. In other words, the relevant question is not what notion is closer to the intuitive idea of "power", but what is a more adequate basis for normative recommendations. And as a base for normative recommendations (e.g., in connection with important issues, as that of the most adequate voting rule in a committee of representatives) it seems more relevant the notion of success than that of decisiveness. If this is taken seriously it seems necessary a revision of the recommendations that have been made so far based on the notion of power as likelihood of being decisive. In this sense the application to the EU Council is a first step in this direction.

References

[1] Banzhaf, J. F., 1965, Weighted Voting doesn't Work: A Mathematical Analysis, *Rutgers Law Review* **19**, 317-343.

[2] Barberà, S. and M. O. Jackson (2004): On the Weight of Nations: Assigning Voting Weights in a Heterogeneous Union, http://www.hss.caltech.edu/~jacksonm/.

[3] Barry, B., 1980, Is it Better to Be Powerful or Lucky?, Part I and Part II, *Political Studies* **28**, 183-194, 338-352.

[4] Benoît, J-P., and L. A. Kornhauser, 2002, Game-Theoretic Analysis of Legal Rules and Institutions, in *Handbook of Game Theory with Economic Applications,* Vol. 3, ed. by R. J. Aumann and S. Hart. Amsterdam: Elsevier Science Publishers - North Holland, 2229-2269.

[5] Brams, S. J., and M. Lake, 1978, Power and Satisfaction in a Representative Democracy, in *Game Theory and Political Science,* ed. by P. C. Ordeshook, 529-562.

[6] Bruckner M., and T. Peters, 1996, Further Evidence on EU Voting Power, *Journal of Theoretical Politics* **8(3)**, 415-419.

[7] Coleman, J. S., 1971, Control of Collectivities and the Power of a Collectivity to Act, in *Social Choice*, edited by B. Lieberman, Gordon and Breach, London.

[8] Coleman, J. S., 1986, *Individual Interests and Collective Action: Selected Essays*, Cambridge University Press.

[9] Dubey, P., and L. S. Shapley, 1979, Mathematical Properties of the Banzhaf Power Index, *Mathematics of Operations Research* **4**, 99-131.

[10] Felsenthal, D. S., and M. Machover, 1998, *The Measurement of Voting Power: Theory and Practice, Problems and Paradoxes*, Edward Elgar Publishers, London.

[11] Felsenthal, D. S., and M. Machover, 2001, The Treaty of Nice and qualified majority voting, *Social Choice and Welfare* **18**, 431-464.

[12] Galloway, D., 2001, The Treaty of Nice and Beyond : Realities and Illusions of Power in the EU, Sheffield Academic Press.

[13] Hosli, M. O., 1993, Admission of the European Free Trade Association States to the European Community: Effects on Voting Power in the European Community Council of Ministers. *International Organization* **47**, 629-643.

[14] Hosli, M. O., and M. Machover, 2004, The Nice Treaty and Voting Rules in the Council: A Reply to Moberg (2002), *Journal of Common Market Studies* **42** (forthcoming).

[15] König, T., and T. Bräuninger, 1998, The inclusiveness of European Decision Rules, *Journal of Theoretical Politics* **10(1)**, 125-142.

[16] Lane, J. E., and R. Maeland, 2000, Constitutional analysis: The Power index approach, *European Journal of Political Research* **37**, 31-56.

[17] Lane, J. E., and R. Maeland, 2002, A note on Nice, *Journal of Theoretical Politics* **14(1)**, 123-128.

[18] Laruelle, A., and F. Valenciano, 2004, Assessing success and decisiveness in voting situations. *Social Choice and Welfare* (forthcoming).

[19] Laruelle, A., and M. Widgrén, 1998, Is the allocation of voting power among the EU states fair? *Public Choice* **94**, 317-339.

[20] Leech, 2002, Designing the Voting System for the EU Council of Ministers, *Public Choice* **113**, 437-464.

[21] Moberg, A., 2002, The Nice Treaty and Voting Rules in the Council, *Journal of Common Market Studies* **40**, 259-282.

[22] Penrose, L. S., 1946, The Elementary Statistics of Majority Voting, *Journal of the Royal Statistical Society* **109**, 53-57.

[23] Rae, D., 1969, Decision Rules and Individual Values in Constitutional Choice, *American Political Science Review* **63**, 40-56.

[24] Shapley, L. S., 1953, A Value for N-person Games, *Annals of Mathematical Studies* **28**, 307-317.

[25] Shapley, L. S., and M. Shubik, 1954, A Method for Evaluating the Distribution of Power in a Committee System, *American Political Science Review* **48**, 787-792.

[26] Straffin, P. D., 1982, Power Indices in Politics. In: *Political and Related Models*, Brams, S. J., W. F. Lucas, and P. D. Straffin, eds, New York, Springer-Verlag, 256-321.

[27] Straffin, P. D., M.D. Davis, and S. J. Brams, 1981, Power and Satisfaction in an Ideologically Divided Voting Body, in *Power, Voting, and Voting Power,* ed. by M. Holler, Würzburg-Wien: Physica-Verlag, 239-253.

[28] Widgrén, M., 1994, Voting Power in the EC and the Consequences of two Different Enlargements. *European Economic Review* **38**, 1153-1170.

	$\Omega_i^{i+}(W, p^*)$			$\Omega_i^{i-}(W, p^*)$			$\Omega_i(W, p^*)$		
Country	W^{Ni}	W^{Sp}	W^{Co}	W^{Ni}	W^{Sp}	W^{Co}	W^{Ni}	W^{Sp}	W^{Co}
Austria	0.046	0.168	0.255	0.975	0.872	0.806	0.510	0.520	0.530
Belgium	0.048	0.171	0.258	0.977	0.876	0.808	0.513	0.523	0.533
Cyprus	0.040	0.157	0.244	0.968	0.862	0.795	0.504	0.510	0.519
Czech	0.048	0.171	0.258	0.977	0.876	0.809	0.513	0.524	0.533
Denmark	0.043	0.164	0.251	0.972	0.868	0.801	0.507	0.516	0.526
Estonia	0.040	0.158	0.245	0.968	0.863	0.796	0.504	0.511	0.520
Finland	0.043	0.164	0.251	0.972	0.868	0.801	0.507	0.516	0.526
France	0.063	0.237	0.334	0.992	0.941	0.884	0.528	0.589	0.609
Germany	0.063	0.269	0.379	0.992	0.973	0.930	0.528	0.621	0.654
Greece	0.048	0.172	0.258	0.977	0.876	0.809	0.513	0.524	0.534
Hungary	0.048	0.171	0.258	0.977	0.875	0.808	0.513	0.523	0.533
Ireland	0.043	0.162	0.249	0.972	0.866	0.799	0.507	0.514	0.524
Italy	0.063	0.235	0.332	0.992	0.939	0.883	0.528	0.587	0.607
Latvia	0.040	0.160	0.246	0.968	0.864	0.797	0.504	0.512	0.522
Lithuania	0.043	0.161	0.248	0.972	0.866	0.799	0.507	0.514	0.524
Luxemburg	0.040	0.157	0.243	0.968	0.861	0.794	0.504	0.509	0.519
Malta	0.039	0.157	0.243	0.967	0.861	0.794	0.503	0.509	0.518
Netherlands	0.050	0.180	0.266	0.978	0.884	0.817	0.514	0.532	0.542
Poland	0.062	0.205	0.303	0.990	0.910	0.854	0.526	0.558	0.578
Portugal	0.048	0.171	0.258	0.977	0.875	0.808	0.513	0.523	0.533
Slovakia	0.043	0.164	0.251	0.972	0.869	0.802	0.507	0.516	0.526
Slovenia	0.040	0.159	0.246	0.968	0.864	0.796	0.504	0.511	0.521
Spain	0.062	0.207	0.304	0.990	0.914	0.854	0.526	0.560	0.579
Sweden	0.046	0.169	0.256	0.975	0.874	0.807	0.510	0.521	0.531
U.K.	0.063	0.238	0.335	0.992	0.942	0.886	0.528	0.590	0.610

Table 1: Probabilities of success in the EU-25 countries under the three rules.