Computation of the Shapley-Owen Power Index in Two Dimensions

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Abstract

The Shapley-Owen power index is reviewed and an algorithm for its computation in two-dimensional spatial voting models is presented. The Shapley-Owen values for ideal point distributions representing the UN Security Council, Bretton Woods institutions, US legislators, as well as Monte Carlo data are considered

1 Introduction

The Shapley-Owen value (SOV) arises in spatial voting models and the solution concepts of cooperative games [Shapley-Owen, 1989]. Within the context of proximity spatial voting models, SOV measures how likely a voter is to determine the location of an adopted proposal, i.e., that the voter be pivotal. As a solution concept for cooperative games, SOV attempts to address perceived shortcomings of the Shapley-Shubik index.

In so far as SOV may be less familiar than other "power indexes" such as those of Shapley-Shubik, Banzhaf-Coleman, Deegan-Packal, Johnston, and Penrose, we review the highlights of SOV. We then present an algorithm for computing in SOV based on a model proposed by Shapley. The algorithm includes cases not encompassed by analytic methods, such as weighted voting, coincident ideal points, and arbitrary dimension.

We apply this algorithm to a number of examples. The examples are designed to illustrate the canonical properties of SOV as well as the impact of its generalization to proximity voting models more general than those through which SOV was introduced by Shapley and Owen.

2 SOV in Cooperative Games

Consider a finite set of n voters, N. Introduce a strict order relation on N, $<<$. Define a set $Q(i,<<)$ to consist of all voters j, such that $j << i$. Finally, let W be a set of subsets of N that we suggestively call *winning coalitions*. A voter, i, is called a *pivot* if and only if

$$Q(i,<<) \notin W \text{ and } Q(i,<<) \cup \{i\} \in W,$$

i.e., the pivot splits the set N into two disjoint sets, one of which is winning, namely

$Q(i,<<) \cup \{i\} \in W$.

Shapley and Owen, in motivating consideration of SOV note that the Shapley value for voter i, $v_i(N,W)$, can be written

$$v_i = \frac{q_i}{n!} \quad (1)$$

where q_i is the number of orderings for which voter i is the pivot. Since n! is the total number of all possible orderings, i.e., the size of the sample space, v_i has a natural interpretation as the probability that voter i will be the pivot in a random draw from a uniform distribution of orderings.

Shapley and Owen note that it seems unlikely all possible coalitions of equal size have an equal probability of forming in actual political situations, as required for the probability interpretation of (1). Accordingly, each has suggested formal modifications of (1), the upshot being to modify (1) to reflect a more realistic sample space.

3 SOV in Proximity Spatial Voting Models

A particular modification of (1) proposed by Shapley involves a spatial voting model [Shapley, 1977]. Shapley's model consists of a finite set of n voters, N, a set of subsets of N, called winning coalitions, W, and a set of n points P_i, $i \in N$, in an m-dimensional affine (or projective) space, $\Re^m$, representing the ideal points of the voters. These points, called *ideal points*, represent the preferred policy outcomes of each voter. The space is assumed to be measurable (Lebesgue) with a Euclidean metric, d(x,y) and inner product, <x,y>. In fact, $d(x,y) = |<x,y>|^{1/2}$.

Shapley considers unit vectors $U \in \Re^m$. These vectors lie on the unit sphere H_{m-1}, each vector defining a direction in the space. Furthermore, except for a set of measure 0, each unit vector, U, induces a order relation << as

$$i <<_U j \iff \langle U, P_i \rangle \leq \langle U, P_j \rangle$$

Those U that do not induce an order Shapley notes form an m-2 dimensional subspace and so have measure 0, i.e., can be neglected when computing probabilities.

Let U be randomly chosen from a uniform distribution, i.e., subsets of H_{m-1} with equal measure have equal probability. Assuming the points P_i are distinct, the pivot for the order induced by U will be unique almost surely. Let ϕ_i denote the probability that i is the pivot under the ordering induced by U. Note that the sum over i of ϕ_i is unity. Then ϕ_i becomes the modified version of (1) for the spatial voting model.

Shapley and Owen use this modified version of (1) to introduce the notion of a *center of power* in a spatial voting game defined by

$$P^* = \sum_{i \in N} \phi_i P_i \,. \qquad (2)$$

The center of power must lie within the convex hull of the points P_i as each $\phi_i \leq 1$.

We will exploit Shapley's model to develop our algorithm for computing SOV. But before proceeding to discuss our algorithm, we further develop the SOV for two-dimensional spatial voting models.

4.1 SOV in Two-Dimensional Proximity Spatial Voting Models

In two-dimensional spaces, m = 2, it is possible to give an explicit prescription for computing ϕ_i. This prescription relies on using (2). The basic idea is to show that the center of power is unique and has the property of minimizing the area of the *win set*. The SOV arises as an essential byproduct of determining the explicit formula for the center of power.

Consider the simplest example of a proximity spatial voting model consisting of three voters located at points P_1, P_2, and P_3 in a two-dimensional issue space, as well as a distinguished point called the status quo. The basic assumption of proximity models is that each voter prefers a proposal "closer" to his/her ideal point than the status quo. Assuming each voter has Euclidean preferences, i.e., "closer" is measured using a Euclidean metric, then indifference curves can be drawn as circles centered on each voter's ideal point passing through the status quo, X. John Nash introduced the status quo, as a formal concept, in bargaining games calling it the *disagreement point*, the outcome under no agreement. In voting games it corresponds to the outcome when a proposal fails to be adopted. The location of the status quo is generally regarded as *independent* of voter ideal points. Along with the dimensions of the space, it serves to "frame" the decision. See [Miller-Grofman-Feld, 1989].

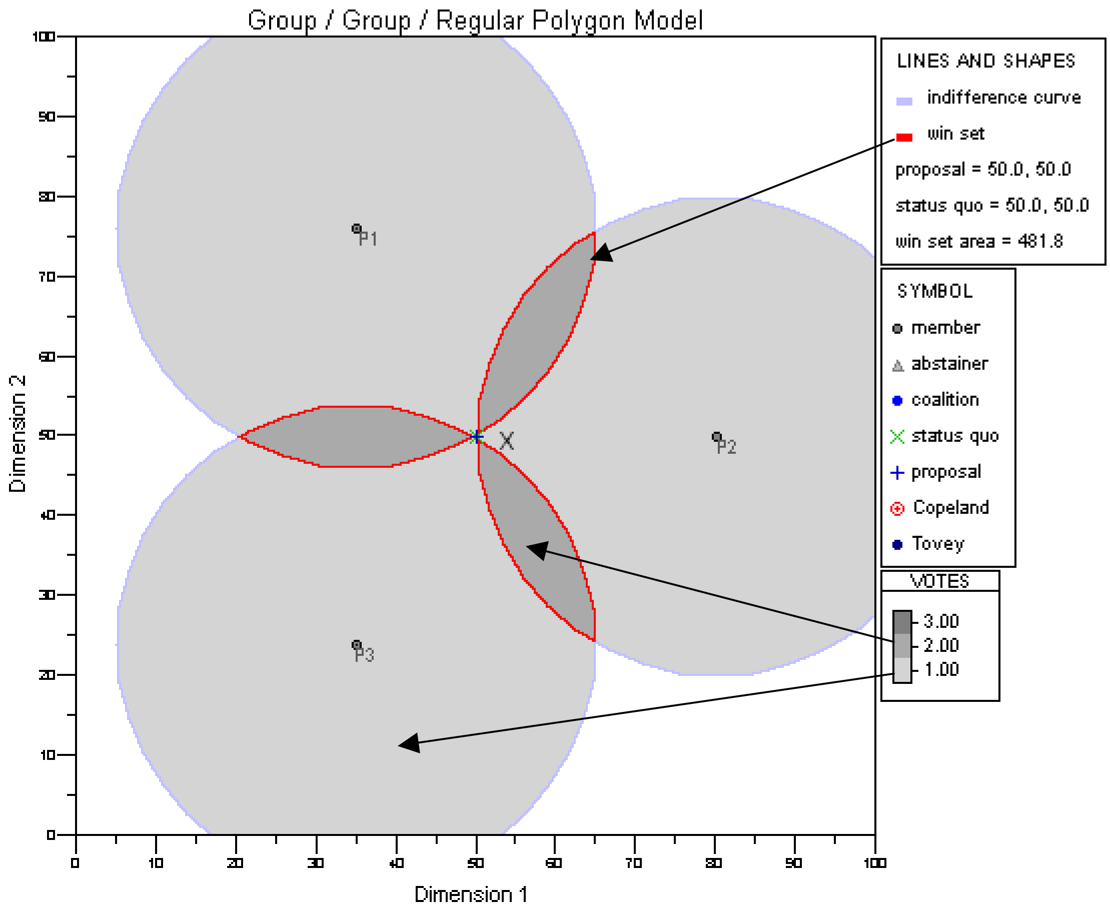


Figure 1 – Elements of Proximity Spatial Models

Under simple majority rule, those areas resulting from the intersection of a majority of areas enclosed by indifference curves, often called *petals*, represent locations where a proposal can be offered that will be majority preferred to the status quo. The union of these areas is generally called the *win set*. In their paper, Shapley and Owen refer to this area with the notation $A(X)$ and call it the *vulnerability*.

Shapley and Owen consider whether there exists a point S that minimizes $A(X)$ and whether this point is unique. Shapley and Owen answer in the affirmative, calling S the strong point. The strong point is also known as the Copeland winner as this point by definition defeats the greatest number of proposals in pairwise votes.

In order to minimize $A(X)$ it is necessary to obtain an analytic expression for $A(X)$. Shapley and Owen accomplish this by developing an expression for computing and aggregating petals. The results is:

$$A(X) = 2\int_0^{\pi} \langle U_\theta, P(\theta) - X \rangle^2 d\theta \qquad (3)$$

where $P(\theta) = P_i$, the ideal point of voter i for those θ for which voter i is the pivot and U_θ is a unit vector in the θ direction. Some intuition for this formula can be gained by considering it from the perspective of polar coordinates, in which case $\sqrt{\langle P(\theta) - X, P(\theta) - X \rangle}$ represents the radial distance between $P(\theta)$ and X, whence (3) is just an integral for area in polar coordinates. The inner product with the unit directional vector handles the angular variation of the petals.

A(X), as expressed in (3), is not readily minimized directly. Instead, Shapley and Owen introduce an auxiliary quantity,

$$B(X) = \frac{1}{\pi}\int_0^\pi \langle P(\theta) - X, P(\theta) - X \rangle \, d\theta$$

and show that $\pi B(X) - A(X)$ is independent of X. Hence, it is sufficient to minimize B(X), which is easier to do.

The minimization of B(X) is left to the reader by Shapley and Owen; we provide the demonstration here. If we vary B(X) with respect to X we find

$$\begin{aligned} \frac{\partial B(X)}{\partial x_\alpha} &= \frac{1}{\pi}\int_0^\pi \frac{\partial}{\partial x_\alpha}\left(\langle P(\theta), P(\theta)\rangle - 2\langle P(\theta), X\rangle + \langle X, X\rangle\right) d\theta \\ &= \frac{1}{\pi}\int_0^\pi \left(-2P_\alpha(\theta) + 2X_\alpha\right) d\theta \\ &= \frac{-1}{\pi}\int_0^\pi P_\alpha(\theta)\,d\theta + X_\alpha \end{aligned}$$

where α indicates differentiation with respect to the x and y coordinates individually. Since we are seeking a minimum, we set $\frac{\partial B(X)}{\partial x_\alpha}$ equal to zero for each coordinate.

$$X_\alpha = \frac{1}{\pi}\int_0^\pi P_\alpha(\theta)\,d\theta$$

As we already noted the $P(\theta)$ are constant except for a finite number of transitions as the pivotal voter changes for specific values of θ. Thus, for each P_i we have

$$\phi_i = \frac{1}{\pi}\int_{C_i} d\theta \tag{4}$$

where C_i denotes the concentration of the angle measure for which $P(\theta) = P_i$. Thus

$$X = \sum_{i \in N} \varphi_i P_i \tag{5}$$

where

$$\sum_{i \in N} \varphi_i = 1 . \quad (6)$$

And so we see that the SOV plays an essential role in determining the strong point.

Observe that the minimization of $B(X)$ leading to (4) depends only on there being an inner product, not the specific form of the metric. The Euclidean form of the metric enters the picture in deriving (5), i.e., establishing that $\pi B(X) - A(X)$ is independent of X. The degree to which (5) is affected by departures from a Euclidean metric assumption is unknown.

There are at least two cases to consider. In the first, one generalizes from Euclidean metric to a Riemannian metric. In the second case, more realistic, one admits a different metric for each voter. Whereas the former may be accessible analytically, the later seems ideally suited for investigation through computer simulation.

On the analytic side we want to make one observation. Suppose (5) depends on the choice of metric and that we are free to vary the metric for each voter separately. Let's suppose all voters except voter i have Euclidean preferences. Then, by virtue of (6), there is no metric i can choose that will yield for i a different value without either losing or gaining value relative to all other voters. This suggests that there is some optimal choice of metric for voter i. Extending this decision to all voters, we have a non-cooperative game in which each voter seeks a metric choice strategy to optimize his/her value given that all other voters are pursuing the same goal.

On the simulation side, note that even in the case of a Euclidean metric the computation of C_i for arbitrary distributions of voter ideal points is hard (and tedious) to do in closed analytic form. There is, however, a simple and intuitive algorithm that does the job.

4.2 An Algorithm for Computing SOV

Our algorithm is a direct translation of Shapley's model discussed earlier. The only material difference concerns the implementation. In place of the direction unit vectors, we fix an origin and rotate a line about the origin. Voter ideal points are projected on to the line for each increment of rotation. The *pivot* is determined as the voter occupying the median position using the natural linear order of the line to order the projected points. The following table summarizes the correspondences.

Shapley Model	Algorithm
Direction angles: θ_i; $i = 1, 2, \ldots, n-1$	Rotation angles: θ_i $i = 1, 2, \ldots, n-1$
Directional unit vector $U(\theta_i)$	Line vector, $L(\theta_i)$
$\langle U, P_i \rangle$	$\langle L, P_i \rangle$
$i <<_U j \Leftrightarrow \langle U, P_i \rangle \leq \langle U, P_j \rangle$	$i <<_L j \Leftrightarrow P_i \leq_L P_j$

Although we could leave our description of the algorithm at this abstract level, it is helpful to consider how one might arrive at it by way of a series of approximations, starting with Median Voter Theorem.

Consider a finite set of n voters, N, in a one-dimensional proximity spatial voting model, i.e., single-peaked preferences, under simple majority rule. Let P_i denote the ideal point for voter i in the issue space, then the linear ordering of points induces a strict order, <<, given by

$$i << j \;\Leftrightarrow\; P_i \leq P_j.$$

In the case of an odd number of voters, the pivot is the member k occupying the median position in the order. This position, P_k, determines the policy outcome.

The pivot both determines the outcome of the vote and gains the full value of the outcome, as the proposal resides at the pivot's ideal point. Considering the particular vote as a game, the full value of the game is allocated to the pivot. We can generalize this observation by borrowing a familiar concept from probability theory. According to Shapley's model, ϕ_i is the probability the i^{th} voter is the pivot. If the total value of the game is 1, then ϕ_i is just the expected payoff voter i receives. A subtle point, however, is that except in one dimension, the outcome may never reside on the ideal point of any voter. So the interpretation is purely formal. Keeping in mind that the notion of value in spatial voting models is purely formal, it nevertheless provides a convenient shorthand for describing the influence of voters on the expected outcome (strong point).

Suppose now we introduce a second dimension to the issue space. As is well known, there is generally no Condorcet winner in such cases, i.e., no voter is pivotal. On the other hand, while it may seem intuitive that even in two dimensions not all voters are equally important, i.e., centrally located voters generally can form more winning coalitions, how are we to quantify this?

If we divide the question in the two-dimensional case there is, by the Median Voter Theorem, a pivot for each dimension. If in one dimension we assign 1 to the pivot, then in two-dimensional games it seems reasonable to assign a ½ to each pivot. If it happens that there is a Condorcet winner, this voter gets the full value of the game. Otherwise, the two distinct pivots split the value evenly. Everyone else gets zero.

Now imagine rotating the space of voters and again dividing the question along some new, albeit, mixed-issue dimensions. In general new voters will be pivotal. In fact, suppose we consider m such rotations. The rotations can be random but it will be easier to understand the limit if we imagine a sequence of rotations by an incremental angle, θ, such that

$$m\,\theta = 2\,\pi$$

i.e., one, complete revolution. Since we are dividing the question we have two pivots for each such increment. But we have a total of m increments. After m incremental rotations, $2\,m$ values will be assigned. So we must divide by $2\,m$ to have a net value of 1. Thus, the value assigned to each voter is the number times the voter is pivotal divided by $2\,m$.

We divided the question to motivate the method of assigning value. And we rotated by 2π to provide a complete revolution. Neither of these, however, is strictly necessary. We may consider pivots with respect to a single axis. Then, once the axis has rotated by π we note that the axis has essentially returned to its initial condition, with voters in reverse order. Whence, it is sufficient to consider rotating a single line through half a revolution, leading to the formula

$$v_i(m) = \frac{q_i}{m}$$

where q_i is the number of times i is the pivot and m is the number of increments used to rotate the line through π radians. Note that q_i depends on m. In the limit that the angular increments become infinitesimal, i.e., $m \to \infty$, we have

$$\phi_i = \lim_{m\to\infty} v_i(m),$$

i.e., $v(m)$ approaches the Shapley-Owen value.

Figure 2 illustrates how the algorithm works for the simple example of three ideal points positioned on a regular triangle. The angle measure concentration for each voter's ideal point is indicated. The dashed lines represent transitions from one voter to another being pivotal. The solid line represents one line increment, showing the projection of each voter on to the line. Observe that voter P_1 is the pivot, i.e., median voter, and will remain so for all lines within the wedge defined by the angle $\pi\phi_1$. Note also that the opposite angle for each concentration is of the same size. The second half of the revolution is identical except that the projections on the line are in reverse order. The pivots, however, remain invariant. Hence, it is sufficient to consider revolution of the line by π, i.e., a half revolution, when computing SOVs, as we noted earlier.

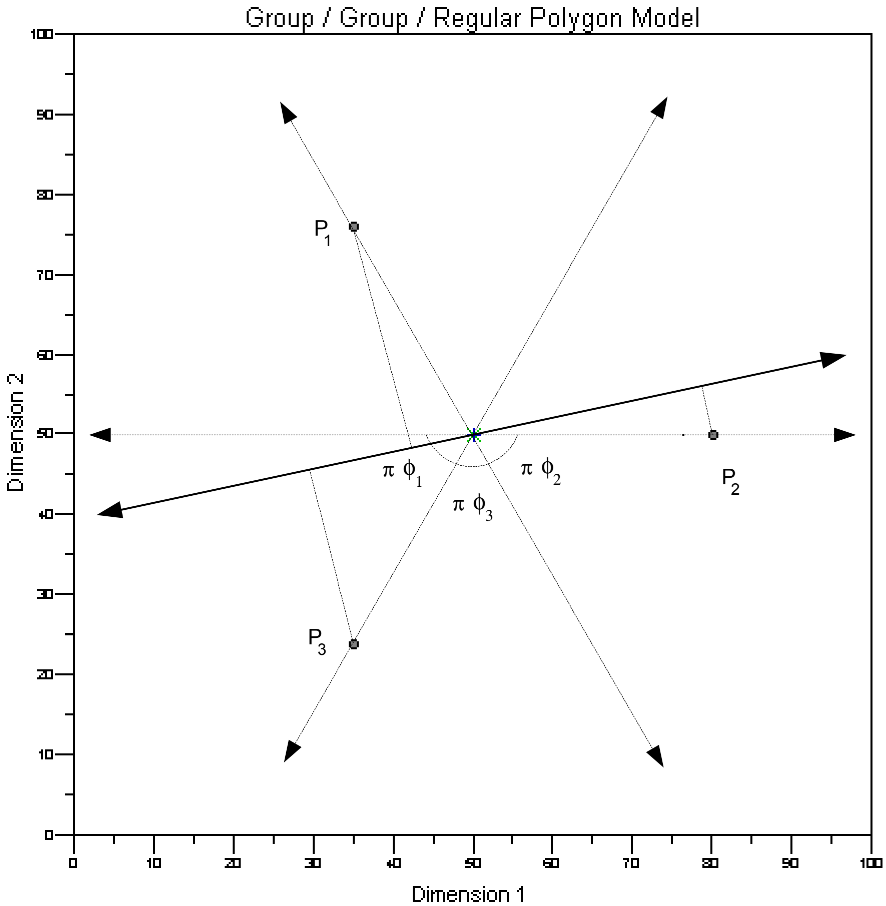


Figure 2 – SOV Computation Based on Rotating Line

The correspondence of our algorithm with Shapley's model is realized when we replace the rotating line with a directional unit vector. A rotating line, however, is computationally more convenient. By translation invariance, the rotating line can be located anywhere in the plane. Indeed, after each increment of rotation the line can be translated anywhere in the plane without affecting the order of voters projected on the line. It is really only the direction of the line that matters, i.e., the directional unit vectors.

The following diagram illustrates the projection of voters on to a line in a given direction for three distinct spatial translations of the line. Observe that the order of voters projected on the line is identical in all three cases. This is what is meant by translation invariance.

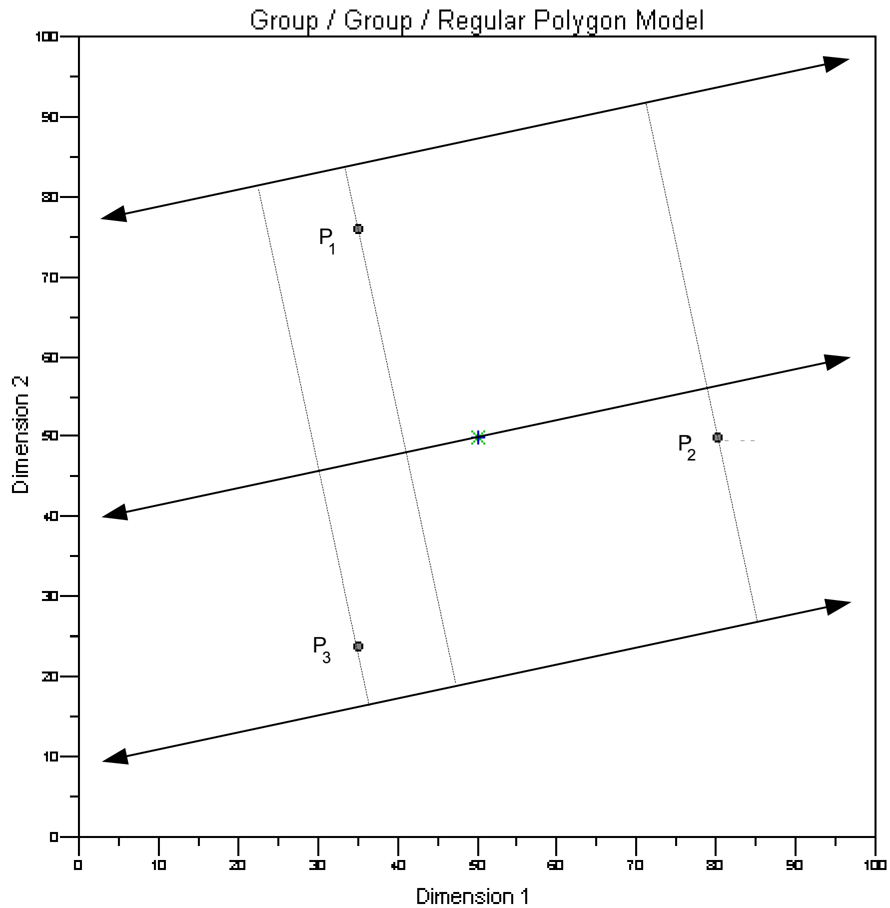


Figure 3 – Translation Invariance of Projections on to a Line

Note that there is nothing about our algorithm that is essentially limited to rotations in the plane. We could extend the algorithm to arbitrary rotations of a line in an n-dimensional space using generalized rotation angles, e.g., Euler angles in three dimensions (roll, pitch, and yaw).

The algorithm can be freed from other restrictions as well, notably, simple majority rule, odd number of voters, and distinct ideal points. In the case when simple majority rule is abandoned in favor of plurality there will in general be two pivots for each angle increment. In the case of an even number of voters there will always be at least two pivots. And in the case where voters have identical ideal points it will be necessary to identify each as a pivot when either is. The algorithm will give unambiguous sensible answers in all these cases, even though these conditions may significantly complicate analytic treatments.

4.3 Monte Carlo – Algorithm Verification

The validity of the algorithm is clear. It remains to verify that the implementation of the algorithm is correct. Apart from the practical value of checking the correctness of the program, the test cases serve to illustrate the behavior of SOV.

3 Voters

We begin by considering the simplest, non-trivial proximity spatial voting model, namely that of three voters located on an equilateral triangle in a two-dimensional issue space (Fig. 4a). We leave the issue dimensions unspecified. A common ordinal scale running from 0 to 100 is used for each dimension. The SOV is displayed next to each ideal point.

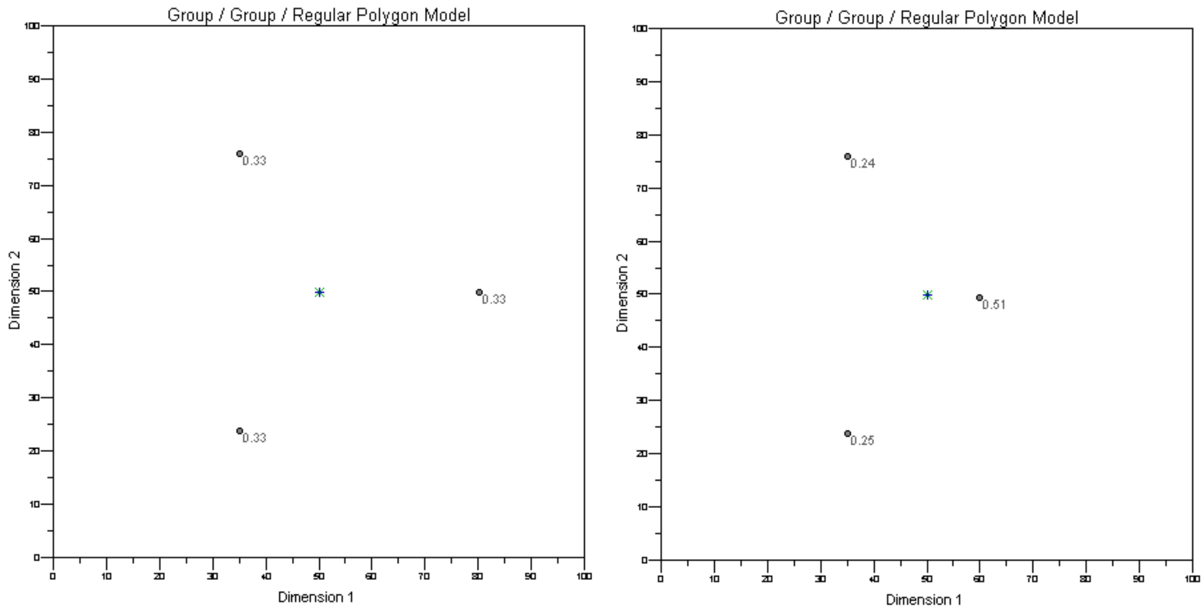


Figure 4a,b – SOVs for Three Voters in a Triangular Configuration

Observe that each voter receives a third of the value of the game, as we would expect from symmetry. The sum of SOV values is 1. This is an important constraint, useful for checking for round-off errors.

Next we move one of the members closer, roughly along a line midway between the two other voters (Fig. 4b). Observe that the more central voter has larger SOV = 0.51, the other two voters having smaller SOVs of 0.24 and 0.25, the total sum being 1. The value of the more central voter continues to rise until reaching the line joining the two other voters, at which point he/she becomes the median voter. This is the behavior we expect to see from the SOVs.

Accordingly we next present the configuration of the Median Voter Theorem where we can anticipate that the SOV for the central voter should be 1, with the other voters receiving no value (Fig. 5). And indeed this is so.

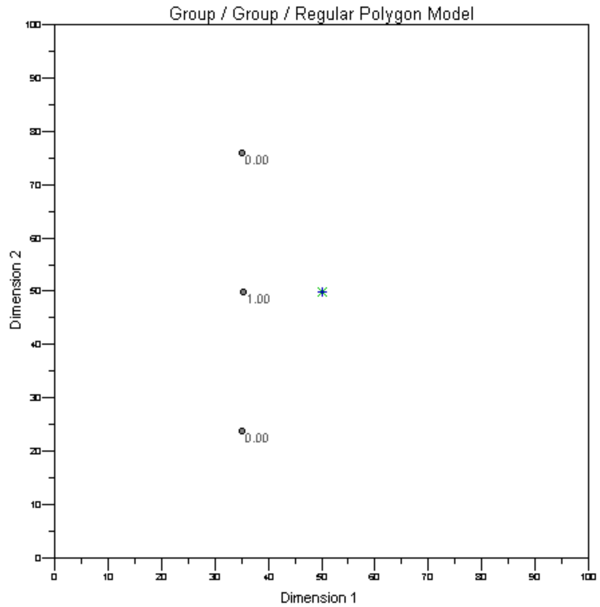


Figure 5 – SOVs for Three Voters in a Linear Configuration

Finally, we check that the SOV's vary symmetrically, as we would expect. This checks whether the implementation has an orientation bias (that would be a bug!).

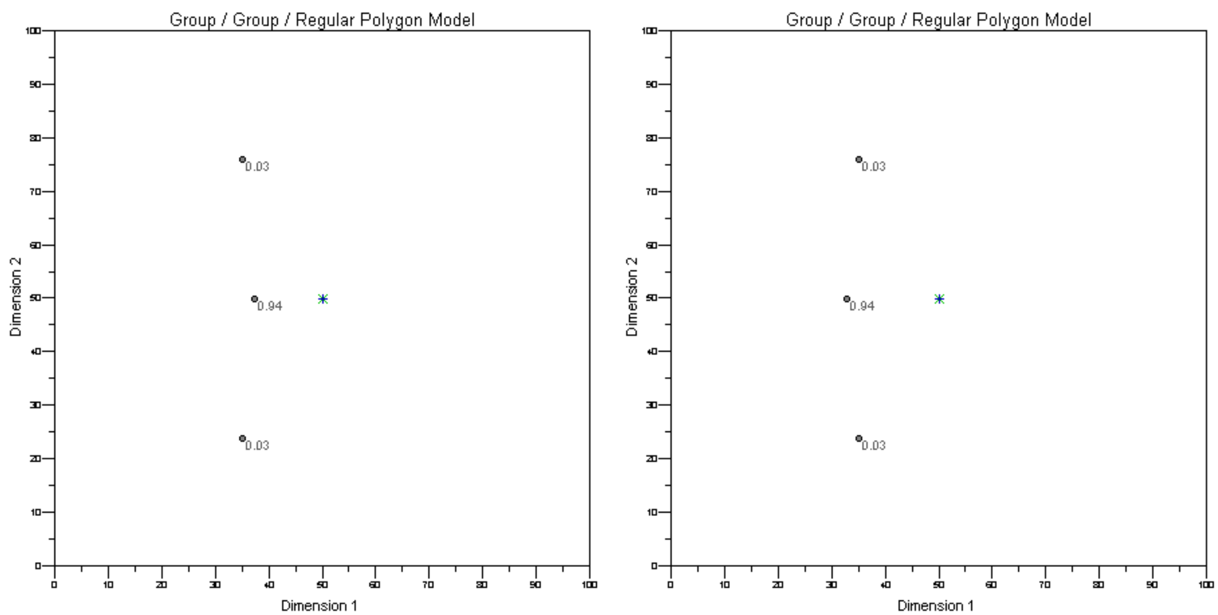


Figure 6a,b – SOVs for Symmetrical Configurations of Three Voters

In the example shown (Fig. 6 a,b) the SOV value is 0.94 for two points opposite each other across the line joining the two other voters.

5 Voters

Consider a more complicated configuration consisting of five voters forming a regular pentagon (Fig. 7a). From symmetry we can expect each voter to receive an equal SOV of 0.20 and indeed each does.

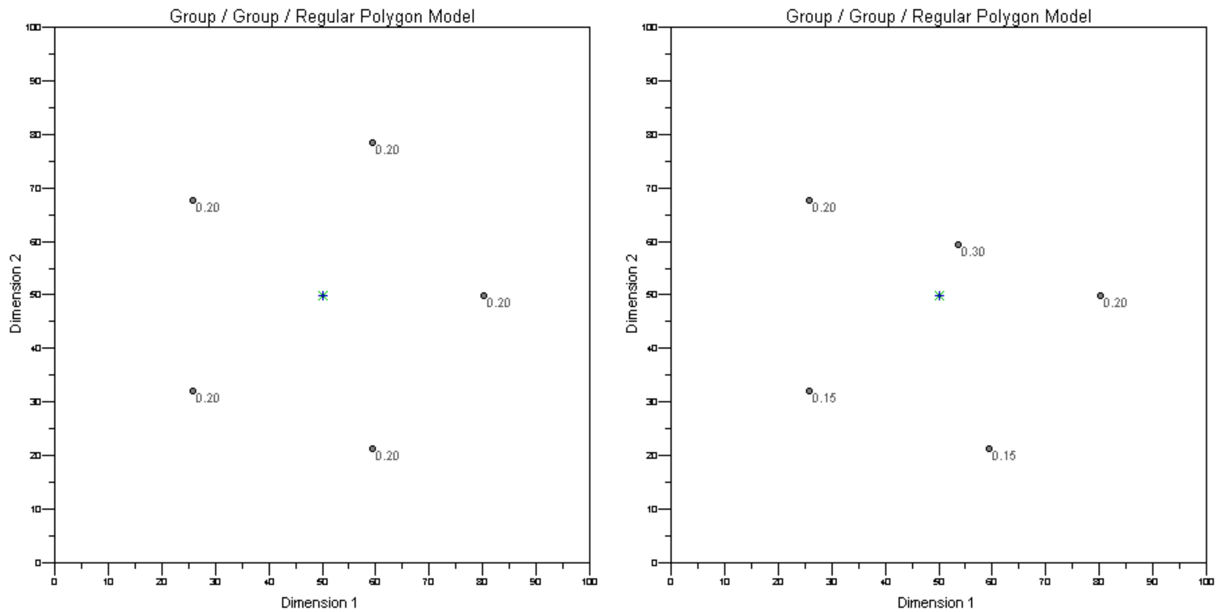


Figure 7a,b – SOVs of Five Voters Arrayed in a Pareto Set

Now we move one of the voters in toward the center (Fig. &b). Observe that the SOV the voter receives increases at the expense of the two voters most opposite, attaining a value of 0.30 while the opposite voters each drop to 0.15.

A more interesting test comes when we move the member inside the Pareto set (Fig. 8). Voters inside the Pareto set have a more central role and so should receive considerably more value.

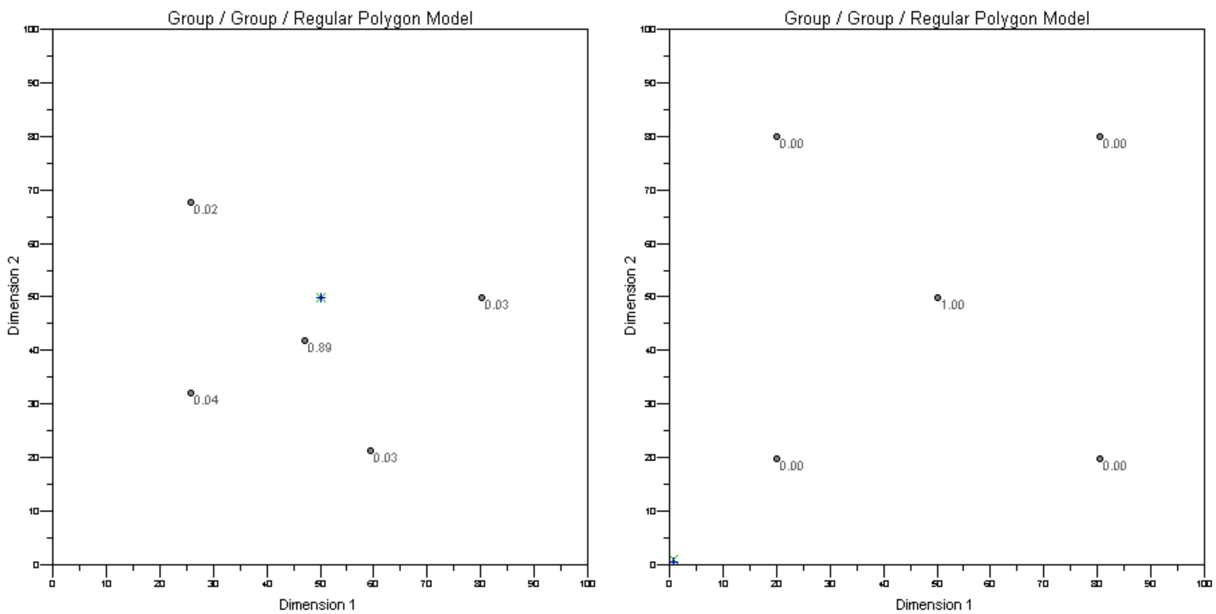


Figure 8a,b - Five Voters Arrayed Near and in a Plott Condition

We have placed the voter so at to approximate a Plott condition (Fig 8a). We therefore expect the central player to command a significant share of the game's value, and indeed the central voter does, with an SOV of 0.89. If we arrange a precise Plott condition, the central voter should receive the entire value of the game, which we confirm (Fig. 8b).

4 Voters

Much attention is given in the literature to the study of odd-sized committees. Even-sized committees are subtler in their properties. In an even committee no single member can be pivotal, i.e., there can be no single median voter. We can, however, define a pair of median voters, corresponding to the voters just before and after the would-be median position; e.g., often the median is *defined* the average of their positions. Thus for even committees we will have the convention of referring to these voters as a median-pair.

We consider now the simplest non-trivial even-sized committee consisting of four voters on a square (Fig. 9a). Each receives a quarter of the game value, as we would expect from symmetry.

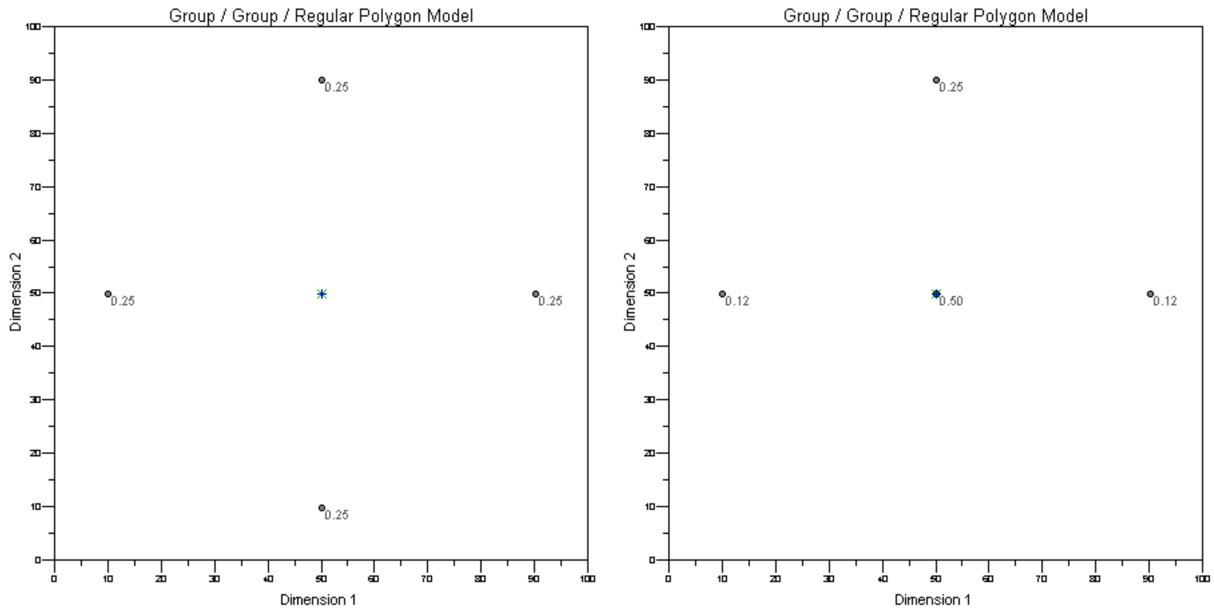


Figure 9a,b - Four Voters in Arrayed in a Pareto Set

As we move one of the voters toward the center of the square the voter's share of the game value increases until it reaches half the value of the game (Fig. 9b).

Remarkably, once the voter steps inside the Pareto set his/her SOV does not change but remains at 0.50 throughout the Pareto set. The following images illustrate this point (Fig 10a,b).

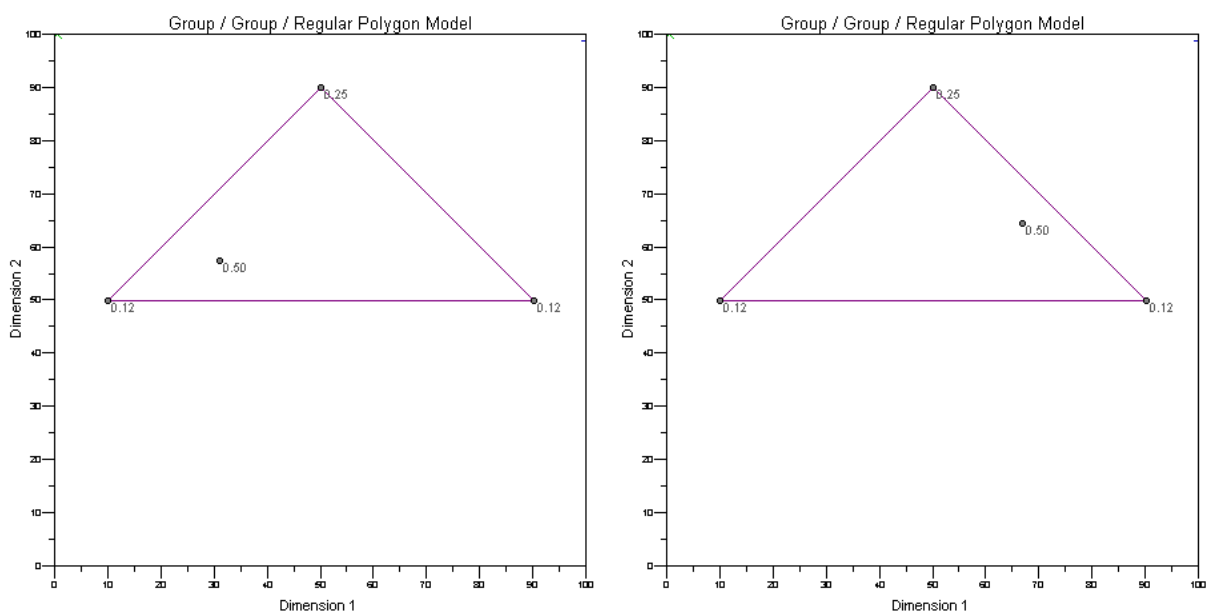


Figure 10a,b - SOV Constant Inside Pareto Set for Four Voters

On reflection this should seem reasonable, even obvious. The voter within the Pareto set in this case will be a member of all median-pairs, thereby taking half the value of the game. So this perhaps counter-intuitive result adds confidence both in the algorithm but also in the utility of the SOV in understanding coalition formation in proximity spatial voting models.

<u>*11 Voters*</u>

As a final verification we wish to make use of a theorem by Feld and Grofman relating the radius of the yolk to SOVs [Feld-Grofman, 1990]. Given a voter i in two dimensions with Euclidean preferences a distance d from a yolk of radius r, then the SOV of the voter is bounded as

$$\phi_i \le \frac{2\arcsin\left(\frac{r}{d}\right)}{\pi}$$

where the yolk is the smallest circle that intersects every line that divides the space into two groups of voters neither of which is the majority, called *median lines*.

The theorem rests on the observation that a voter can be pivotal only if he/she is on a median line. As all median lines pass through the yolk, the only angles for which the voter can be pivotal correspond to lines through the yolk. The two most extreme angles are tangent at opposite sides of the yolk; the SOV angle can be no greater than the angle subtended by these two tangents meeting at the voter.

Let d be the distance of the voter from the yolk center along some ray. Let θ be the angle formed between this ray and each of the lines tangent to the yolk. From elementary geometry

$$\sin\theta = \frac{r}{d}$$

where r is the radius of the yolk. And so

$$\theta = \arcsin\left(\frac{r}{d}\right)$$

The total angle is just twice this and the SOV is just this angle divided by π, which proves the theorem.

We can use this theorem to check that no computed SOV exceeds the Feld-Grofman bound. Consider 11 voters distributed randomly according to uniform distributions in Dimensions 1 and 2 (see Fig. 11). Next to each voter is the corresponding SOV.

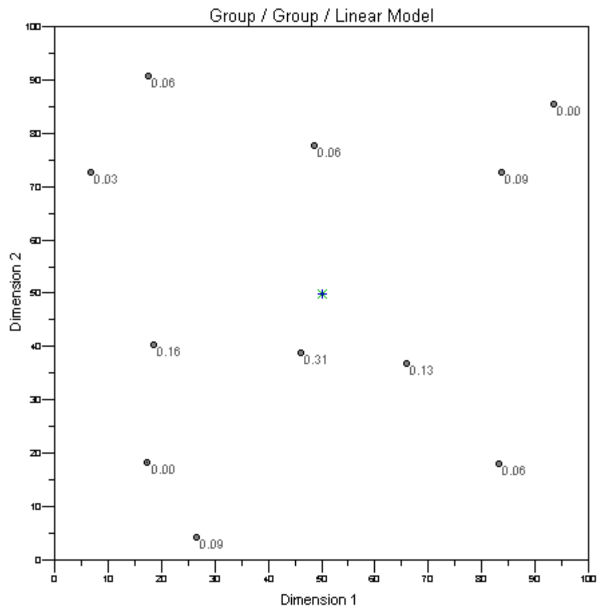


Figure 11 - A Random Distribution of 11 Voter Ideal Points

Next, we compute the yolk. This is a two-step process. First we determine the smallest circle intersecting all *limiting* median lines, i.e., median lines that pass through one voter and infinitesimally close to another. Usually this step suffices to determine the yolk, but in some cases, as pointed out by Tovey, exceptions occur. The second step consists in identifying whether a "Tovey anomaly" exits, which voters are responsible, and adding non-limiting median lines to represent their circumstance. In the present example there is no anomaly (Fig 12a).

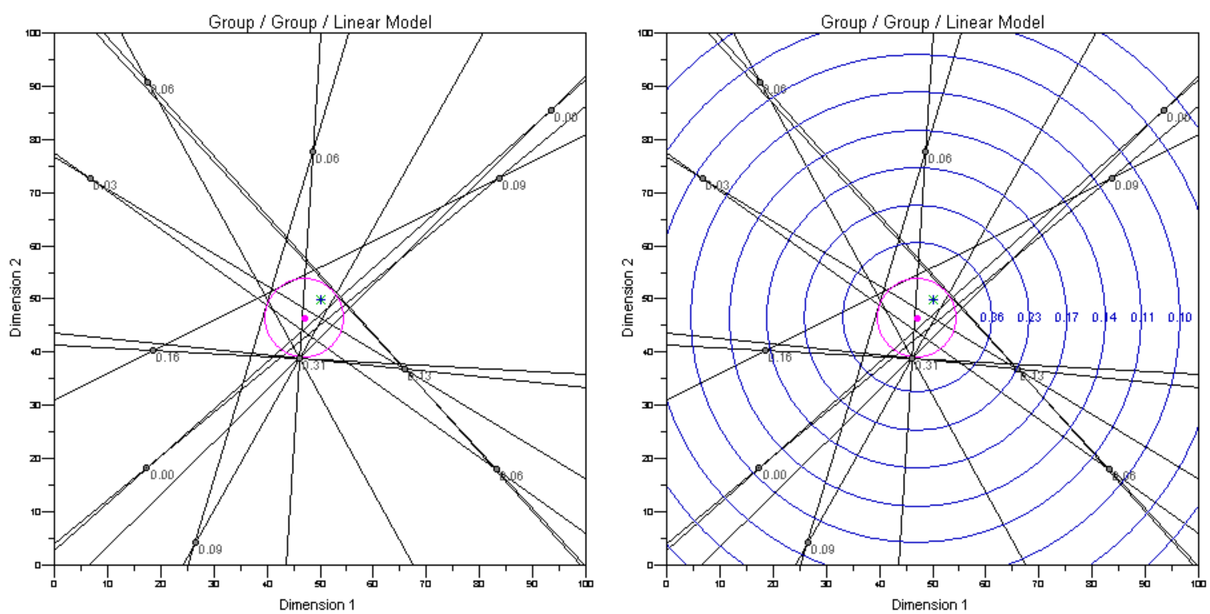


Figure 12a,b -Yolk and Feld-Grofman Bounds on SOV

With the yolk determined, we can compute the Feld-Grofman bounds. We display representative values as concentric rings centered on the yolk (Fig 12b). Or, for greater legibility, we display representative Feld-Grofman bounds without the median lines and yolk displayed (Fig 13).

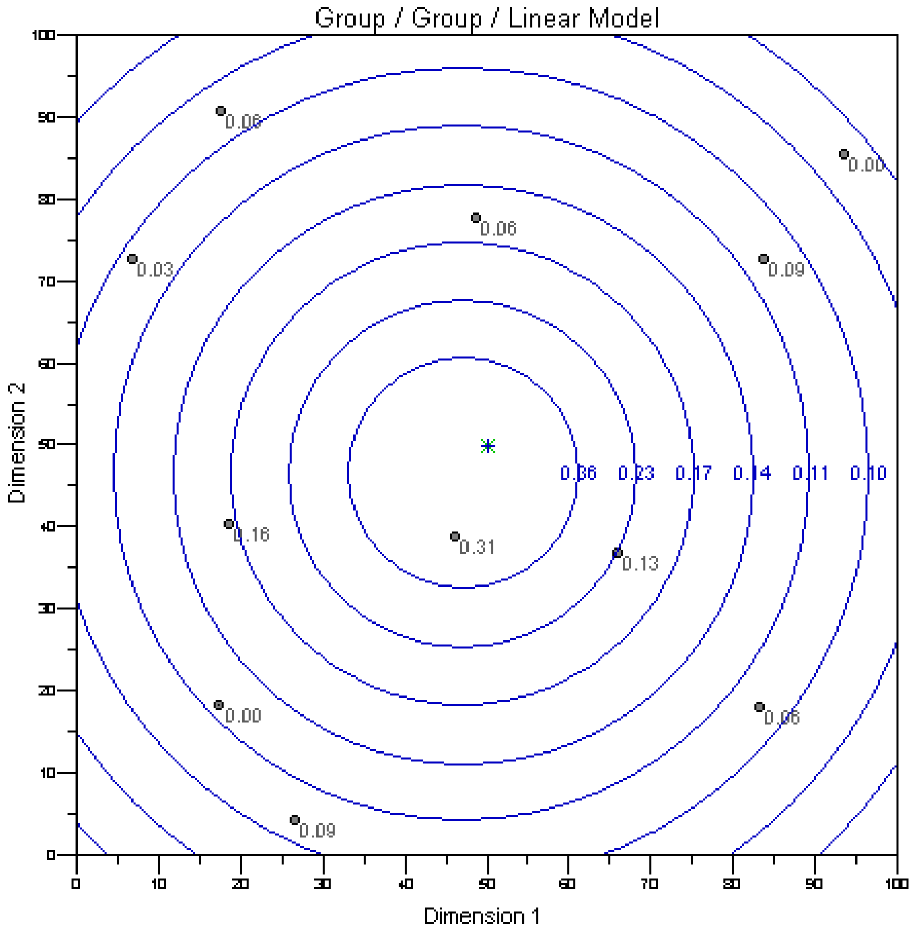


Figure 13 - Feld-Grofman Bounds on SOV

Observe that few voters challenge the Feld-Grofman bound. In fact, only one voter, with an SOV 0.16, appears to attain the bound.

Weighted Voting

As a final example we take the case of weighted voting. This problem goes beyond the warrants of the Shapley-Owen theory, but is tractable under the algorithm in a way we hope this example illustrates.

We consider first the case of four voters. We take the voter from the previous example and move him/her to be coincident with another voter (Fig. 14a). This violates an assumption of Shapley-Owen that excludes the case of coincident ideal points. Observe that the coincident voters have equal values of 0.37, the total of which is 0.74.

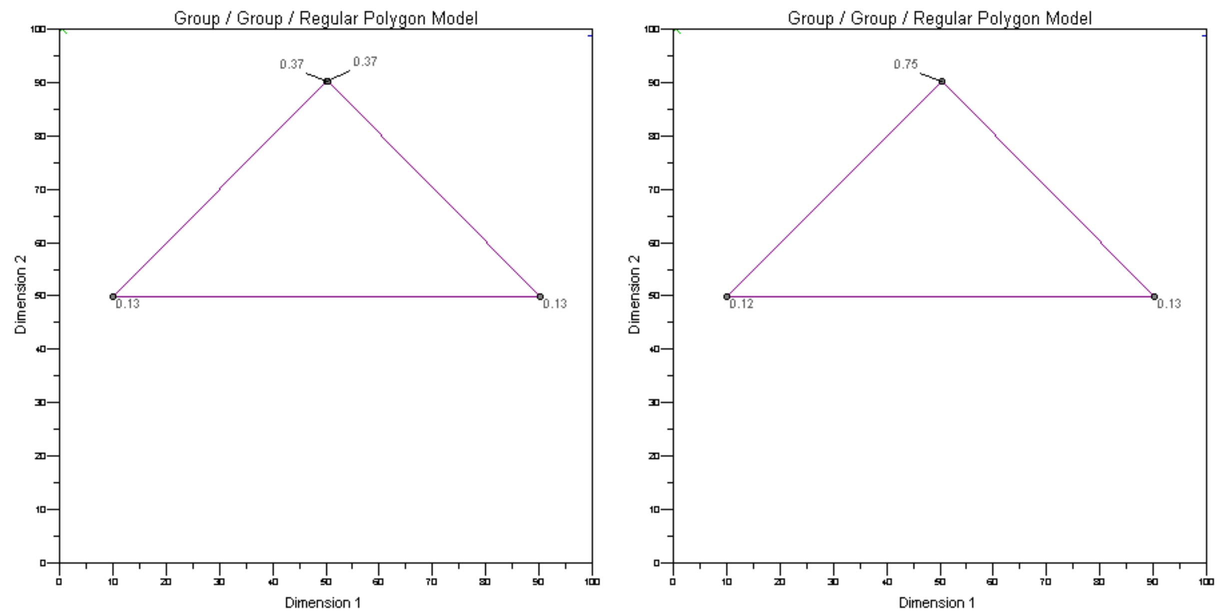


Figure 14 a,b - Superposition of Two Voters Compared with One Weighted Voter

Suppose we remove one of the two coincident voters and grant the remaining voter a vote equal to the vote of the two voters jointly, i.e., if each voter has a vote of 1 the voter will have a vote of 2 (Fig 14b). This is an example of weighted voting.

Observe that the single voter with twice the vote weight has essentially twice the voting power. The slight discrepancy in numbers is due to the approximations involved. Note, as a quality check, that the sum of SOVs is one.

The Shapley-Owen theory does not apply when voters have different vote weights. Nor does it apply when some voters occupy the same ideal point. So it is not immediately clear in what sense these models represent SOVs. Our algorithm, based on Shapley's model, rigorously implements SOV when the Shapley-Owen assumptions hold. The same algorithm, without modification, can be used to compute values for voters whose voting weights differ or who occupy the same ideal point. This suggests that our algorithm is a natural extension of the Shapley-Owen theory.

But there is a difficulty. To draw it out, observe that any weighted voting model with a *finite* number of voters can be transformed into a weighted voting model with integer-valued votes by rescaling the weights. Weighted voting thus appears very similar to voters with coincident ideal points. The main difference concerns assigning value to voters at the same ideal point whose votes differ in weight. The problem is acute if some subset of the voters determines the quota. This voting problem is non-spatial, i.e., zero-dimensional, the solution for which is to use a combinatorics-based index such as Shapley-Shubik. The suggestion, therefore, would be to compute the value of ideal points according to a weighted voting model and distribute that value to coincident voters on a combinatorial basis. The algorithm reported in this paper shares value equally; the case when coincident voters have different weights does not arise in any example considered. Handling the general case just described is beyond the scope of this paper.

5.1 Case Study: US Congress – Empirical Evidence for Shapley-Owen

The following case study considers SOV in the context of significant legislative battle in the Congress of the United States that took place in 1993 shortly after Clinton was first elected President.

Health care was and continues to be a major domestic issue. Clinton aimed to dominate the legislative agenda with a bold initiative in health care reform. The Republicans, recognizing the political threat, mounted a fierce opposition and succeeded in defeating the measure. Arguably this defeat set the stage for an even greater debacle, resulting in the eventual dominance of the Republican Party in US politics by the turn of the millennium. So this is a particularly interesting piece of legislation to consider from a "power" perspective.

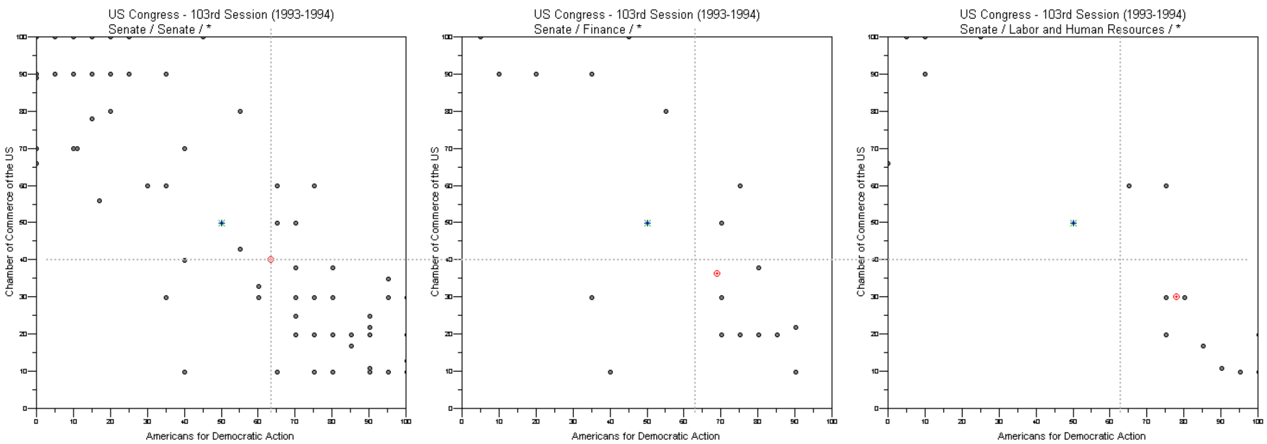
The data for this case study is taken from Kenneth Goldstein's study of grassroots lobbying [Goldstein, 1999]. The data is based on interviews with 21 lobbying firms (special interest groups or SIGs), as well as major stakeholders in the political process. What makes Goldstein's data particularly useful for an SOV analysis is that he reports on whom SIGs perceived to be the pivotal legislators on a particular issue. We are thus able to contrast SOV values with the mobilization behavior of SIGs. Goldstein did not collect his data, however, with spatial modeling in mind. Thus the spatial analysis we offer here represents an extension of his findings.

Based on his interviews and analysis, Goldstein determined that the political debate could be framed in terms of two dimensions, a political dimension of party affiliation and a second of (small) business affinity. He used for these dimensions ratings assembled by two SIGs, the Americans for Democratic Action (ADA), well recognized as a measure of party affiliation, and the National Federation of Independent Businesses (NFIB), an association advocating for the interests of small businesses.

Without getting into the complexities of the US legislative process, it suffices for our purposes to note that the health reform measure was referred to a number of committees for markup. Not all committees carry equal weight politically, however, as has been pointed out by Fenno [Fenno, 1973]. Committees may be classified as influence, policy, and re-election committees, with the influence committees being considered the most powerful.

We can get a sense of Fenno's thesis by contrasting the full Senate with two Senate committees that considered Clinton's bill, Finance and Labor and Human Resources, using the strong point as a basis of comparison (Fig 15a,b,c). We assume for this comparison Euclidean preferences for all members in regard to the health bill. This may not be realistic for all Senators. Some Senators, for example, may feel more strongly about their party affiliation than their small business interests. Nevertheless, Goldstein reports that for the health bill debate both party affiliation and business dimensions were salient for a majority of Senators (and Representatives). Furthermore, in the absence of specific preference data, the assumption of Euclidean preferences is the least biased.

Subject to these model assumptions and caveats, observe that the Finance strong point is much closer to the Senate value than that of Labor and Human Resources. If we regard the strong point as the expected outcome of each committee, we see that Finance will produce an outcome more favorable to the Senate than will Labor and Human Resources. Indeed, Goldstein identifies Finance as an influence committee and Labor and Human Resources as a policy committee.



	Senate	Finance	Labor and Human Resources
ADA	65	70	80
COC*	40	37	30

*Goldstein does not have NFIB data for the full Senate; Chamber of Commerce ratings used as a surrogate.

Figure 15a,b,c - The Strong Points of the Senate, Finance, and Labor and Human Relations

Accordingly, SIGs seeking to influence the final outcome of the health bill would be expected to invest more of their resources on Finance than on Labor and Human Resources. This is in fact what Goldstein observed. Of the 21 SIGs, 5 mobilized for Labor and Human Resources with remainder, 16, mobilizing for Finance.

In so far as the strong point is computed through weighted ideal points by SOVs, the discrimination between Senate committees serves as one example of applying the Shapley-Owen theory in an empirical context. We turn now to a more explicit application.

We focus our attention of the influence committees, Senate Finance, House Ways and Means, and House Energy and Commerce. We begin with Senate Finance (Fig 16).

<u>*Senate Finance Committee*</u>

There are two parties, Republicans and Democrats. Republicans are generally in the upper left corner and are more pro-NFIB. The Democrats are generally in the lower right corner and are less pro-NFIB. The color-coding reflects the number of SIGs mobilizing to influences the given Senator. Next to each ideal point is the Senator's name and SOV.

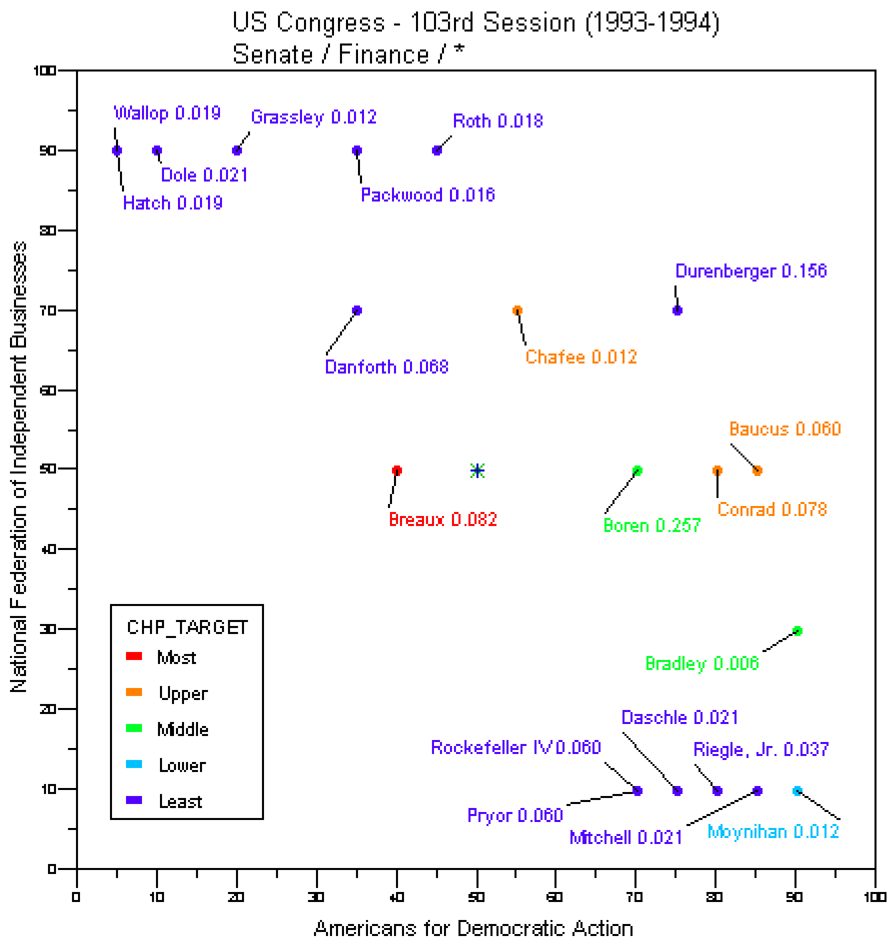


Figure 16 – Senate Finance: SIG Mobilization Targets

There appears to be a rough correspondence between mobilization and SOV, but there are evident anomalies, such as Durenberger, Danforth, and Rockefeller for whom no SIG mobilized. The attention to Moynihan is understandable in so far as he was the committee chair. The Pearson correlation between SOV and SIG is 0.23.

State	Party	Name	ADA	NFIB	SOV	SIG
OK	D	Boren, David	70	50	0.257	5
MN	IR	Durenberger, Dave	75	70	0.156	0
LA	D	Breaux, John	40	50	0.082	11
ND	D	Conrad, Kent	80	50	0.078	8
MO	R	Danforth, John	35	70	0.068	0
MT	D	Baucus, Max	85	50	0.060	9
AR	D	Pryor, David	70	10	0.060	0
WV	D	Rockefeller IV, John	70	10	0.060	0
MI	D	Riegle, Jr., Donald	80	10	0.037	0
SD	D	Daschle, Thomas	75	10	0.021	0
KS	R	Dole, Robert	10	90	0.021	0
ME	D	Mitchell, George	85	10	0.021	0
UT	R	Hatch, Orrin	5	90	0.019	0
WY	R	Wallop, Malcolm	5	90	0.019	0
DE	R	Roth, William	45	90	0.018	0

OR	R	Packwood, Bob	35	90	0.016	0
RI	R	Chafee, John	55	70	0.012	8
IA	R	Grassley, Charles	20	90	0.012	0
NY	D	Moynihan, Daniel	90	10	0.012	3
NJ	D	Bradley, Bill	90	30	0.006	5

We plot the SIG mobilization count against SOV value, using a log scale for SOV (Fig. 17).

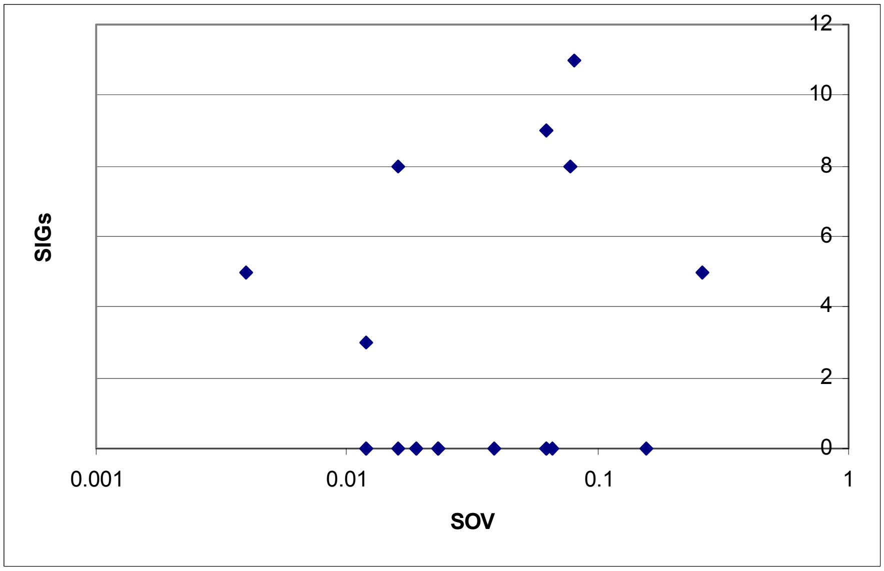


Figure 17 – Senate Finance: Number of SIG Mobilized Versus Shapley-Owen Value

There appear to be two distinct groups, from a mobilization perspective: those that warrant mobilization and those that don't. Among those that do warrant mobilization there appears to be some relationship between SIGs and SOV. The relationship would be striking but for the relative neglect of Boren: (SOV, SIG) = (0.26,5).

Goldstein notes that Senators such as Durenberger, while having exceptionally high SOV, were not up for re-election. SIGs in such cases believe they have relatively little influence and so expend their resources elsewhere. In point of fact, Durenberger voted with his party and against the bill. Danforth, however, more Republican politically than Durenberger, voted for the bill, as did Chafee (who did receive considerable attention). Conrad, on the other hand, was up for re-election, was heavily targeted, and voted against bill and against his party.

Moynihan, while personally favoring a government-sponsored health plan, believed that for the health bill to survive a filibuster on the Senate floor bipartisan support was required. He therefore worked to amend the bill in such a way as to attract moderate Republicans. He did so by removing certain language offensive to small business interests (so called *employer mandates*). The result was a bill positioned closed to the Finance committee strong point and, therefore, the Senate strong point (Fig. 18).

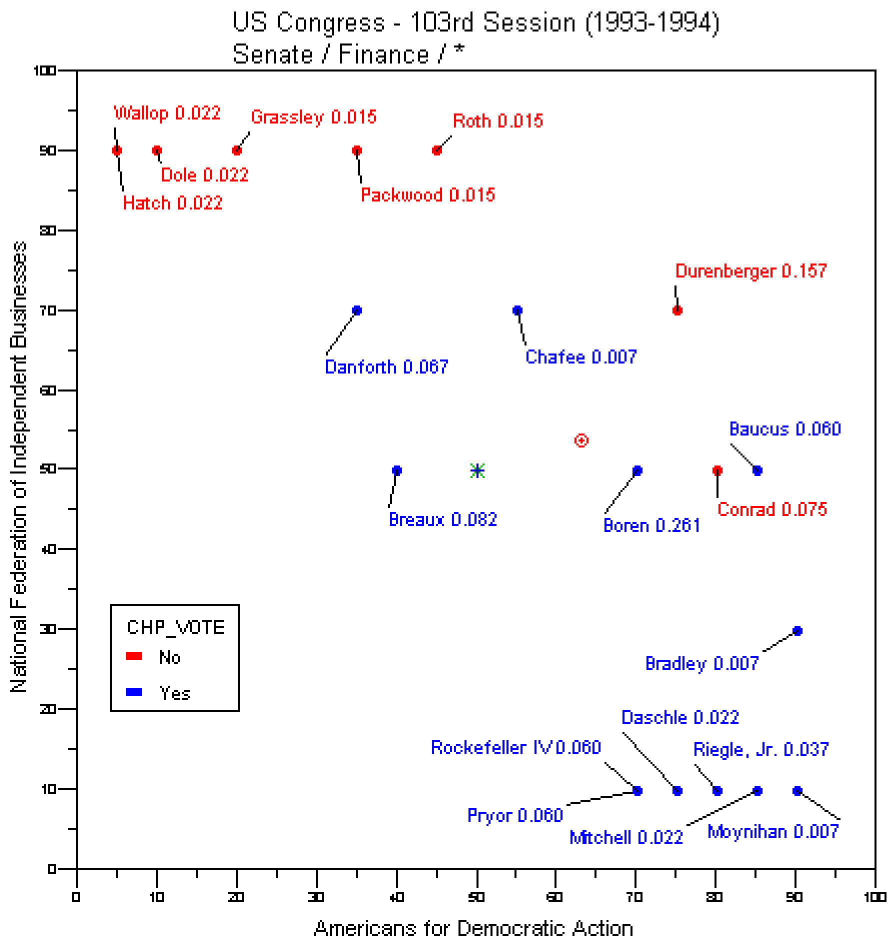


Figure 18 – Senate Finance: Final Vote with Strong Point

In order to reproduce the vote as shown and have the proposal reported near the strong point, Danforth, Chafee, Breaux, and Conrad would have to have voted as if their ideal points were shifted from the positions reported by Goldstein. For example, in one possible explanation, Danforth, Chafee, and Breaux would have voted as if further to the lower right and both Durenberger and Conrad as if further to the upper left. In the diagram the green X marks the original location of the strong point, while the blue cross + represents the adopted proposal (Fig 19).

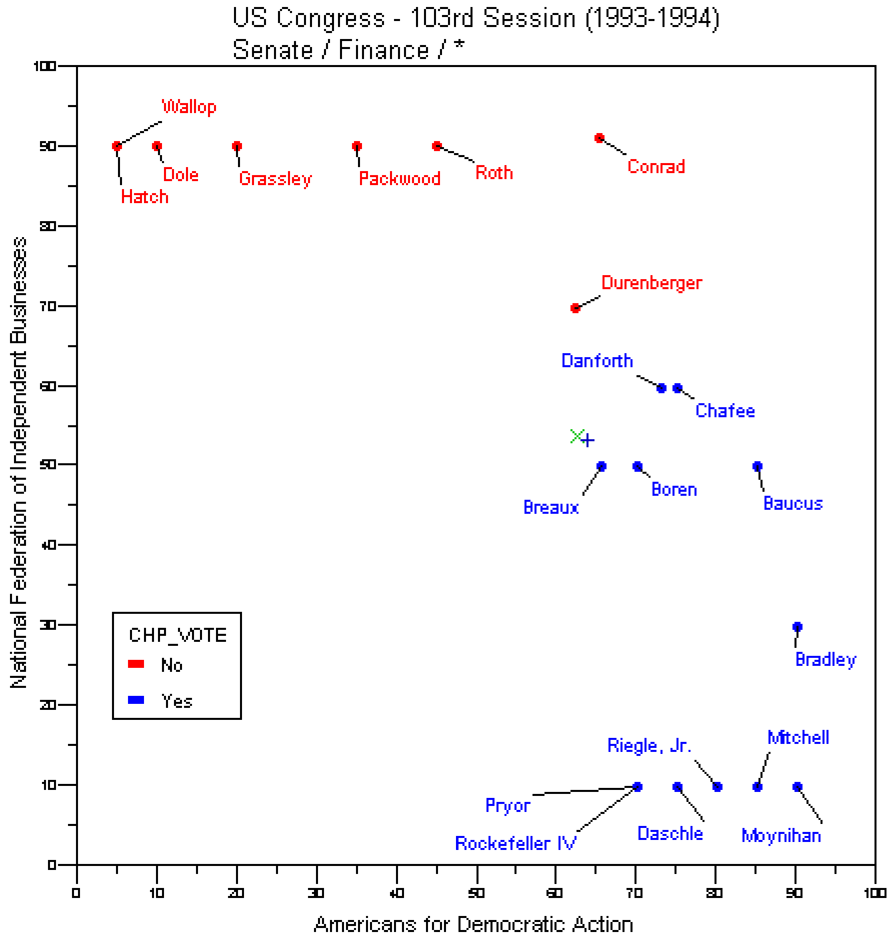


Figure 19 – Senate Finance: Final Vote "Explained"

<u>House Ways and Means Committee</u>

Next we consider the House Ways and Means committee. This committee experienced some turnover during the markup process but nothing that fundamentally affects the analysis. The plotting conventions follow those of Senate Finance (Fig 20).

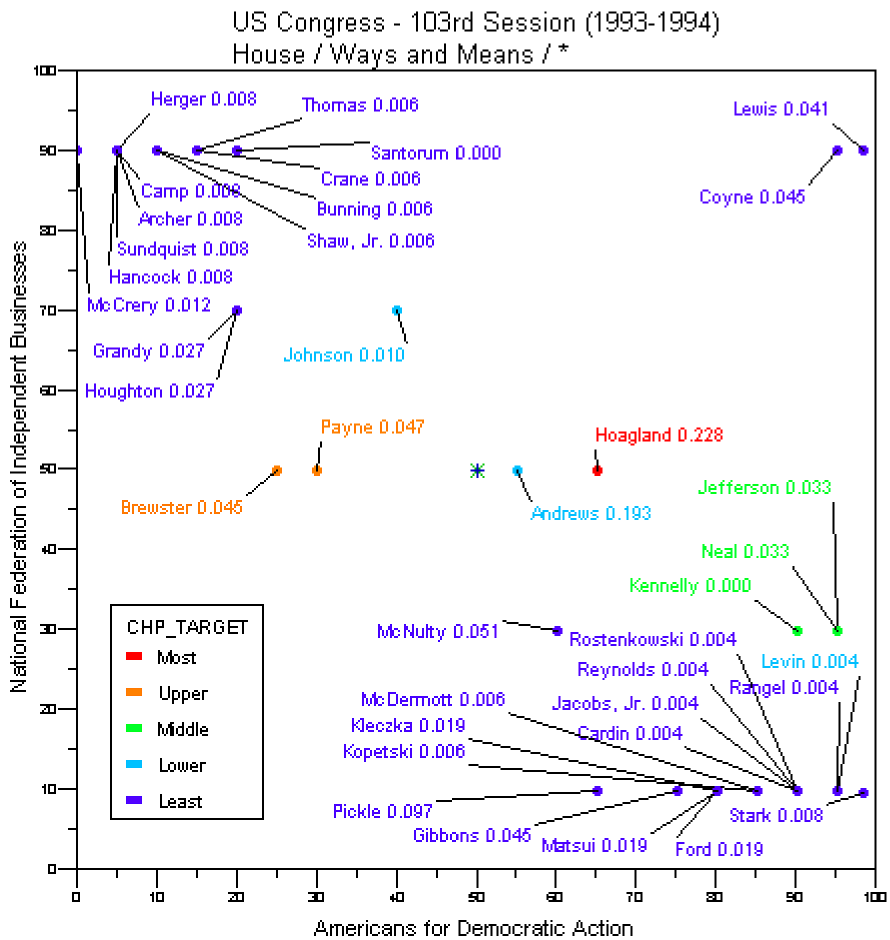


Figure 20 – House Ways and Means: SIG Mobilization Targets

In this committee the most heavily targeted legislator was also the one with highest SOV. In general, if we list Representatives sorted in descending order by SOV, legislators with higher SOV tend to have higher SIG mobilizations. The Pearson correlation between SOV and SIG is 0.54.

District	Party	Name	ADA	NFIB	SOV	SIG
NE-2	D	Hoagland, Peter	65	50	0.228	12
TX-25	D	Andrews, Michael	55	50	0.189	2
TX-10	D	Pickle, J.	65	10	0.099	0
NY-21	D	McNulty, Michael	60	30	0.054	0
VA-5	D	Payne, L.	30	50	0.048	10
OK-3	D	Brewster, Bill	25	50	0.045	10
PA-14	D	Coyne, William	95	90	0.045	0
FL-11	D	Gibbons, Sam	75	10	0.045	0
GA-5	D	Lewis, John	100	90	0.039	0
LA-2	D	Jefferson, William	95	30	0.033	6
MA-2	D	Neal, Richard	95	30	0.033	3
IA-5	R	Grandy, Fred	20	70	0.024	0
NY-31	R	Houghton, Amo	20	70	0.024	0
TN-9	D	Ford, Harold	80	10	0.021	0
WI-4	D	Kleczka, Gerald	80	10	0.021	0

CA-5	D	Matsui, Robert	80	10	0.021	0
LA-5	R	McCrery, Jim	0	90	0.012	0
TX-7	R	Archer, Bill	5	90	0.009	0
MI-4	R	Camp, Dave	5	90	0.009	0
MO-7	R	Hancock, Mel	5	90	0.009	0
CA-2	R	Herger, Wally	5	90	0.009	0
CT-6	R	Johnson, Nancy	40	70	0.009	2
CA-13	D	Stark, Fortney	100	10	0.009	0
TN-7	R	Sundquist, Don	5	90	0.009	0
MD-3	D	Cardin, Benjamin	90	10	0.006	0
IL-8	R	Crane, Philip	15	90	0.006	0
IN-10	D	Jacobs, Jr., Andy	90	10	0.006	0
OR-5	D	Kopetski, Mike	85	10	0.006	0
MI-12	D	Levin, Sander	95	10	0.006	2
WA-7	D	McDermott, Jim	85	10	0.006	0
NY-15	D	Rangel, Charles	95	10	0.006	0
IL-2	D	Reynolds, Mel	90	10	0.006	0
IL-5	D	Rostenkowski, Dan	90	10	0.006	0
CA-21	R	Thomas, William	15	90	0.006	0
KY-4	R	Bunning, Jim	10	90	0.003	0
FL-22	R	Shaw, Jr., E.	10	90	0.003	0
CT-1	D	Kennelly, Barbara	90	30	0.000	3
PA-18	R	Santorum, Rick	20	90	0.000	0

If we plot the SIG mobilization count against SOV value, using a log scale for SOV, we see a similar pattern to Senate Finance (Fig. 21).

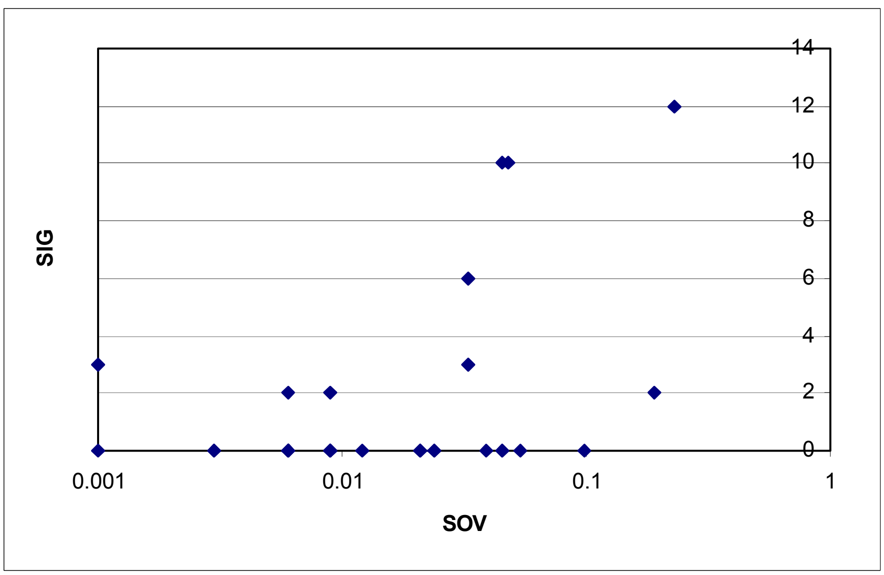


Figure 21 – House Ways and Means: Number of SIG Mobilized Versus Shapley-Owen Value

Again, there are apparently two distinct groups, from a mobilization perspective, those that warrant mobilization and those that don't. Among those that do warrant mobilization there appears to be some relationship between SIGs and SOV.

There is more spatial structure in this committee than Senate Finance. This becomes apparent when we look at the final vote on the bill by Ways and Means. The final bill after markup resided fairly close to the strong point of the committee, roughly between Hoagland and Andrews (Fig 22).

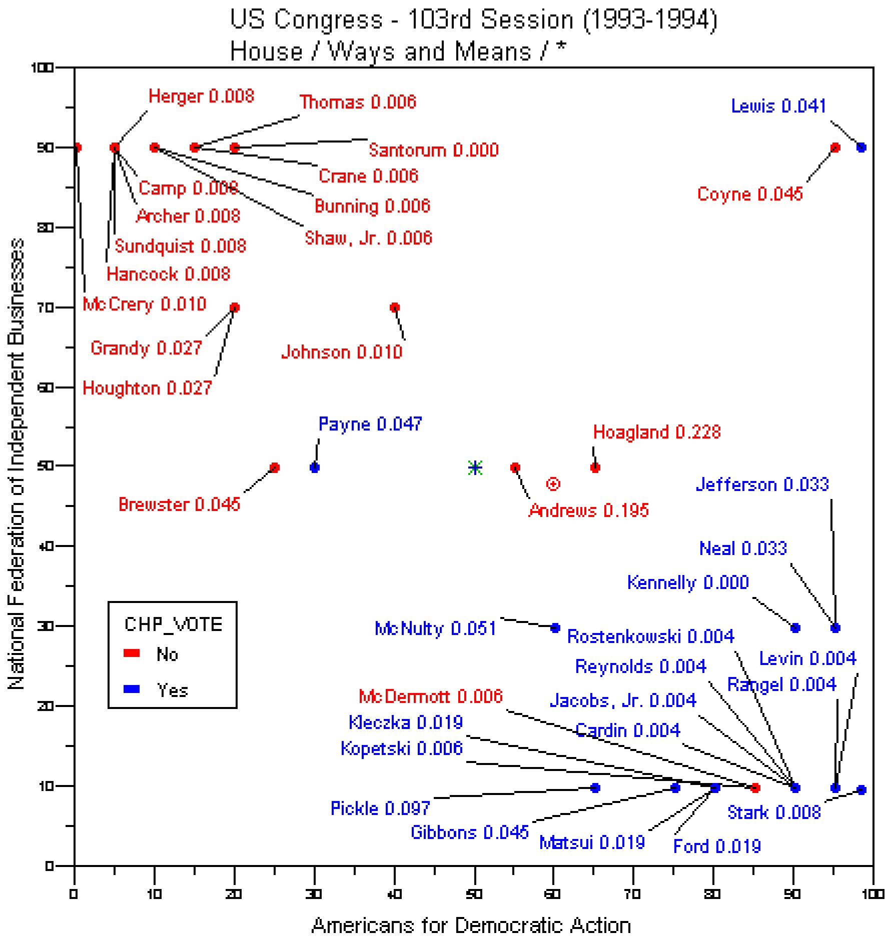


Figure 22 – House Ways and Means: Final Vote

Noticeable anomalies are Payne, McDermott, and Coyne/Lewis. Coyne is all the more remarkable because he was a co-sponsor of the bill, whereas Lewis was not – yet their votes are flip-flopped! Payne and McDermott are explained fairly easily using a conventional narrative explanation. Coyne and Lewis provide an opportunity to apply SOV.

Payne was from Virginia and objected to certain tax provisions against tobacco in the bill. In exchange for removing those provisions he supported the amended bill.

McDermott insisted on a heavily subsidized public health issuance plan and would settle for nothing less. His preferences cannot be modeled as Euclidean but as *absolute* – his ideal point or nothing. His SOV can still be computed using our algorithm. The interpretation of the strong point suffers, however, as McDermott does not contribute to the area of the win set. We do not attempt to account for this perturbation in the present analysis.

Now let's consider Coyne and Lewis more closely. First, we compute the regression line for the committee (Fig. 23).

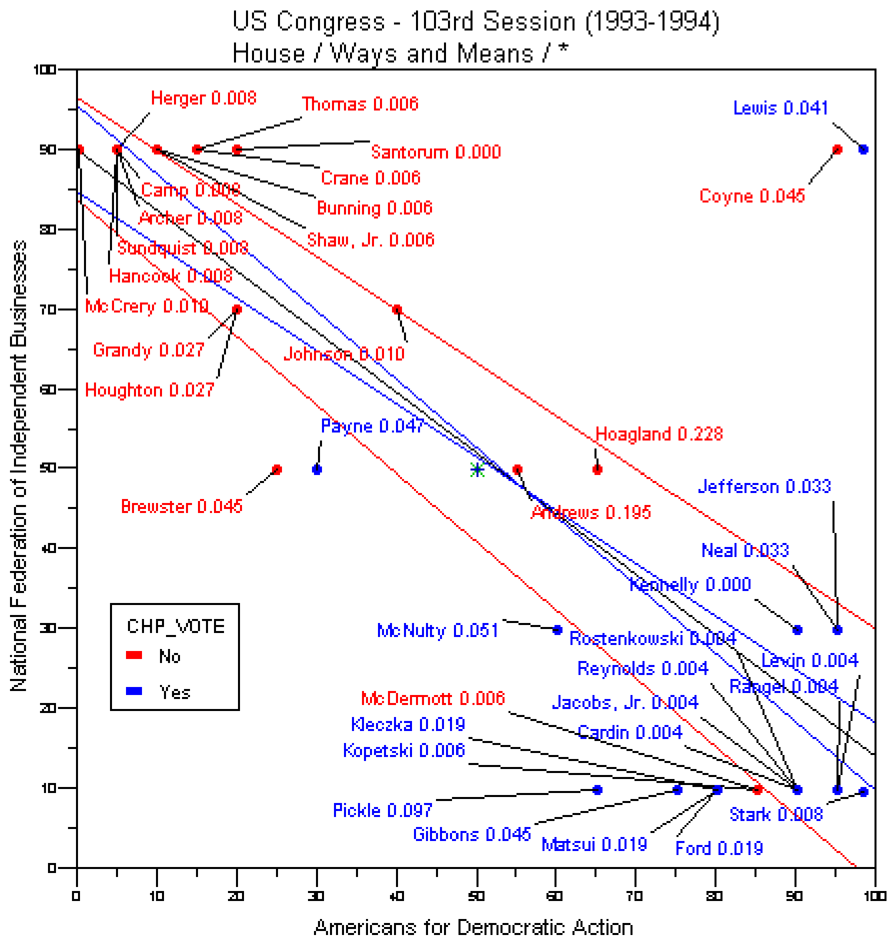


Figure 23 – House Ways and Means: Regression Line

The blue lines represent the error range for the slope of the regression line; the red lines represent the 1-sigma corridor in which the regression line most likely falls; and the black line is the best estimate for the regression line. Evidently there is a pretty good correlation along the regression line.

The strong correlation suggests that we can approximate the decision in this committee as a one-dimensional decision along the line of correlation. Accordingly we are particularly interested in identifying legislators near the median position. We accomplish this by projecting all legislators on to the correlation line and draw a perpendicular line through the median position (*cut line*), depicted as a red dashed line (Fig 24).

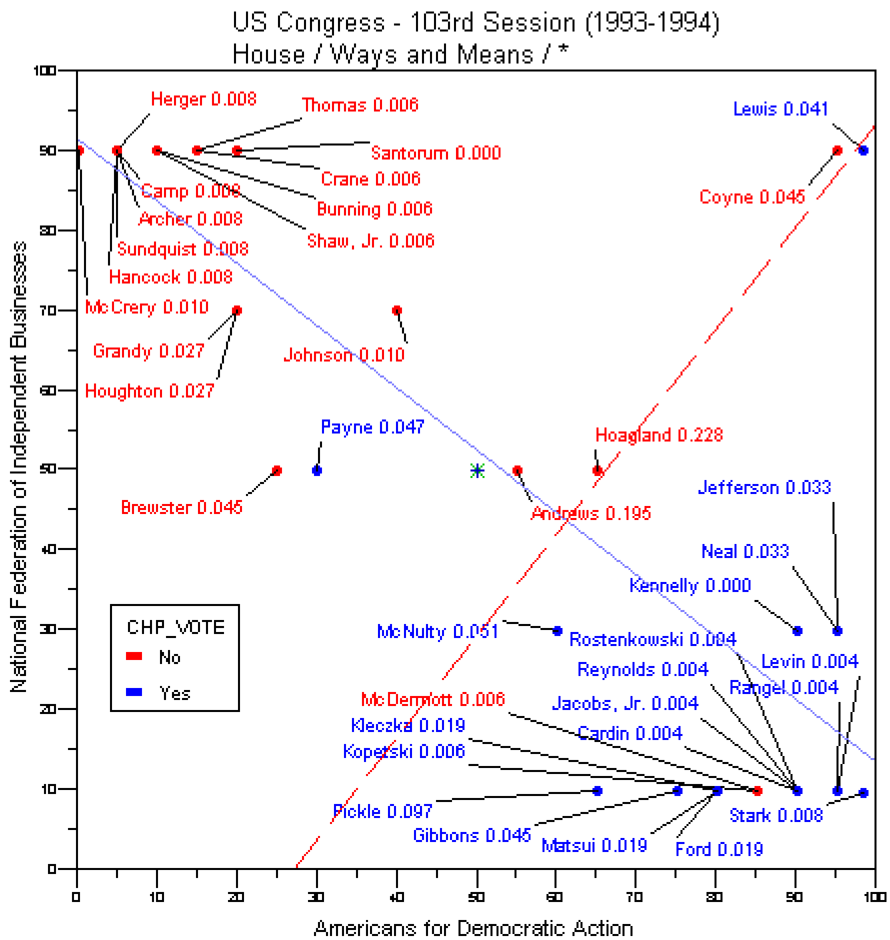


Figure 24 – House Ways and Means: Cut Line

We discover that Coyne and Lewis, along with Hoagland and Andrews, are all very close to median position. Indeed, Coyne is on the "Republican" side and Lewis is on the "Democratic" side, consistent with their final votes.

The significant take away here is that the SOVs of both Coyne and Lewis are as large as Payne and Brewster, both of whom attracted significant SIG attention, but no SIG paid attention to Coyne and Lewis. Even Goldstein makes no mention of these two! Judging from their spatial positions, neither Coyne nor Lewis conforms to either party's conventional profile, so perhaps these legislators were perceived as marginal and discounted by SIGs. A formal SOV analysis, however, is free of this bias and so would have identified Coyne and Lewis to be as significant targets as Payne and Brewster.

Finally, to illustrate Payne's negotiating position, if we adjust Payne's position to reflect his actual vote, i.e., Payne votes "as if" his ideal point were in the new position, and recompute SOVs (Fig. 25).

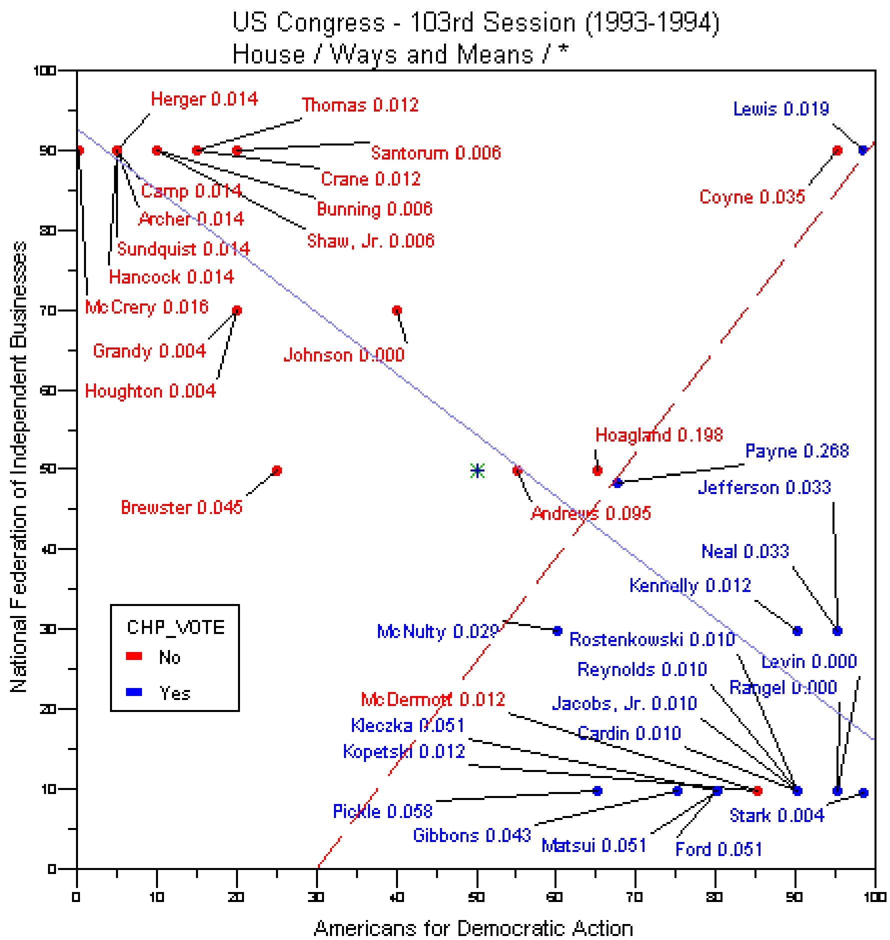


Figure 25 – House Ways and Means: Final Vote Explanation

Voting with the Democrats, Payne now has the single largest SOV. This reflects, perhaps, the challenge Republicans had in keeping Payne from defecting.

Finally we consider the House Energy and Commerce Committee. This committee never voted on the bill so all we can do is report the data, not the final outcome.

House Energy and Commerce Committee

The Energy and Commerce committee was one in which the chairman, Dingle, wanting to push through the health bill with minimal markup, was unable to gain much cooperation. An SOV analysis makes the challenge particularly clear (Fig 26). Note that both Boucher and Lehman have the same ideal point, each receiving an SOV of 0.422. Between the two them they have 0.844 of the total value. (In fact there appears to be some systematic round-up error in this committee from the SOV algorithm; it is an even committee with a many members sharing the same ideal point. However, experimenting with minor adjustments of ideal points to avoid the compounding of coincidence errors, Boucher and Lehman continue to command well over 70% of the game value. So qualitatively, at least, the representation is accurate.) To report the bill unchanged, Dingle would have had to ask Boucher and Lehman to give up a considerable amount of value. And he would still have had Slattery and Cooper to persuade, whose positions would be strengthened if Boucher and Lehman surrendered value.

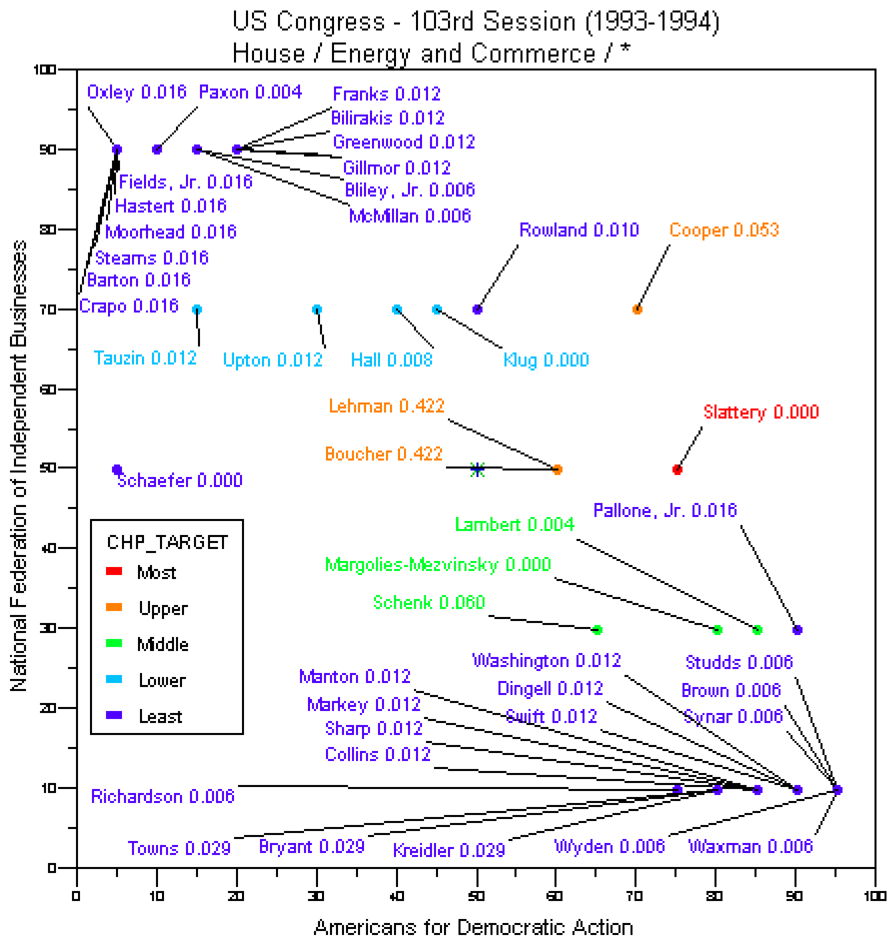


Figure 26 – House Energy & Commerce: SIG Mobilization Targets

Similar to Ways and Means, if we list Representatives sorted in descending order by SOV, legislators with higher SOV tend to have higher SIG mobilizations. Unlike Ways and Means, however, the legislator receiving the most SIG attention, Slattery, received a vanishing SOV. The Pearson correlation between SOV and SIG is 0.51.

Rep	Party	Name	ADA	NFIB	SOV	SIG
VA-8	D	Boucher, Rick	60	50	0.422	9
CA-19	D	Lehman, Richard	60	50	0.422	9
CA-49	D	Schenk, Lynn	65	30	0.059	0
TN-4	D	Cooper, Jim	70	70	0.054	10
NY-10	D	Towns, Edolphus	80	10	0.029	0
TX-5	D	Bryant, John	80	10	0.029	0
WA-9	D	Kreidler, Mike	80	10	0.029	0
CA-27	R	Moorhead, Carlos	5	90	0.016	7
FL-6	R	Stearns, Clifford	5	90	0.016	2
ID-2	R	Crapo, Michael	5	90	0.016	0

IL-14	R	Hastert, Dennis	5	90	0.016	0
NJ-6	D	Pallone, Jr., Frank	90	30	0.016	0
OH-4	R	Oxley, Michael	5	90	0.016	0
TX-6	R	Barton, Joe	5	90	0.016	0
TX-8	R	Fields, Jr., Jack	5	90	0.016	7
MI-6	R	Upton, Fred	30	70	0.013	0
CT-5	R	Franks, Gary	20	90	0.012	0
FL-9	R	Bilirakis, Michael	20	90	0.012	0
IL-7	D	Collins, Cardiss	85	10	0.012	0
IN-2	D	Sharp, Philip	85	10	0.012	0
MA-7	D	Markey, Edward	85	10	0.012	0
NY-7	D	Manton, Thomas	85	10	0.012	0
OH-5	R	Gillmor, Paul	20	90	0.012	0
PA-8	R	Greenwood, Jim	20	90	0.012	3
GA-8	D	Rowland, J.	50	70	0.011	0
LA-3	D	Tauzin, W.	15	70	0.011	4
MI-16	D	Dingell, John	90	10	0.011	0
TX-18	D	Washington, Craig	90	10	0.011	0
WA-2	D	Swift, Al	90	10	0.011	0
TX-4	D	Hall, Ralph	40	70	0.009	0
NM-3	D	Richardson, Bill	75	10	0.006	0
CA-29	D	Waxman, Henry	95	10	0.005	0
MA-10	D	Studds, Gerry	95	10	0.005	1
NC-9	R	McMillan, Alex	15	90	0.005	0
NY-27	R	Paxon, Bill	10	90	0.005	0
OH-13	D	Brown, Sherrod	95	10	0.005	0
OK-2	D	Synar, Michael	95	10	0.005	0
OR-3	D	Wyden, Ron	95	10	0.005	0
VA-7	R	Bliley, Jr., Thomas	15	90	0.005	0
AR-1	D	Lambert, Blanche	85	30	0.002	9
CO-6	R	Schaefer, Dan	5	50	0.000	0
KS-2	D	Slattery, Jim	75	50	0.000	13
PA-13	D	Margolies-Mezvinsky, Marjorie	80	30	0.000	0
WI-2	R	Klug, Scott	45	70	0.000	0

If we plot the SIG mobilization count against SOV value, using a log scale for SOV, we see again a pattern of two groups (Fig 27).

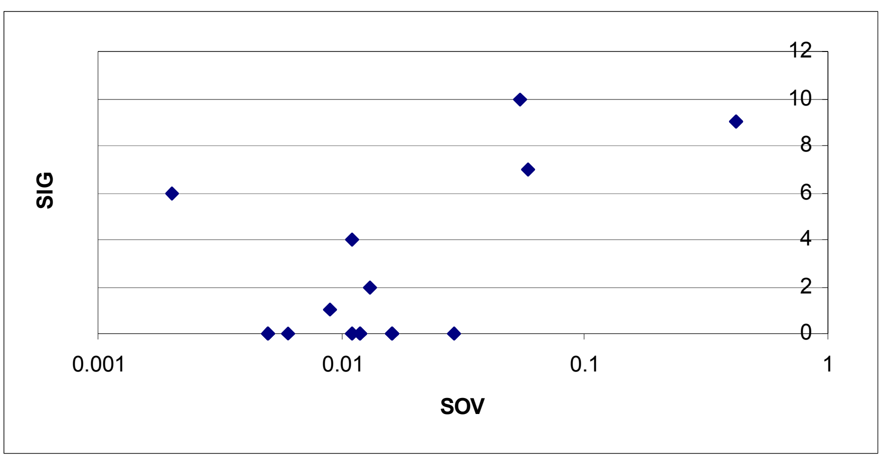


Figure 27 – House Energy & Commerce: Number of SIG Mobilized Versus Shapley-Owen Value

When compute the strong point, we find that it coincides almost exactly with the ideal point of Boucher and Lehman (Fig. 28). If we believe the strong point estimates the likely outcome of the committee, this means that Boucher and Lehman were effectively median voters for this committee. It was wholly unrealistic for Dingle to insist on reporting the bill with minimal markup. The best Dingle could have hoped to accomplish was literally ask Boucher and Lehman to mark up the bill to their satisfaction, perhaps asking Boucher and Lehman to consult with Slattery and Schenk to draw the ideal point of the proposal more toward the Democratic corner.

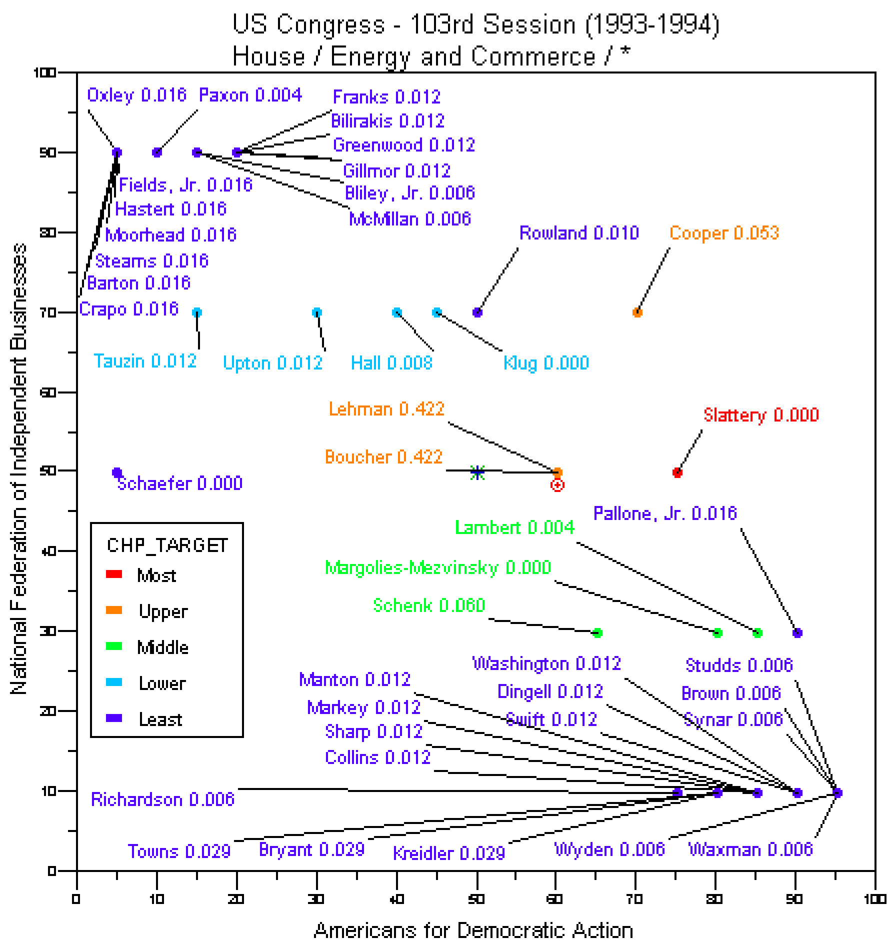


Figure 28 – House Energy & Commerce: Strong Point

Had Dingle effectively delegated markup to Boucher and Lehman the result would have been a bill whose ideal point matched fairly closely the corresponding bills reported by Senate Finance and House Ways and Means. Instead no bill was reported.

The result of the impasse in Energy and Commerce resulted in a breakdown of the usual legislative process. The House asked the Senate to work on the bill. The Senate leadership chose to disregard the bill reported by Senate Finance and restored the provisions Moynihan had, through compromise removed, ignoring the political information encoded in the Finance committee's strong point. The Senate leadership, in effect, followed Dingle in refusing to compromise. The result was a stillborn bill that ensured the defeat of the Clinton health initiative, a defeat with far reaching political implications for the Democratic Party.

Comparison with Goldstein's Model

Before moving on to our next case we consider a model developed by Goldstein for predicting SIG mobilization. According to Goldstein SIGs are more likely to mobilize for undecided legislators who occupy positions on influence committees and who are facing re-election. His reasoning is that legislators not facing re-election do not require the political information that SIGs can provide to help them form their opinion about an issue. Being undecided is a necessary condition for influence to matter. Finally, legislators not on influence committees are essentially irrelevant to shaping the final form of the bill before consideration on the floor of the House and Senate.

We have seen how a strong point analysis supports the differentiation between influence and policy committees, a component of Goldstein's model. On a heuristic basis we can argue that being undecided also translates into an SOV representation. A legislator could be undecided for any number of reasons, but surely a common or significant reason is that he/she cannot determine the relative positions of the proposal and status quo. Compounding this uncertainty, the location of proposal is subject to change under markup. Being undecided could be a strategic posture to exercise influence on the markup. But even so such legislators are not outliers in the issue space. Such legislators generally (though not always) have larger SOVs, per the Feld-Grofman bound. Legislators far away from the yolk center cannot have large SOVs, so the predominant contribution to the total sum, which must be 1, must come from legislators closer to the yolk center. Put simply, we expect the undecideds to reside closer to the yolk than those whose legislators who have made up their mind.

The one piece of information not available through SOV is whether or not a legislator is up for re-election. Surely this is an important political consideration. As Goldstein demonstrates, SIGs do not mobilize for legislators that are not up for re-election. From an efficiency perspective this makes sense. But there may be important exceptions representing a lost opportunity for SIGs. The examples of both Danforth and Coynes come to mind, particularly Coynes. Coynes reversed his position from being a co-sponsor to being an opponent. Somehow he was influenced, but not by mobilization.

Thus we see in all three committees that the SOV and strong point provide varying degrees of insight on the preferences shaping the Clinton health bill debate. Spatial models are only one way of considering these problems and may be subject to overly simplistic assumptions about legislator preferences. Nevertheless, an SOV-based analysis appears able to parallel Goldstein's model and identify/explain features of the data either not considered or not explained by Goldstein's model, the most dramatic example being the voting behavior of Coynes and Lewis.

We turn next to examples for which comparably rich preference and voting data are not available and furthermore rely on the extensions of SOV involving weighted voting models and plurality decision rules.

5.2 IMF – Weighted Voting Model

The IMF, formed by the Bretton Woods Conference of 1945, is a 184-nation body with all but two members enjoying voting rights. Two member nations currently have their voting rights suspended, Liberia and Zimbabwe. The IMF was founded after WWII to promote international financial practices to avoid a repeat of the Great Depression, but has become increasingly involved in loans to developing countries.

The IMF is a weighted, "Yes-No" voting system. Each nation receives a single vote, weighted according to the percentage contribution of that nation to the world GDP. A single proposal is considered at a time, upon which members vote "yes" or "no." Proposals are adopted according to simple majority rule. Constitutional changes and special majorities require a majority of 85%. The United States, with 17.1% of the votes, has an effective veto in such cases, a topic we do not explorer further in this case study. We do consider the relationship of veto power and weighted voting, however, in our next case study concerning the UN Security Council.

Unfortunately, as stated on the IMF web site (http://www.imf.org/external/pubs/ft/exrp/what.htm accessed 6/19/2005), "the Board rarely makes decisions based on formal voting; rather, most decisions are based on consensus among its members and are supported unanimously." This raises the level of difficulty in applying formal methods to analyzing the IMF. Due to the lack of empirical data and the unwieldy nature of analyzing the full body of the IMF, we will concentrate instead on the smaller, and administratively more significant IMF Executive Directors, i.e., the IMF Board.

Our aim will be to provide a plausible distribution of IMF Board member ideal points from which to develop hypotheses regarding voting weights in the context of a proximity spatial model (and relying on SOV measures). Much more research would be required to develop an empirically well-founded proximity spatial model of the IMF Board, assuming such a model can be developed and found compelling.

The IMF Board consists 24 directors, 5 appointed representatives from the United States, Japan, Germany, France, and Britain, and 19 elected representatives from 19 groups of countries. These groups are summarized below. The lines in gray identify the group, below which are enumerated the group members. Each of the elected groups is identified by the country of the currently elected representative of the group, followed by the geography of the majority of group members – except in cases where the group has only one member, Russia, China, and Saudi Arabia. The elected groups have formed more or less organically, outside any formal IMF framework, but do generally share a common geography. [Leech, 2002]

APPOINTED		**VOTE (WEIGHT)**
United States		17.08
United States		
Japan		6.13
Japan		
Germany		5.99
Germany		
France		4.95
France		

Britain		4.95
Britain		

ELECTED		VOTE WEIGHT
Belgium & Eastern Europe		5.13
Austria	Hungary	Slovak Republic
Belarus	Kazakhstan	Slovenia
Belgium	Luxembourg	Turkey
Czech Republic		
Netherlands & East Europe		4.84
Armenia	Cyprus	Moldova
Bosnia and Herzegovina	Georgia	Netherlands
Bulgaria	Israel	Romania
Croatia	Macedonia	Ukraine
Mexico & Central America		4.27
Costa Rica	Honduras	Spain
El Salvador	Mexico	Venezuela
Guatemala	Nicaragua	
Italy & South Europe		4.18
Albania	Malta	San Marino
Greece	Portugal	Timor-Leste
Italy		
Canada & North Atlantic		3.71
Antigua and Barbuda	Canada	Jamaica
Bahamas	Dominica	St. Kitts and Nevis
Barbados	Grenada	St. Lucia
Belize	Ireland	St. Vincent and the Grenadines
Norway & Northern Europe		3.51
Denmark	Iceland	Norway
Estonia	Latvia	Sweden
Finland	Lithuania	
Korea & South Pacific		3.33
Australia	Mongolia	Philippines
Kiribati	New Zealand	Samoa
Korea	Palau	Seychelles
Marshall Islands	Papua New Guinea	Solomon Islands
Micronesia		
Egypt & Arabia		3.26
Vanuatu	Kuwait	Qatar
Bahrain	Lebanon	Syria
Egypt	Libya	United Arab Emirates
Iraq	Maldives	Yemen
Jordan	Oman	
Saudi Arabia		3.22
Saudi Arabia		
Malaysia & South East Asia		3.17
Brunei Darussalam	Lao People's Democratic Republic	Singapore
Cambodia	Malaysia	Thailand
Fiji	Myanmar	Tonga
Indonesia	Nepal	Vietnam
Tanzania & Africa		3.00
Angola	Lesotho	South Africa
Botswana	Malawi	Sudan
Burundi	Mozambique	Swaziland
Eritrea	Namibia	Tanzania
Ethiopia	Nigeria	Uganda
Gambia	Sierra Leone	Zambia
Kenya		

China		2.94
China		
Switzerland & W & C Asia		2.84
Azerbaijan	Serbia and Montenegro	Turkmenistan
Kyrgyz Republic	Switzerland	Uzbekistan
Poland	Tajikistan	
Russia		2.74
Russian Federation		
Iran & C Asia & N Africa		2.47
Afghanistan	Iran	Pakistan
Algeria	Morocco	Tunisia
Ghana		
Brazil & South America		2.46
Brazil	Ecuador	Panama
Colombia	Guyana	Suriname
Dominican Republic	Haiti	Trinidad and Tobago
Indian Subcontinent		2.39
Bangladesh	India	
Bhutan	Sri Lanka	
Argentina & South America		1.99
Argentina	Chile	Peru
Bolivia	Paraguay	Uruguay
Equatorial Guinea & Africa		1.41
Benin	Congo, Republic of	Mali
Burkina Faso	Côte d'Ivoire	Mauritania
Cameroon	Djibouti	Mauritius
Cape Verde	Equatorial Guinea	Niger
Central African Republic	Gabon	Rwanda
Chad	Guinea	São Tomé and Príncipe
Comoros	Guinea-Bissau	Senegal
Congo, Democratic Republic of	Madagascar	Togo

Based on the general charter of the IMF and much of the public debate surrounding the IMF, we will assume the principal issue dimensions concern "free trade," the degree to which trade is regulated between member nations, and "easy money," the degree to which credit is made available to member nations.

We conjecture the location of member ideal point based on various trade reports, notably the Global Competitiveness Report 2003-2004 (GCR), news reports, and personal judgment [Salai-i-Martin, 2004]. The goal here is not so much to provide an accurate picture of the IMF as to assess the value potential to member states of policy positions they might adopt given their respective vote weights. In a particular, we want to explore the extent to which the United States can wield exceptional power and/or to which that power can be blocked by strategic behavior of other IMF members.

For the sake of concreteness and reproducibility we took two survey questions from the GCR and used these as surrogates for the "easy money" and "free trade" dimensions. The GCR was chosen largely due to its breadth of coverage and availability. The GCR is produced semi-annually based on an annual Expert Opinion Survey (EOS) conducted by the World Economic Forum (see http://www.weforum.org/ for more details) involving 7,500 business leaders and entrepreneurs.

The first question, identified as "2.07 Ease of access to loans," reads

"How easy is it to obtain a bank loan in your country with only a good business plan and no collateral (1=impossible, 7 = easy)."

We use this score as surrogate for "easy money" in the following sense. The higher respondents score a given country then the more is capital available in that country. Such countries will not have as high a demand for access to loans as countries where the score is lower. There could be, and likely are, other factors driving demand for/against "easy money." The biggest challenge in using this surrogate is that some countries, e.g., "developed countries," may recognize the need for "easy money" beyond their immediate national business needs. So using these data as a basis for "easy money" may be biased low for "developed countries."

The second question, identified as "10.09 Control of international distribution, " reads

" International distribution and marketing from your country (1= takes place through foreign companies, 7 = is owned and controlled by local companies.)"

We use this score as a surrogate for "free trade" in the following sense. The higher respondents score a given country then the fewer foreign companies operate within the country, giving natives less cause to feel threatened by foreign competition. This in turn will reduce the political pressure for protectionist trade policies. Conversely, in countries more heavily penetrated by foreign companies, we expect greater political pressure against free trade policies. Again, there could be, and likely are, other factors driving demand for/against "free trade."

In order to use the GCR data, some data preparation was needed. First, scores were converted from a 7 point Likert scale to the 100 point scale being used consistently throughout this paper. Second, not all IMF members are included in the GCR. Rather than attempt imputation, however, the average of available scores for members in each of the IMF Board groups was used. Generally this seemed appropriate. The largest gaps concerned groups of Arabian states and African states. The states for which data were available seemed representative based on news reports and personal judgment. One imputation was required, namely, for Saudi Arabia. Values close to the Arabian group were used for this purpose.

The final scales are defined as follows:

```
Easy Money = 100 *(1 – [Q02.07]/7)
Free Trade = 100 *(     [Q10.09]/7)
```

Below is a diagram of these data prepared as described (Fig 29). The IMF Board members are identified along with their SOVs. A color-code is also used to indicate the SOV magnitude, with light green in this case being the highest values and blue the lowest. Observe that the largest SOVs are associated with *elected*, not appointed IMF Board members. Thus, despite the combinatorics advantage the United States has in its voting power, it does not actually realize that advantage under the posited configuration.

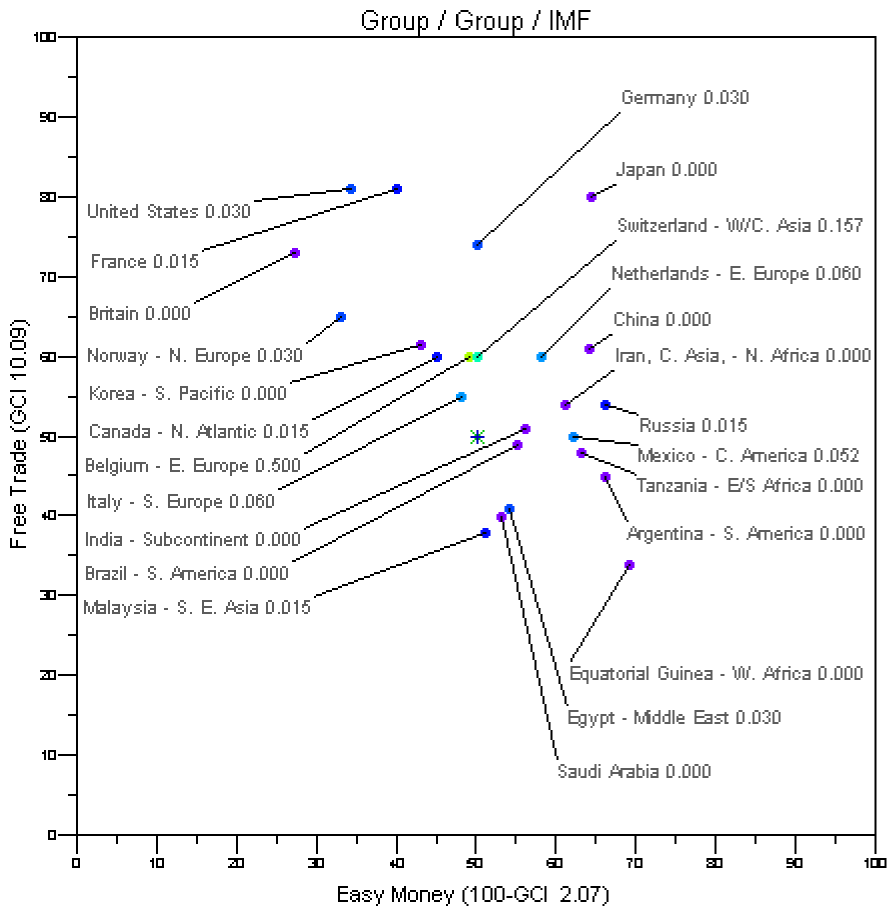


Figure 29 – IMF Board: Free Trade vs. Easy Money - Initial

Is this configuration reasonable? In general placement of ideal points seems plausible. Some adjustments might be called for, particularly for the developed countries as regards "easy money." Britain's recent encouragement of the United States to extend more aid/loans to Africa, for example, suggests that Britain's ideal point is too far left. Similarly, Northern European countries have a strong reputation for foreign aid. So the Northern European ideal point may be too far left as well. And finally, France's recent rejection of the EU Constitution based in part on free trade concerns, suggests that France's ideal point may be too high on the free trade scale. On the other hand, the positioning of developing members seems generally accurate. In particular, members who have recently experienced economic hardship have ideal points further down and to the right.

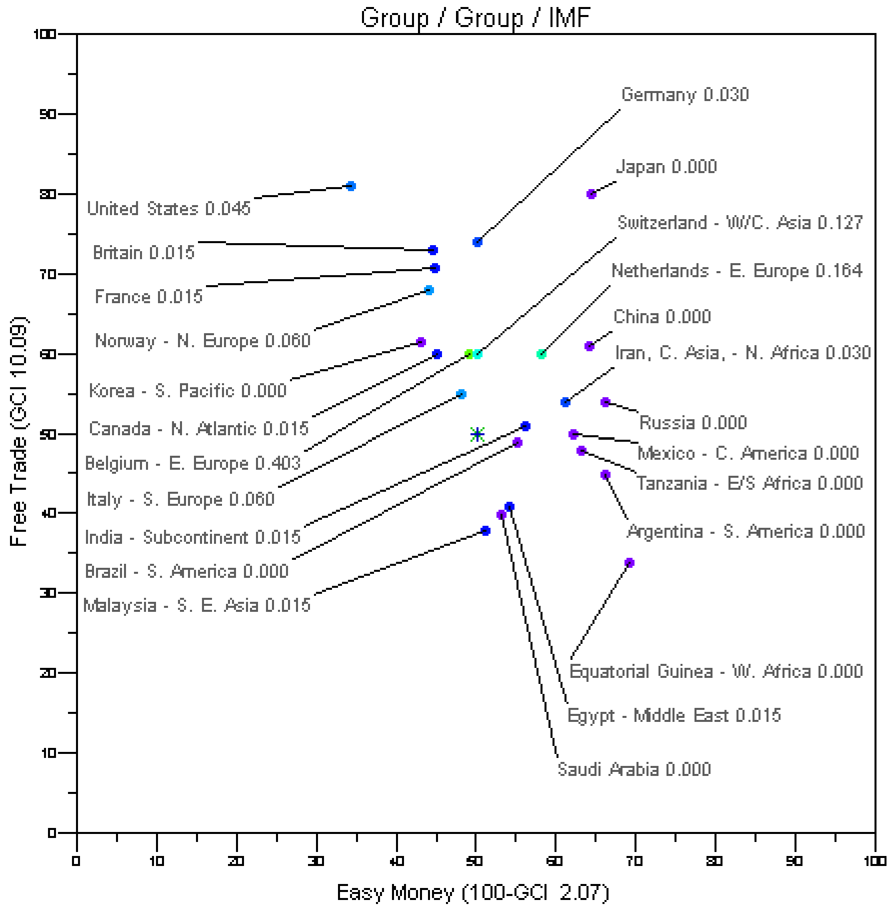


Figure 30 – IMF Board: Free Trade vs. Easy Money - Improved

Making these adjustments, the United States' SOV has actually increased, but is still relatively low due to its marginal position relative to other ideal points (Fig 30).

Undoubtedly arguments can be made for further refinements. Unfortunately, as noted earlier, we cannot hope with the data available to have an accurate/precise model for IMF Board member preferences. Our aim here is only to provide a plausible distribution of IMF Board member ideal points; which we believe we have done.

Let us now consider various modifications of the IMF Board member model. First, suppose we move the United States to the strong point. Observe that the United States commands nearly all the value (Fig 31).

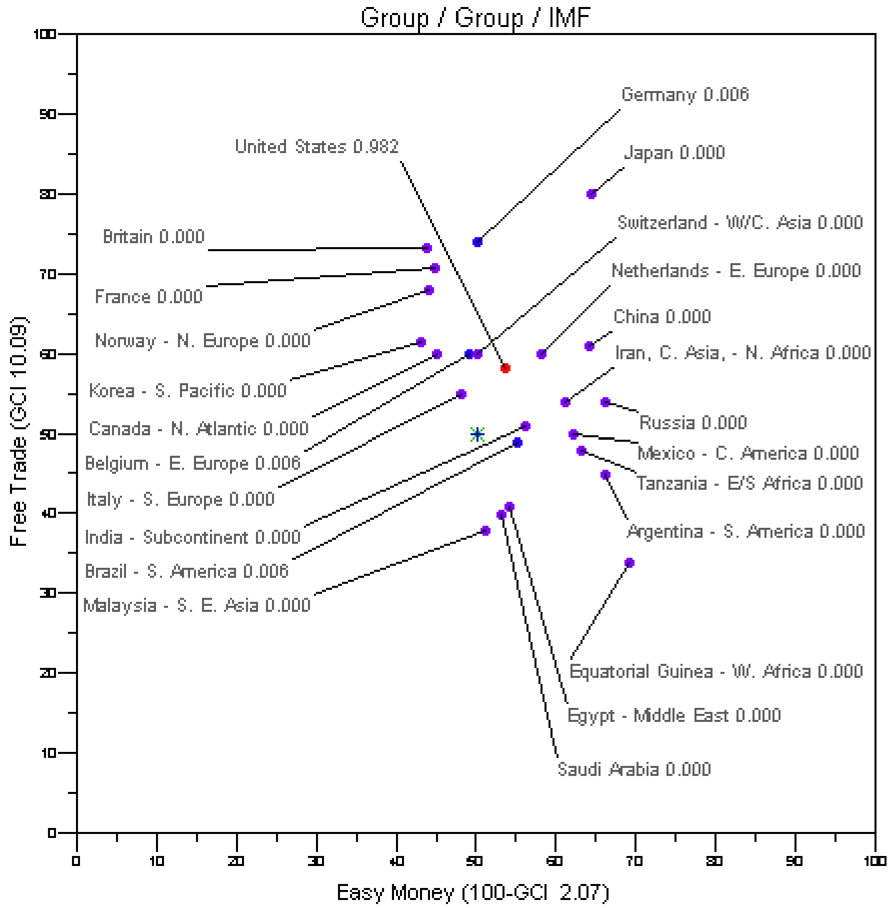


Figure 31 – IMF Board: Hypothetical Adjustment of United States' Ideal Point

Instead, suppose we move to the strong point another appointed member, one with a quarter of the vote weight, e.g., Britain. Observe that Britain only gains about a quarter of the total SOV and that the strong point is shifted – toward the United States (Fig. 32). Thus while the United States does not command a significant amount of value in this scenario, the United States nevertheless has a powerful influence on the value other players can realize and the location of the strong point.

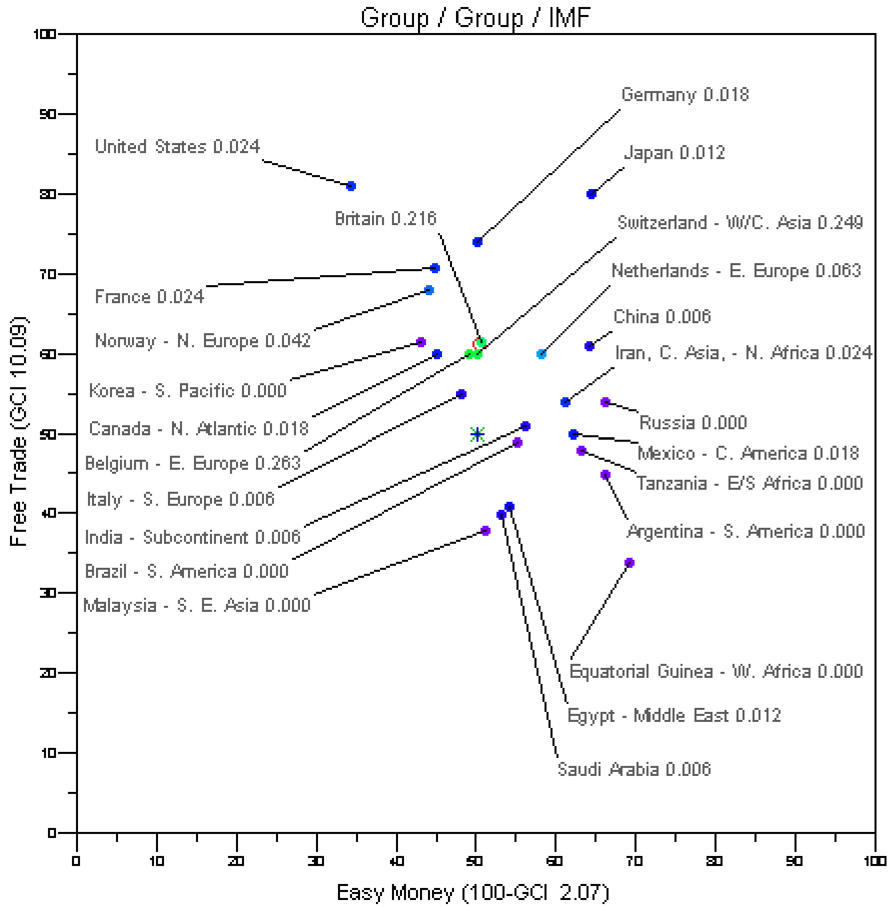


Figure 32 – IMF Board: Hypothetical Adjustment of Britain's Ideal Point

A number of scholars and critics of the Bretton Woods plan, notably Keynes, have argued that the IMF weighting system is flawed, particularly the large vote weight accorded the United States. While on its face weighting votes according to the financial contribution each member makes to the IMF (a percentage of GDP) seems fair, voting power, the ability to form coalitions and influence policy outcomes, does not necessarily scale linearly with such weighting schemes. This, at least, is the basis for a positive critique. Normative critiques may consider other factors such a population and development status.

Leech, using the Banzhaf-Coleman index, has performed a positive analysis and offers these conclusions [Leech 2002]:

1. Countries' voting powers over ordinary decisions are much more unequal than their financial contributions; the power of the USA is much greater than its nominal 17% of the votes.
2. The effect of the special 85% majority requirement for major decisions is to severely limit the effectiveness of the decision-making system.
3. The use of the 85% majority requirement is counterproductive to the US pursuing an active role in the IMF by limiting its power to have its policies accepted.

4. The IMF should make all decisions by simple majority and scrap special majorities. That would make its democratic decision making system most effective.
5. The United States should support the use of simple majorities for all decisions if it wishes to increase its influence within a democratic IMF.
6. Votes of all members and executive directors should be reweighted in order to give the desired share of voting power to each country and director.

Others, notably Timothy Lane (from the IMF), have suggested reweighting taking into account population and development status [Lane, 2004]

We consider next a redistribution of voting power. We begin with Leech's prescription of re-apportioning weights and conclude with considerations of population as suggested by Lane. We re-apportion weights using exponential smoothing, resulting in a drop of the United States vote weight from 17.1% to 12.5%. Much of the difference is taken up by Japan, Germany, Belgium & Eastern Europe, France, and Britain, but the lowest weight members, Argentina – South America and Equatorial Guinea - Africa also gain a small amount. We use our initial configuration of ideal points as a convenient, reproducible baseline for comparison. (Fig. 33)

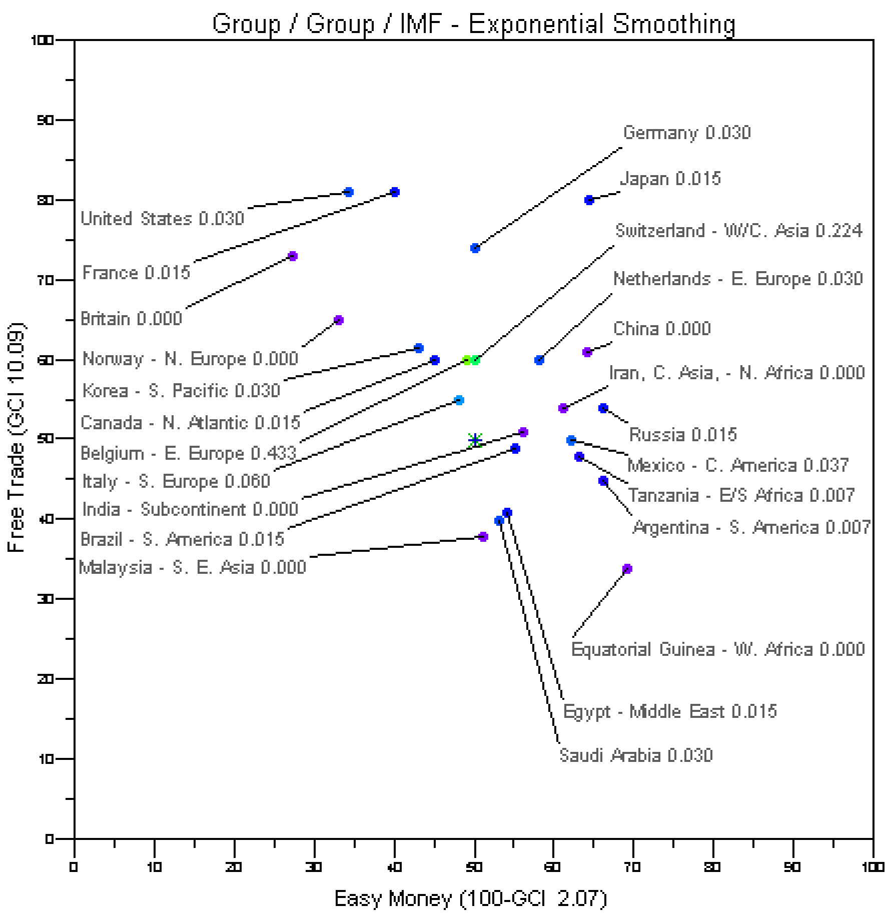


Figure 33 – IMF Board: Vote Weight Adjustment by Exponential Smoothing

Fundamentally nothing changes. The United States has exactly the same SOV as it did in the initial configuration. Argentina – South America has acquired a slight non-zero SOV, but Equatorial Guinea – Africa remains at zero. Thus, despite the United States giving up nearly a third of its voting share, a substantial redistribution of value has not occurred. There is, however, a major difference. In this scenario the United States cannot veto. Perhaps under such a dramatic re-apportionment greater use would be made of simple majority, as opposed to special majority, for voting decisions. This might actually favor the interests of United States, as suggested by Keynes, given that the United States' SOV does not change between scenarios.

Suppose now we adjust by hand for the populations of India and China. We consider this in addition to the re-apportionment, as that reform did not have an appreciable effect. Perhaps this one will. In this scheme 0.5% is taken from the United States, 0.3% from Japan, and 0.2% from German with 0.5% being given to China and India respectively (Fig. 34).

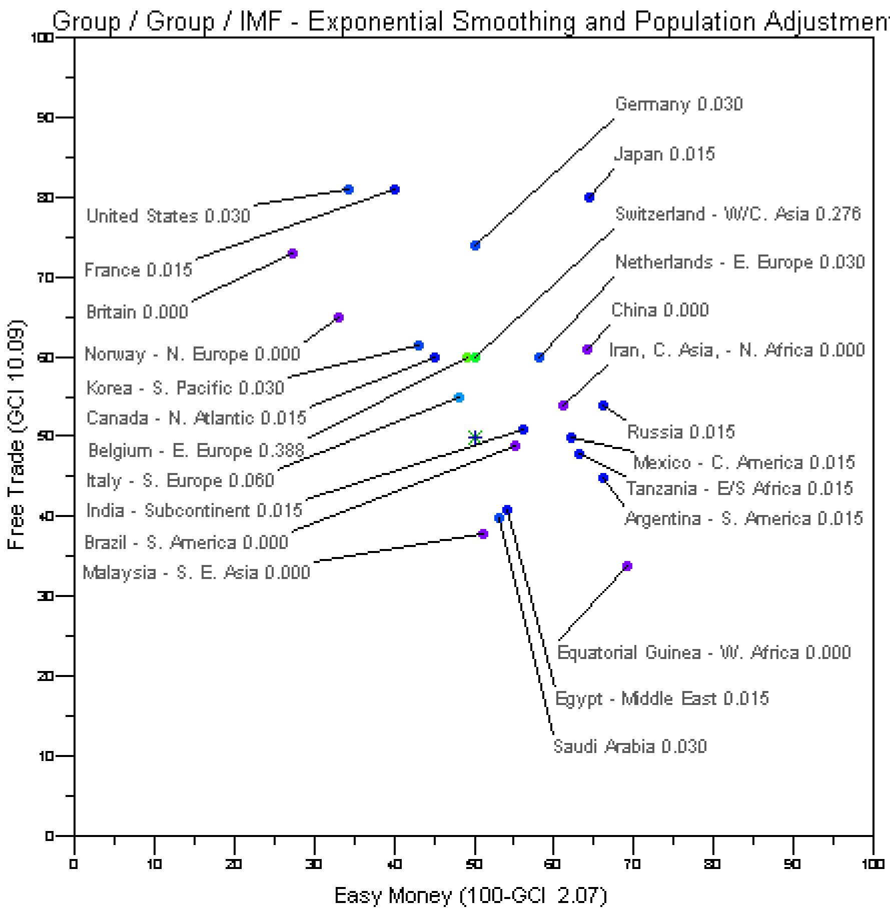


Figure 34 – IMF Board: Vote Weight Adjustment by Population

Again, this has no appreciable affect on the SOVs of the United States, Japan, and Germany, but India – Subcontinent has benefited. China, on the other hand, continues to have zero SOV.

What can we conclude? It appears that the spatial context, the *spatial constraint* on coalition formation, is a significant determinant of value. Here the Shapley-Owen value, and other power indexes of its conception, differentiates from purely combinatorics-based indexes such as the Shapley-Shubik or Banzhaf-Coleman index. The value associated with a combinatorics-based index is fixed regardless of any strategies adopted by players. The Shapley-Owen value, on the other hand, can vary dramatically depending on the strategy choices of players. Thus whereas the United States has substantial, fixed combinatorics-based value, the value realized in decisions modeled accurately by proximity spatial voting models can be much less depending on the strategic choices of the United States and other voters.

The one consideration that cannot be overcome through strategic activity in a spatial context is when a single voter has enough votes to defeat any measure it finds objectionable. If the IMF vote allocation were dynamic such that as nations states increased their GDP there resulted a redistribution of votes, the veto power would be dissipated.

5.3 UN Security Council – Weighted Voting Model Used to Represent Veto Power

For our final case study we look more closely at the question of veto power. For this purpose we use of UN Security Council as a subject. This is probably one of the most closely watched deliberative bodies. We aim to show that the Shapley Owen value algorithm accurately describes the decision-making characteristics of this body.

The UN Security Council consists of 5 permanent members and 10 non-permanent members elected by the UN General Assembly for a two-year term.

Permanent Members	Non-Permanent Members
United States, Russia, China, France, Britain	Argentina, Benin, Brazil, Denmark, Greece, Japan, Philippines, Romania, Tanzania, Algeria

Each council member has one vote. Procedural decisions require 9 affirmative votes. Substantive decisions require 9 votes and concurrence of all 5 permanent members, i.e., a non-concurrence by any permanent member is a "veto."

The UN Security Council is "Yes-No" voting system, with veto. Such systems can be represented as a weighted "yes-no" voting system [Taylor, 1995]. We are interested in such a representation so we may apply our SOV algorithm to the UN Security Council to see what we might learn.

Let us assert that non-permanent members are assigned a voting weight of 1 and the permanent members a voting weight of x. Let q denote the number of votes required to adopt a proposal, the quota. Consider the case of substantive issues. Suppose one of the permanent members exercises the veto then, even if all other permanent and non-permanent members vote affirmative, the UN Security Council will be one vote short of quota.

$4x + 10 = q-1.$

On the other hand, suppose 5 permanent members and 4 non-permanent members vote affirmative, meeting the Charter threshold of 9 affirmative votes, then

$5x + 4 = q.$

These equations are readily solved, yielding $x = 7$ and $q = 39$. Thus, assigning a weight of 7 to permanent members, 1 to non-permanent members, and a quota of 39, the votes of the UN Security Council can be faithfully reproduced.

Upon plotting the UN Security Council in an abstract two-dimensional issue space and experimenting with ideal point location, we quickly discover that value generally resides near the boundary of the Pareto set, not its interior. The following graphic in which each permanent member occupies a corner of a regular pentagon illustrates the point (Fig. 35).

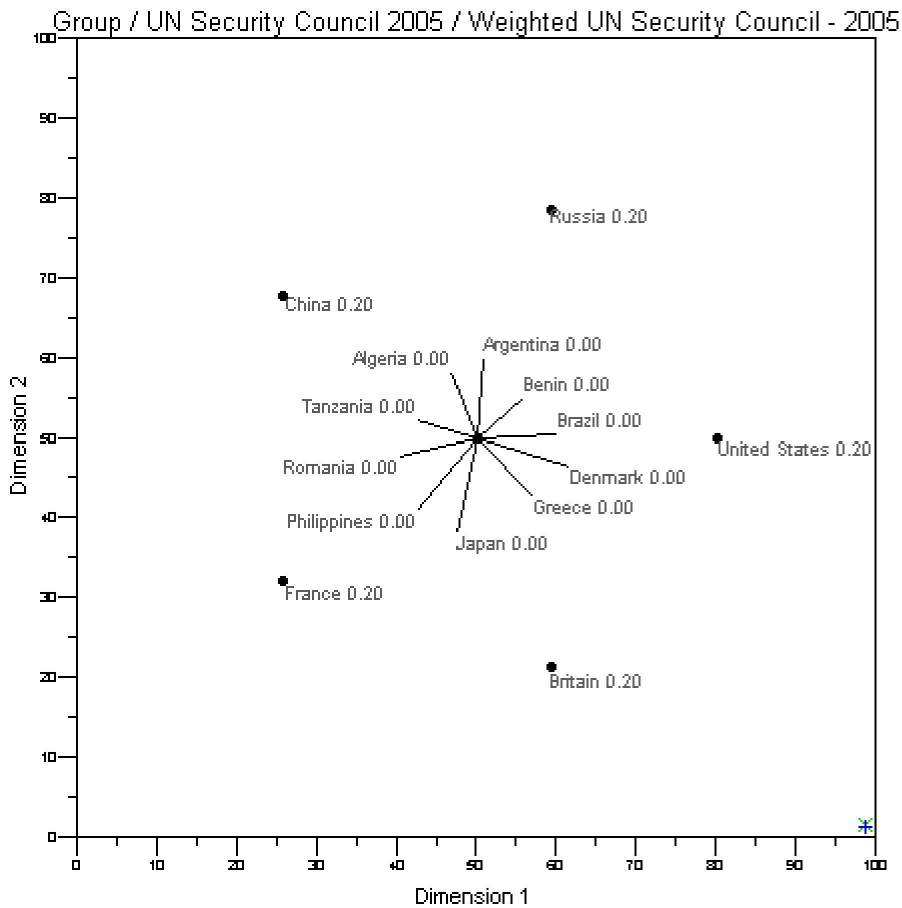


Figure 35 – UN Security Council: Shapley Owen Values

It is possible for a member to realize higher value than the symmetric value of 0.20 found in this scenario. To do so, the member must be a permanent member and be an outlier among the permanent members, as reckoned by a measure of central tendency such as the yolk center. For example, in the following scenario we have moved Britain into the lower right corner (Fig. 36).

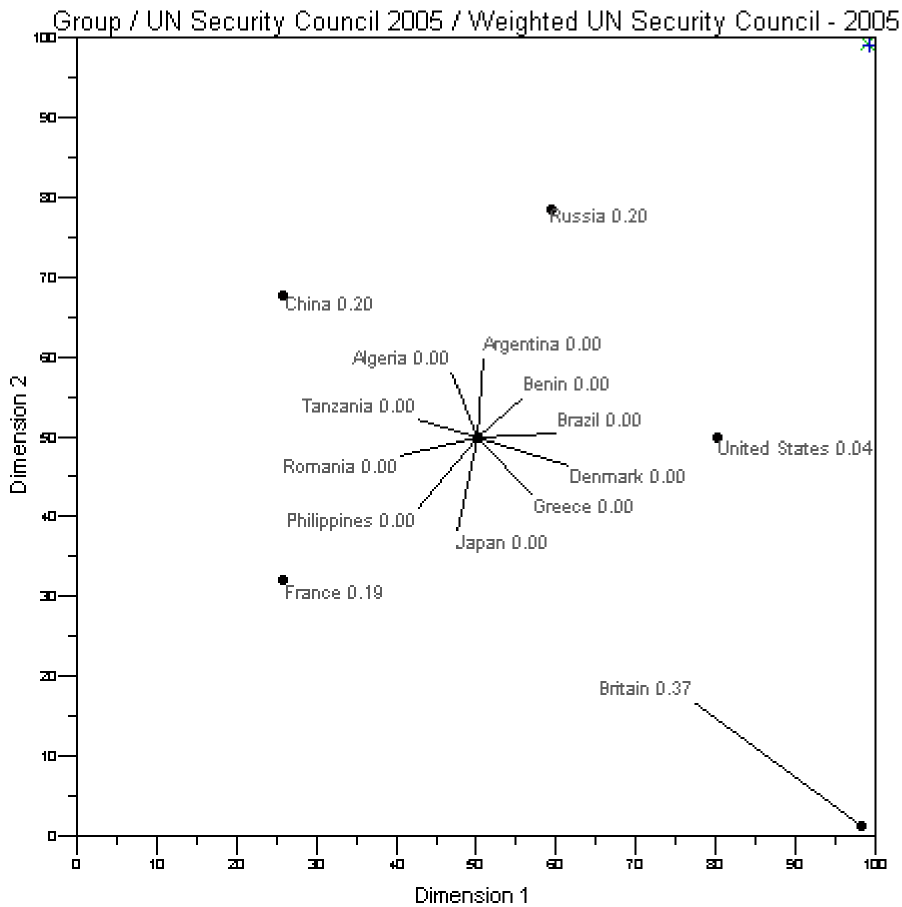


Figure 36 – UN Security Council: Veto Favors Extreme Positions

Observe that Britain has nearly doubled its SOV, principally at the expense of the United States, but also France.

Experiments such as this suggest that value in the UN Security Council comes from staking out unique, relatively extreme positions. We can understand this effect as being due in large part to a decision rule that requires a super-majority. Thus for any direction in the decision space there are two pivots, unlike the case of a simple majority in which there is only one. Relative to the median these pivots are more toward the tails of the voter distribution.

In the diagram below, voters are projected on to the blue line (Fig. 37). The red dashed lines indicated the locations of the pivots on the blue line and pass through the ideal point of the corresponding voter as a way to identify the pivot graphically.

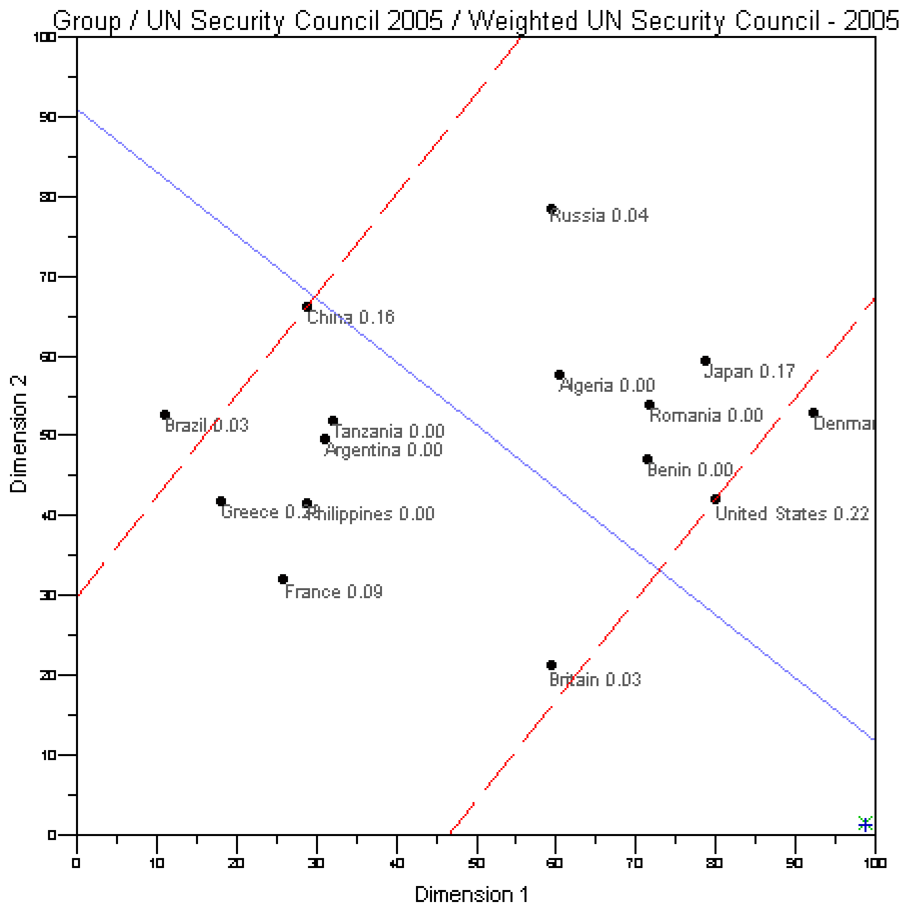


Figure 36 – UN Security Council: Super-Majority Pivots

In this case China and the United States are pivots. Their large SOV is not due to being pivots on *this* line, but by being pivots on a large percentage of *all* lines. Observe that Japan also has a large SOV, comparable to China.

Which pivot position will win in any given vote depends, of course, on the location of the status quo. If the status quo is between the pivots, neither pivot will win. The status quo will prevail. If the status quo, however, is more extreme than either pivot, the pivot nearer the status quo will prevail. Thus, the highest payoffs are for relatively extreme positions. Extreme positions are only tenable, however, when the status quo has for some reason shifted dramatically outside the Pareto set.

Our SOV analysis of the UN Security Council leads us to the conclusion that the decision rule adopted by the UN for the Security Council is very conservative. Apart from routine decisions in which consensus can be achieved, the only circumstances in which the UN Security Council will vote for a proposal against the status quo is when the status quo has dramatically changed just prior to the period of deliberation, e.g., a crisis, usually outside the Pareto set. The status quo must be outside the permanent members' Pareto set or a permanent member would veto. There are configurations involving non-permanent members where the status quo can be inside the Pareto set and still be defeated.

This description agrees with the behavior of the UN Security Council, which gives some confidence the SOV algorithm, extended to weighted voting models and to super-majority decision rules behaves reasonably, at least in broad terms.

6 Conclusions

The Shapley-Owen value represents an important example of a probabilistic generalization to the Shapley-Shubik value. The Shapley-Owen value takes into account the preferences of voters in determining the likelihood of coalition formation according to a proximity spatial voting model. The spatial proximity serves as a *constraint* that greatly reduces the sample space.

We have shown how to interpret the Shapley-Owen value through the familiar model of the median voter. Using the metaphor of a line rotating at a constant speed in a two-dimensional space on to which all voters are projected for each increment of rotation, the Shapley-Owen value of a voter is just the amount of time that voter occupies the pivotal position on the line. We have taken this model and used it as the basis for implementing an algorithm for computing Shapley-Owen values in two-dimensional spatial voting games.

We argued that our algorithm was on its face valid. We then checked that the implementation functioned properly by considering a number of test cases. These test cases also served to illustrate properties of the Shapley-Owen value, some perhaps unexpected. We then pushed the envelope and considered a weighted voting model, a problem that goes beyond the theoretical underpinnings of Shapley-Owen. The extension appears to work except in the case when coincident voters have different vote weights. This case was not considered in the paper.

Finally we applied our algorithm to three examples, one for which we had sufficient data for a basic analysis, a second for which we could hypothesize data, and a third essentially speculative but useful for structural analysis. These cases we believe provide evidence that the Shapley-Owen value can be a powerful tool for understanding coalition formation in two-dimensional group decision-making. Specifically, we found evidence that in weighted voting, issue framing and strategic behavior can be used to offset the advantages of voters with vote weight advantages. We also observed that plurality decision rules (often equivalent to a veto when combined with weighted voting) have a tendency to promote extreme policy positions.

We close by offering some general questions and observations:

1. Much of the theoretical literature is based on Euclidean preferences. But even in the one case study we could offer based on actual data, this assumption did not hold for all voters. How critical is this assumption?
2. How far can the Shapley-Owen theory be generalized by weakening requirements on ideal point topology, metric types, etc.? How useful is the algorithm in exploring these questions?

3. There appear to be two separate but related issues with the Shapley Owen value, calculation of the strong point (some sort of expectation value for a proximity spatial voting model), and the probability a voter is pivotal in any given vote. It seems intuitive that the more likely a voter is to determine the outcome, i.e., be pivotal, the more influence that voter should have on the outcome. This is roughly what the strong point formula says, and accords with Shapley's model. It would be interesting, however, to have this result for more complex models than those in which every voter has Euclidean preferences.
4. The algorithm seems to generalize nicely to situations not covered by the analytic assumptions of Shapley and Owen. The UN Security Council results may seem trivial in the sense that these findings would be clear without a Shapley-Owen analysis. However, the fact that Shapley-Owen produces the same result shows that weighted voting and plurality rules can be successfully analyzed using Shapley-Owen values. The challenge now is to find cases studies with real data!
5. Much of the work in spatial vote models is theoretical/hypothetical. There is a great need for data such as produced by Goldstein to put these ideas to rigorous empirical testing.

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