

## “Voting power: notes for a revision of the paradigm”

By Annick Laruelle & Federico Valenciano

### Extended abstract

It seems that it was Shubik who called Shapley's attention to the fact that as a voting rule can be described as a simple TU game the Shapley value could be applied to it. The 'slight of hands' consisted of producing a game where there were no players! Then applying to it the recently introduced Shapley value the Shapley-Shubik index was born. They interpret it as an evaluation of a priori 'voting power' in the committee. As the marginal contribution of a player to a coalition in this game can only be 0 or 1, being 1 only when the presence/absence of a player in a coalition makes it winning/losing, they also propose an interpretation in terms of likelihood of being pivotal or decisive. Hence the seminal duality or ambiguity:

*The Shapley-Shubik index is a 'value' of a sort of bargaining situation, or an assessment of the likelihood of being decisive?*

Later Banzhaf adheres consistently the point of view of power as decisiveness, and criticizes the Shapley-Shubik index in view of the unnatural probability underlying its probabilistic interpretation in the context of voting. If power meant being decisive, then a measure of power is the probability of being it. Thus an a priori evaluation of power, if a priori all vote configurations are equally probable, is the probability of being decisive under this assumption. (Independently, Penrose in 1946 reached basically the same conclusion in a narrower formal setting.) In fact Banzhaf only 'almost' says so for he destroys this clean probabilistic interpretation by 'normalizing' this vector.

So the old dispute is served: *Which is better Shapley-Shubik index or the Banzhaf index? What are more relevant axioms or probabilities?*

But in order to solve the dilemma a more basic issue is to be addressed: *What are we talking about?* In lieu of proceeding from abstract terms as 'power' related to an exceedingly broad class of situations (any collective body that makes decisions by vote) and getting entangled with big words, we think it wiser to

1. *start by setting the analysis of voting situations as the central goal*

Collective decision-making by vote may include an extremely wide and heterogeneous constellation of voting situations: a parliament law-making, a parliament vote for the endorsement of a government, a referendum, a presidential election, a governmental cabinet decision-making, a shareholders' meeting, an international or intergovernmental council, and a huge et cetera enter it. By setting the analysis of voting situations as the central goal, instead of the abstraction 'voting power', we mean

2. *starting from clear-cut models of well specified clear-cut voting situations,*

instead of starting from words denoting hardly specified abstractions in vaguely specified situations. For instance, it is not the same a committee with capacity to bargain the proposal before voting, that one only allowed to accept or reject proposals by vote. It is not the same millions of voters than a few, etc.

A dichotomy (only provisional, and open to become part of a wider ‘multichotomy’), consistent with the above principles would distinguish between two types of voting situations or committees: ‘take-it-or-leave-it’ committees vs. ‘bargaining’ committees.

2.1 *In a ‘take-it-or-leave-it’ committee:* (i) The committee votes upon different independent proposals over time; (ii) proposals are submitted to the committee by some external agency; and (iii) the committee is only entitled to accept or reject proposals, but cannot modify them.

A ‘take-it-or-leave-it’ committee raises among others the following issues:

-The question of ‘success’: Positive and normative assessment of the likelihood of a voter being in the winning side in a vote. This is the central issue from the point of view of the voters, instead of their ‘decisiveness’ in what traditional voting power has mainly concentrated.

-The question of the optimal (and in what sense) voting rule in a ‘take-it-or-leave-it’ committee of representatives in which every member acts on behalf of a constituency of different size.

A natural way to enlarge the framework is including (in addition to the voting rule) a second input in the specification of a voting situation of this type: a probability distribution over all possible vote configurations. This simple step allows for a clear differentiation of the notions of decisiveness and success, a trivialization of paradoxes, and a formal connection with a non a priori approach.

2.2. *In a ‘bargaining’ committee:* (i) The committee deals with different issues over time; (ii) for every issue a different configuration of preferences emerges in the committee over the set of feasible agreements concerning the issue at stake; and (iii) the committee bargains about every issue in search of a unanimous agreement, in search of which is entitled to adjust the proposal.

A ‘bargaining’ committee raises among others the following issues:

-The question of ‘bargaining capacity’ and the precise meaning of this term: Positive and normative assessment of the bargaining capacity that the voting rule in the committee confers to its members.

-The question of the optimal voting rule in a bargaining committee of representatives. To begin with: Optimal in what sense?

A way to approach this type of voting situation is by introducing a second ingredient in the model (in addition to the voting rule): a set of feasible payoff vectors associated with the configuration of preferences over the feasible agreements. This, assuming rational players in the sense of von Neumann-Morgenstern, yields a generalization of classical Nash’s bargaining model, and allows for addressing a game theoretic, cooperative and non cooperative, foundation of an assessment of the ‘bargaining power’ (in the precise game-theoretic sense of this term) that the voting rule confers to the voters. This provides also grounds for a normative recommendation for the choice of rule in a bargaining committee of representatives.

Though in reality it is often the case that a same committee acts sometimes like a ‘take-it-or-leave-it’ committee, and others like a bargaining committee, this crisp

differentiation of two clear-cut types of situation provides benchmarks for a better understanding of many mixed real world situations.