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# The underlying assumptions of electoral systems (Extended abstract)*

My main thesis is that we have to make a strict distinction between two kinds of consideration in choosing a voting/election procedure:

- Political: including criteria ranging from the pragmatic to the philosophical.
- Social-choice criteria in the narrow sense: the logical–mathematical properties of a voting system, the pathologies and paradoxes that afflict it.

Social-choice theorists must realize that political considerations are paramount in choosing a voting procedure. It is the politicians who usually make the choice; and even when the choice is made by referendum, the question put to referendum is framed by politicians.

But politicians and their advisors must be aware of the logical–mathematical properties of the voting procedures in question; otherwise they can easily walk into a trap. So it is wrong to dismiss these matters as of interest only to geeks.

I would like to highlight two dichotomies. In each of them, the choice is political.

1. **Proportional Representation (PR) v District Representation (DR)**

(This dichotomy is relevant only to electing an assembly, not a holder of an individual post, such as a president.)

The choice here is based on two distinct concepts of representation: PR conceives of the assembly as a microcosm of the electorate. A member of the assembly represents an ideologically homogeneous but geographically dispersed constituency. DR conceives of a member as an agent of an ideologically diverse but geographically contiguous constituency, of which s/he is in some sense the "best" representative.

2. **Deterministic voting v probabilistic (lottery) voting.**

(Note: in fact even deterministic systems use a lottery to resolve ties; but this is an extremely unlikely event.)

Thus we have a four-fold division.

What does social choice have to offer in these four cases?

## *PR + Determinism*

The only electoral system that really satisfies this choice (as far as possible) is the list system.

(Contrary to frequently heard claims, STV is not a PR system.)

## *PR + Lottery*

There is a solution to this choice: the lottery voting procedure proposed by A R Amar (1984). It has many of the virtues of DR, but produces, with very high probability, a very good approximation to proportionality. (Proof uses Kolmogorov's Law of Large Numbers.)

Turning to DR systems, we introduce a subsidiary politically-based dichotomy:

**Majoritarianism (majority rule) v aggregation of voters' degree of approval/preference.**

Degree of approval can be ordinal, cardinal or "in between" (as in grading by marks that are not reducible to cardinal numbers.) But in any case they require or imply at least a transitive weak ordering of the candidates by each voter

Aggregation systems pose two distinct problems.

First, can degrees of approval/preference of different voters be meaningfully aggregated? (This problem is familiar in relation to utilities, which are cardinal preferences; but it is more general.)

Second, loss of information in aggregating. Arrow's theorem is but a manifestation of this. It applies only to systems that try to aggregate ordinal preferences (preference orderings) into a "social" ordering. But the problem is more general.

Majoritarian systems are based on the political view that regards majority rule (MR) as a paramount principle.

MR is clear enough when there are just two candidates. The straightforward natural generalization of this is Condorcet's Principle: If candidate A dominates candidate B (ie, A is preferred to B by a majority of the voters), then A is socially preferable to B.

Note that in order to apply this rule, it is not necessary in principle for a voter to order the candidates in a (transitive) preference ordering. Only pairwise comparisons are needed. And a voter's comparisons may well be cyclic. (Contrary to widespread view, this need not be irrational.)

## *MR + determinism*

If one candidate needs to be elected, this provides a solution only if a Condorcet winner exists. Otherwise, MR needs to be supplemented by some concept of aggregation – opening the problems that afflict aggregation.

## *MR + Lottery*

Here there is a unique optimal solution, provided by a theorem proved in 1991/2 by Laffond, Laslier and Le Breton; and independently by Fisher and Ryan.

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