Note on the Background and Purpose of the 2010 VPP Workshop

Assessing Alternative Voting Procedures

1. Foreword

Nearly six decades have now elapsed since Kenneth Arrow (1950, 1951) proved his rather pessimistic impossibility theorem. And nearly four decades have passed since Allan Gibbard (1973) and Mark Satterthwaite (1975) proved that all reasonable voting procedures involving three or more candidates and two or more voters are susceptible to strategic manipulation.

Although it has since been clear that any majoritarian voting procedure that can be devised for electing one out of three or more options must suffer from certain deficiencies (often referred to as 'paradoxes'), it has also been intuitively clear:
1) that not all possible deficiencies that may afflict such voting procedures are equally undesirable;
2) that not all voting procedures are susceptible to the same deficiencies;
3) that, *ceteris paribus*, the likelihood of at least some deficiencies occurring under several voting procedures which are susceptible to them is not the same;
4) and that some voting procedure(s) may be considered as more desirable than others in satisfying certain criteria.

Defenders of a particular voting procedure may argue that the paradoxes afflicting this procedure are not a problem because seemingly they can occur only very infrequently. They would, we presume, claim that a few contrived examples should not deter us from using this procedure. Moreover, some authors believe that there exists sufficient homogeneity of voters' opinions to get rid of paradoxes while using majority rule (see Dasgupta and Maskin, 2004, 2008).

As a result of such arguments, one encounters in the social-choice literature five different methods for estimating the likelihood of the various paradoxes under selected voting procedures. All these methods invoke simplifying assumptions and suffer from various drawbacks.

The first, and most prominent, method is to use mathematics to compute exactly the likelihood of the pathologies (paradoxes) of various voting procedures under several basic probabilistic assumptions regarding voters' preferences. This method stems from the works of Fishburn and Gehrlein (see, for instance Fishburn, 1974; Fishburn and Gehrlein, 1982). The employed mathematics can be rather advanced. For instance, results were recently obtained via Ehrhart polynomials (see Lepelley, Louichi, and Smaoui, 2008). However, the limitations of this method are that the underlying probabilistic assumptions regarding voters' preferences are usually restricted to two frameworks: Impartial Culture (IC) or Impartial Anonymous Culture (IAC), both based on some kind of equiprobability – which is not very realistic.

The second, and most common, of these methods uses computer simulations (see, for instance, Bordley, 1983; Hoffman, 1983; Merrill, 1984, 1985; Niemi and Frank, 1985; Nitzan, 1985; Niemi and Wright, 1987; Felsenthal, Maoz, and Rapoport, 1990; Mitchell and Trumbull, 1992). Similar to the first method, the main drawbacks of this method are that it, too, usually assumes all possible preference orderings that voters may have among the competing candidates to be both complete and equally likely; or if there are too many possible orderings, that the ones under investigation constitute a random sample from a well-defined population. Clearly, these simplifying assumptions are not necessarily realistic.

A third method is to conduct controlled laboratory experiments where voters' preference orderings are held constant and their behavior under various voting procedures is observed (see, for example, Felsenthal, Rapoport, and Maoz, 1988; Rapoport, Felsenthal, and Maoz, 1991). The main problems with this method are that it must be limited to small (and usually unrepresentative) samples, and the voters' preference orderings must be induced artificially.

A fourth method is to conduct a survey in which a representative sample of voters are asked how they would vote under various procedures for a given set of candidates (see, for example, Chamberlin, Cohen, and Coombs, 1984; Felsenthal, Maoz, and Rapoport, 1986; Fishburn and Little, 1988; Rapoport, Felsenthal, and Maoz, 1988; Felsenthal, 1992; Brams and Fishburn, 2001; Balinski and Laraki, 2007b; Laslier and Van der Straeten, 2008; Baujard and Igersheim, 2009). The main problem with this method is the one common to survey research in general: answers given to hypothetical questions may not be a good predictor of real action.

Under the fifth method one extrapolates from real outcomes obtained under one voting procedure the likely outcomes, *ceteris paribus*, that would be obtained under other voting procedures (see, for example, Feld and Grofman, 1992; Felsenthal, Maoz, and Rapoport, 1993; Felsenthal and Machover, 1995). Two difficulties are associated with this method. First, a prerequisite for conducting extrapolations from an observed procedure to other procedures is that the voters' preference orderings among the candidates under the procedure actually used are known. Since most real elections are conducted under non-ranked procedures, data on actual preference orderings rarely exist for relatively large electorates. Moreover, in the relatively few instances where ranked procedures are used (mostly the STV procedure), the individual voters' ballots – as distinct from the aggregate results – are usually not made available to researchers. Second, and similar to the other four research methods, the reasonableness of the extrapolation results may depend crucially on the reasonableness of the underlying assumptions; for example, the assumption that the (observed) voters' preference orderings under one procedure are unlikely to change under other procedures. It may be objected that, because of strategic reasons, some voters might decide to misrepresent their true preference orderings – in different manners – under different procedures.

Moreover, most of the studies conducted under all of the above five research methods, attempted to verify the likelihood of what we call below '*simple*' or '*straightforward*' paradoxes, e.g., the relative frequency of a top cycle in the social preference ordering, or the relative frequency in which a Condorcet winner is not elected under various procedures when such a winner exists. To the best of our knowledge, hardly any studies were conducted in order to estimate the relative occurrence of the type of paradoxes we call below '*conditional*' paradoxes. Thus, for example, is it indeed true that serious flaws such as lack of monotonicity or the no-show paradox afflicting the often used plurality with runoff procedure, or the alternative vote procedure, are sufficiently rare as to cause no practical concern? Is non-monotonicity equally likely under Dodgson's and Nanson's procedures, both of which are Condorcet-efficient procedures?

Thus the purpose of this workshop is twofold:

a) *To try and reach a consensus among the participants regarding the relative degree of severity which may be attributed to the main paradoxes afflicting voting procedures designed to elect one out of three or more candidates*. For example, *ceteris paribus* should we prefer a voting procedure which guarantees the election of a Condorcet winner when one exists but is susceptible to lack of monotonicity (like Nanson's procedure), or should we prefer a procedure which is not susceptible to non-monotonicity but may not elect even a strong Condorcet winner when one exists, i.e., a candidate who constitutes the top preference of an absolute majority of the voters (like Borda's count procedure)?

b) *To try and formulate necessary and/or sufficient condition(s) for the occurrence of the main paradoxes under each voting procedure listed below that is susceptible to it, or at least outline a research program that would ultimately result in the formulation of such conditions*. For example, it is known that a necessary condition for both Nanson's and Dodgson's procedures to display instances of non-monotonicity is that a top cycle exists in the social preference ordering. But this does not yet tell us which of these procedures is more likely, *ceteris paribus*, to display instances of non-monotonicity. Similarly, it is known that, *ceteris paribus*, a necessary and sufficient condition for a Condorcet winner not to be elected under the plurality procedure (when one must elect one out of three or more candidates) is that there exists another candidate who is ranked first by a plurality of the voters, but that this condition is necessary, but insufficient, for a Condorcet winner not to be elected under the STV procedure; consequently it can be concluded that, *ceteris paribus*, the likelihood of a Condorcet winner not being elected under the plurality procedure is larger than under the STV procedure.

We therefore list below the main paradoxes with which we wish to be concerned. Thereafter we list the main voting procedures proposed in the literature for electing one out of three or more candidates and indicate in a summary table, with respect to each of these procedures, the paradoxes to which it is susceptible.[1]

2. Voting Paradoxes

We define a 'voting paradox' as an undesirable outcome that a voting procedure may produce and which may be regarded, at least by some people, as surprising or as counter-intuitive at first glance.

We distinguish between two types of voting paradoxes:

a) '*Simple*' or '*straightforward*' paradoxes: These are paradoxes where the relevant data leads to a 'surprising' and arguably undesirable outcome. (The relevant data include, *inter alia*, the voting procedure used, the number of voters, the number of candidates, the number of candidates that must be elected, the preference ordering of every voter among the competing candidates, the amount of information voters have regarding all other voters' preference orderings, the order in which voters cast their votes if it is not simultaneous, the order in which candidates are voted upon if candidates are not voted upon simultaneously, whether voting is open or secret, the manner in which ties are to be broken).

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[1] We focus our attention on paradoxes associated with voting procedures designed to elect a *single* candidate because most of the literature on voting paradoxes is concerned with these procedures. However, we welcome also contributions participants may wish to make regarding paradoxes associated with voting procedures that are designed to elect more than one candidate.

b) '*Conditional*' paradoxes: These are paradoxes where changing one relevant datum while holding constant all other relevant data leads to a 'surprising' and arguably undesirable outcome.

An array of paradoxes of one or both types are described and analyzed by McGarvey (1953); Riker (1958), Smith (1973), Fishburn (1974, 1977, 1981, 1982), Young (1974), Niemi and Riker (1976), Doron and Kronick (1977), Doron (1979), Richelson (1979), Gehrlein (1983), Fishburn and Brams (1983), Saari (1984, 1987, 1989, 1994, 2000), Niou (1987), Moulin (1988a), Merlin and Saari (1997); Brams, Kilgour and Zwicker (1998), Scarsini (1998); Nurmi (1998a, 1999, 2007); Lepelley and Merlin (2001); Merlin, Tataru, and Valognes (2002); Merlin and Valognes (2004), among others.

The most well known 'simple' paradoxes that may afflict voting procedures designed to elect one out of three or more candidates are the following:

1. *The Condorcet (or voting) paradox* (Condorcet, 1785; Black, 1958): Given that the preference ordering of every voter among the competing candidates is transitive, the (amalgamated) preference ordering of the majority of voters among the competing candidates may nevertheless be intransitive. All known voting procedures suffer from this paradox.

2. *The Condorcet Winner paradox* (Condorcet, 1785; Black, 1958): An alternative *x* is not elected despite the fact that *x* is preferred by a majority of the voters over each of the other competing alternatives.

3. *The Condorcet Loser or Borda paradox* (Borda, 1784; Black, 1958): An alternative *x* is elected despite the fact that a majority of voters prefer each of the remaining alternatives to *x*.

4. *The Absolute Majority paradox*: An alternative *x* may not be elected despite the fact that it is the only alternative ranked first by an absolute majority of the voters.

5. *The Absolute Loser paradox*: An alternative *x* may be elected despite the fact that it is ranked last by a majority of voters.

6. *The Pareto (or Dominated Candidate) paradox* (Fishburn, 1974): An alternative *x* may be elected while alternative *y* may not be elected despite the fact that *all* voters prefer alternative *y* to *x*.

Similarly, the most well known 'conditional' paradoxes that may afflict voting procedures designed to elect one out of three or more candidates are the following:

1. *Additional Support (or Lack of Monotonicity) paradox* (Smith, 1973): If candidate *x* is elected under a given distribution of voters' preferences among the competing candidates, it is possible that, *ceteris paribus*, *x* may not be elected if some voter(s) *increase(s) his (their) support for x* by moving *x* to a higher position in his (their) preference ordering.

2. *Reinforcement (or Inconsistency or Multiple Districts) paradox* (Young, 1974): If *x* is elected in each of several districts, it is possible that, *ceteris paribus*, *x* will not be elected if all districts are combined into a single district.

3. *Truncation paradox* (Brams, 1982; Fishburn and Brams, 1983): A voter may obtain a more preferable outcome if, *ceteris paribus*, he lists in his ballot only part of his preference ordering among some of the competing candidates than listing his entire preference ordering among all the competing candidates.

4. *No-show paradox* (Fishburn and Brams, 1983; Ray, 1986; Moulin, 1988b, Holzman, 1988/9). This is an extreme version of the truncation paradox. A voter may obtain a more preferable outcome if he decides not to participate in an election than, *ceteris paribus*, if he decides to participate in the election and vote sincerely for his top preference(s).

5. *Twin paradox* (Moulin, 1988b): This is a special version of the no-show paradox. Two voters having the same preference ordering may obtain a preferable outcome if, *ceteris paribus*, one of them decides not to participate in the election while the other votes sincerely.

6. *Violation of the WARP axiom* (Richter, 1966): Candidate x may be elected when there are m candidates ($m > 2$), but may not be elected if, *ceteris paribus*, some candidate(s) other than x drop(s) out of the race. Such a paradox is considered as violation of an axiom called Weak Axiom of Revealed Preferences (WARP).

7. *Lack of Path Independence paradox* (Farquharson, 1969; Plott, 1973): If the voting on the competing candidates is conducted sequentially rather than simultaneously, it is possible that candidate x will be elected under a particular sequence but not, *ceteris paribus*, under an alternative sequence.

8. *Strategic voting paradox* (Gibbard, 1973; Satterthwaite, 1975): *Ceteris paribus*, a voter may obtain a preferred outcome if he votes strategically, i.e., not according to his true preferences. All known voting procedures suffer from this paradox.

3. Voting Procedures for Electing One out of Three or More Candidates

A. Non-Ranked Voting Procedures

1. *Plurality* (or *first past the post*) *voting procedure*: This is the most common procedure for electing a single candidate. Under this procedure every voter casts one vote for a single candidate and the candidate obtaining the largest number of votes is the elected candidate.

2. *Plurality with a Runoff*: Under the usual version of this procedure up to two voting rounds are conducted. In the first round each voter casts one vote for a single candidate. In order to be declared a winner in the first round a candidate must obtain a minimal percentage of the votes (usually at least 40%). If no candidate is declared the winner in the first round then a second round is conducted. In this round only the two candidates who obtained the highest number of votes in the first round participate, and the one among them who obtains the majority of votes is declared the winner.

3. *Approval Voting* (Brams and Fishburn, 1978, 1983): Under this procedure every voter has a number of votes which is equal to the number of competing candidates, and every voter can cast one vote or no vote for every candidate. The candidate obtaining the largest number of votes is the elected candidate.

4. *Successive Elimination* (Farquharson, 1969): This procedure is common in parliaments when voting on alternative versions of bills. According to this procedure voting is conducted in a series of rounds. In each round two alternatives compete; the one obtaining fewer votes is eliminated and the other competes in the next round against one of the alternatives which has not yet been eliminated. The alternative winning in the last round is the ultimate winner.

B. Ranked Voting Procedures that are Condorcet-Consistent

All voting procedures listed in this section require that voters rank order all competing candidates. Thereafter all $m(m-1)/2$ pairwise comparisons (where m is the number of candidates) are conducted between each candidate and every other candidate. If there exists a candidate such that an absolute majority of the voters prefer it to each of the other candidates then this candidate (called the *Condorcet winner)* is elected; otherwise there exist various (deterministic) proposals as to what ought to be the social preference ordering. We list below some of these proposals.

1. *Condorcet's procedure*: Condorcet's procedure is a maximin procedure since it chooses that candidate whose worst showing against the others is as good as possible.

2. *Dodgson's procedure* (Black, 1958, pp. 222-234; McLean and Urken, 1995, pp. 288-297): This procedure is named after the Rev. Charles Lutwidge Dodgson, a.k.a. Lewis Carroll, who proposed it in 1876. It elects the Condorcet winner when one exists. If no Condorcet winner exists it elects that candidate who requires the fewest number of switches (i.e. inversions of two adjacent candidates) in the voters' preference orderings in order to make him the Condorcet winner.

3. *Nanson's Method* (Nanson, 1883; McLean and Urken, 1995, ch. 14). Nanson's method is a recursive elimination of Borda's method. In the first step one calculates for each candidate his Borda score. In the second step the candidate(s) whose Borda score does not exceed the average Borda score of the candidates in the first step are eliminated from all ballots and a revised Borda score is computed for the uneliminated candidates. The elimination process is continued in this way until one candidate is left. If a (strong) Condorcet winner exists then Nanson's method elects him.[2]

4. *Copeland's Method* (Copeland, 1951): Every candidate x gets one point for every pairwise comparison with another candidate y in which an absolute majority of the voters prefer x to y, and half a point for every pairwise comparison in which the number of voters preferring x to y is equal to the number of voters preferring y to x. The candidate obtaining the largest number of points is the winner.

5. *Black's Method* (Black, 1958): According to this method one first performs all pairwise comparisons to verify whether a Condorcet winner exists. If such a winner exists then he is elected. Otherwise the winner according to Borda's count (see below) is elected.

[2]Although Nanson's procedure satisfies the strong Condorcet condition, i.e., it always elects a candidate who beats every other candidate in pairwise elections, this procedure may not satisfy the weak Condorcet condition which requires that if there exist(s) candidate(s) who is (are) unbeaten by any other candidate then this (these) candidate(s) – and only this (these) candidate(s) – ought to be elected. For an example of violation of the weak Condorcet condition by Nanson's procedure see Niou (1987).

6. *Kemeny's Method* (Kemeny, 1959): Kemeny's method specifies that up to $m!$ possible social preference orderings should be examined (where m is the number of candidates) in order to determine which of these is the "most likely" true social preference ordering. The selected "most likely" social preference ordering according to this method is the one where the sum of voters that prefer every alternative x over each of the alternatives ranked below x in the social preference ordering is maximized.

7. *Schwartz's Method* (Schwartz, 1972): Schwartz's method is based on the notion that a candidate x deserves to be listed ahead of another candidate y in the social preference ordering if and only if x beats or ties with some candidate that beats y, and x beats or ties all candidates that y beats or ties.

8. *Young's method* (Young, 1995): Young's method is like Dodgson's in the sense that it is based on altered profiles that have candidates who lose to no other candidate under simple majority. But unlike Dodgson, Young deletes voters rather than inverting preferences to obtain the altered profiles. His procedure suggests that one remains most faithful to Condorcet's Principle if the elected candidate is the one who can become a simple majority nonloser with removal of the fewest number of voters.

C. Ranked Voting Procedures that are Not Condorcet-Consistent

1. *Borda's Count* (Borda, 1784; Black, 1958): Each candidate x is given a score equal to the sum of voters who prefer x to each of the other candidates, and the candidate with the largest score is elected. Equivalently, each candidate x gets no points for each voter who ranks x last in his preference ordering, 1 point for each voter who ranks x second-to-last in his preference order, and so on, and m-1 points for each voter who ranks x first in his preference order (where m is the number of candidates). Thus if all n voters have linear preference orderings among the m candidates then the total number of points obtained by all candidates is equal to the number of voters multiplied by the number of pairwise comparisons, i.e., to $n\ [m(m-1)/2]$.

2. *Single Transferable Vote* (STV). This procedure was first proposed by Thomas Hare in England and Carl George Andrae in Denmark in the 1850s. When used for electing a single candidate (in which case this procedure is called *Alternative Vote* or *Instant Runoff*) it works as follows. In the first step one verifies whether there exists a candidate who is ranked first by an absolute majority of the voters. If such a candidate exists he is declared the winner. If no such candidate exists then, in the second step, the candidate who is ranked first by the smallest number of voters is deleted from all ballots and thereafter one again verifies whether there is now a candidate who is ranked first by an absolute majority of the voters. The elimination process continues in this way until a candidate who is ranked first by an absolute majority of the voters is found.

3. *Coombs' Method* (Coombs, 1976; Straffin, 1980; Coombs, Cohen, and Chamberlin, 1984). This procedure is similar to STV except that the elimination in each round under Coombs' method involves the candidate who is ranked last by the largest number of voters (instead of the candidate who is ranked first by the smallest number of voters under STV).
4. *Range Voting*: The suitability (or level of performance) of every candidate is assessed by every voter and is assigned a (cardinal) grade (chosen from a pre-specified range) reflecting the candidate's suitability or level of performance in the eyes of the voter. The candidate with the highest average grade is the winner.

5. *Majority Judgment* (Balinski and Laraki, 2007): The suitability (or level of performance) of every candidate is assessed by every voter and is assigned an ordinal grade (chosen from a pre-specified range) reflecting the candidate's suitability or level of performance in the eyes of the voter. The candidate with the highest median grade is the winner.

Summary Table: Susceptibility of Several Voting Procedures to Various Voting Paradoxes

Paradox \ Procedure	Plurality	Plurality with Runoff	Approval Voting	Successive Elimination	Range Voting	Majority Judgment	Condorcet	Dodgson	Black	Copeland	Kemeny	Nanson	Schwartz	Young	Borda	STV	Coombs
Condorcet Pdx (Cyclical Majorities)	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
Condorcet Winner Pdx	+	+	+	–	+	+	–	–	–	–	–	–	–	–	+	+	+
Absolute Majority	–	–	+	–	+	+	–	–	–	–	–	–	–	–	+	–	–
Condorcet Loser	+	–	+	–	+	+	–	–	–	–	–	–	–	–	–	–	–
Absolute Loser	+	–	+	–	+	+	–	–	–	–	–	–	–	–	–	–	–
Pareto	–	–	+	+	–	–	–	–	–	–	–	–	–	–	–	–	–
Lack of Monotonicity	–	+	–	–	–	–	–	+	–	–	–	+	–	–	–	+	+
Reinforcement	–	+	–	+	–	+	+	+	+	+	+	+	+	+	–	+	+
No-Show	–	+	–	+	–	+	+	+	+	+	+	+	+	+	–	+	+
Twin	–	+	–	+	–	+	+	+	+	+	+	+	+	+	–	+	+
Truncation	–	–	–	+	+	+	+	+	+	+	+	+	+	+	+	+	+
WARP	+	+	+	+	–	–	+	+	+	+	+	+	+	+	+	+	+
Path Independence	–	–	–	+	–	–	–	–	–	–	–	–	–	–	–	–	–
Strategic Voting	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

Note: A plus sign indicates that a procedure is vulnerable to the specified paradox and a minus sign indicates that a procedure is not vulnerable to the specified paradox. It is assumed that all voters have linear preference ordering among all competing alternatives.

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