


Voting Power in Practice Symposium  
20-22 March 2011, London School of Economics  
A prob. re-view on F&M's Meas. of Voting Power

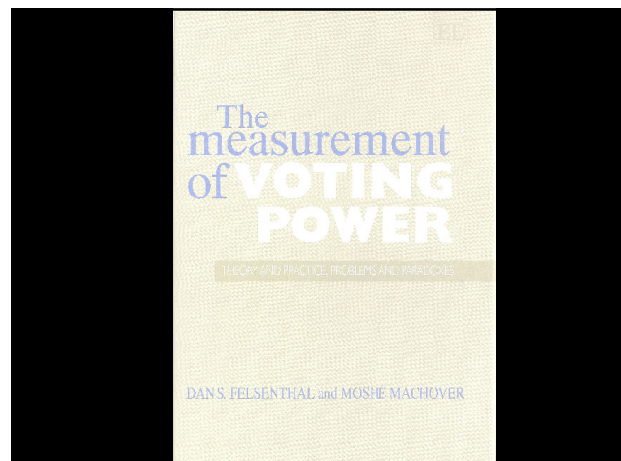
Professor Dr. Friedrich Pukelsheim  
Lehrstuhl für Stochastik und ihre Anwendungen  
Universität Augsburg



A probabilistic 're-view' on  
F&M's *Measurement of Voting Power*

1. The Book  
2. Probability models

2




Contents

List of Tables	ix		
List of Abbreviations	xi		
Preface	xiii		
1 Introduction	1		
1.1 What this Book is About	1		
1.2 Historical Sketch	6		
2 Groundwork of the Theory	11		
2.1 Simple Voting Games	11		
2.2 Comments on the Basic Definition	16		
2.3 Supplementary Definitions	22		
3 Power as Influence	35		
3.1 I-Power: A Probabilistic Notion	35		
3.2 The Banzhaf Measure	38		
3.3 Sensitivity	52		
3.4 The Two Square-Root Rules	63		
4 Weighted Voting in the US	79		
4.1 One Person, One Vote	79		
4.2 The Impact of John Banzhaf	91		
4.3 Local Government	96		
4.4 Basic Ratings	102		
4.5 Nassau County, NY: A Case Study	117		
4.6 Concluding Comments	122		
	vii		
		viii	Contents
		5	Weighted Voting in the CMEC
		5.1	Legislative Process of the EC
		5.2	Evolution of Weighted Voting in the CM
		5.3	Weight, Population and Voting Power
		6	Power as a Prize
		6.1	P-Power: A Game-Theoretic Notion
		6.2	The Shapley Value
		6.3	The Shapley-Shubik Index
		6.4	Other Proposed Indices
		7	Paradoxes and Postulates
		7.1	Preliminaries
		7.2	The Paradox of Large Size
		7.3	The Paradox of Redistribution
		7.4	The Paradox of New Members
		7.5	The Quarrelling Paradoxes
		7.6	Dominance, The Paradox of Weighted Voting
		7.7	The Meet Paradox
		7.8	The Transfer Paradoxes: Donation and Bloc
		7.9	The Blocker's Share and Added Blocker Postulates
		7.10	Concluding Comments
		8	Taking Abstention Seriously
		8.1	Why Bother?
		8.2	Terrary Voting Rule
		8.3	Voting Power under TVR's
		A	Appendix: Numerical Examples
		B	Appendix: Axiomatic Characterisations
			List of US Court Cases Cited
			Bibliography
			Technical Index
			General Index
			319

Voting Power in Practice Symposium  
20-22 March 2011, London School of Economics  
A prob. re-view on F&M's Meas. of Voting Power

Professor Dr. Friedrich Pukelsheim  
Lehrstuhl für Stochastik und ihre Anwendungen  
Universität Augsburg




Achievements of *The Book*:

- Conceptual meaning
- Mathematical theory
- Real applications

6

Voting Power in Practice Symposium  
20-22 March 2011, London School of Economics  
A prob. re-view on F&M's Meas. of Voting Power

Professor Dr. Friedrich Pukelsheim  
Lehrstuhl für Stochastik und ihre Anwendungen  
Universität Augsburg



Voting weight      Stimmgewicht


↓                      ↓

Voting power      Stimmkraft

7

Voting Power in Practice Symposium  
20-22 March 2011, London School of Economics  
A prob. re-view on F&M's Meas. of Voting Power

Professor Dr. Friedrich Pukelsheim  
Lehrstuhl für Stochastik und ihre Anwendungen  
Universität Augsburg



1. The Book  
2. Probability models

8

Voting Power in Practice Symposium  
20-22 March 2011, London School of Economics  
A prob. re-view on F&M's Meas. of Voting Power

Professor Dr. Friedrich Pukelsheim  
Lehrstuhl für Stochastik und ihre Anwendungen  
Universität Augsburg

- Assembly
- Profile space
- Regions of growing acceptance

$N = \{1, 2, \dots, n\}$

$\Omega_N = \{-1, 0, +1\}^N \ni a = (a_j)_{j \in N}$

$[a, 1_N] = \{b \in \Omega_N \mid a \leq b \leq 1_N\}$

9

Voting Power in Practice Symposium  
20-22 March 2011, London School of Economics  
A prob. re-view on F&M's Meas. of Voting Power

Professor Dr. Friedrich Pukelsheim  
Lehrstuhl für Stochastik und ihre Anwendungen  
Universität Augsburg

$W_N \subseteq \Omega_N$  is a "decision rule" when

- (1)  $1_N \in W_N$  and (2)  $0 \notin W_N$  and
- (3)  $\forall a \in W_N : [a, 1_N] \subseteq W_N$

Events of interest:

$C_j = \{a \in \Omega_N : j \text{ is critical in } a\}$

$A_j = \{a \in \Omega_N : j \text{ agrees with } a\}$

10

Voting Power in Practice Symposium  
20-22 March 2011, London School of Economics  
A prob. re-view on F&M's Meas. of Voting Power

Professor Dr. Friedrich Pukelsheim  
Lehrstuhl für Stochastik und ihre Anwendungen  
Universität Augsburg

Probabilities of interest:

$P(W_N)$  efficiency of  $W_N$

$P(C_j)$  influence probability of  $j$

$\sum_{j \in N} P(C_j)$  influence sensitivity of  $W_N$

$P(C_j) / \sum$  power share of  $j$

$P(A_j)$  success probability of  $j$

11

Voting Power in Practice Symposium  
20-22 March 2011, London School of Economics  
A prob. re-view on F&M's Meas. of Voting Power

Professor Dr. Friedrich Pukelsheim  
Lehrstuhl für Stochastik und ihre Anwendungen  
Universität Augsburg

Penrose/Banzhaf  $P_N$  (no abstentions):

Theorem (4 in Ruff/P. 2010)

If  $P$  is a partitioning of  $N$  and each bloc  $B$  in  $P$  has its decision rule  $W_B$  then, for all  $j \in A \in P$ :

$P_N(C_j) = P_A(C_j | W_A) \cdot Q_P(C_A | W_P)$

where  $Q_P = \otimes_{B \in P} \text{Bernoulli}(P_B(W_B))$

12

Voting Power in Practice Symposium  
20-22 March 2011, London School of Economics  
A prob. re-view on F&M's Meas. of Voting Power

Professor Dr. Friedrich Pukelsheim  
Lehrstuhl für Stochastik und ihre Anwendungen  
Universität Augsburg

Penrose/Banzhaf  $P_N^t$  (with abstentions):

Theorem (5.2 in Birkmeier/Käufel/P. 2011)

With abstention probability  $t \in [0, 1)$ :

$$\sum_{j \in N} P_N^t(C_j(W_N)) = \frac{1}{1-t} E_{P_N^t}[\sigma_{W_N}]$$

13

Voting Power in Practice Symposium  
20-22 March 2011, London School of Economics  
A prob. re-view on F&M's Meas. of Voting Power

Professor Dr. Friedrich Pukelsheim  
Lehrstuhl für Stochastik und ihre Anwendungen  
Universität Augsburg

Shapley/Shubik  $S_N^t$  (with abstentions):

Theorem (in Diss. Birkmeier 2011)

With abstention probability  $t \in [0, 1)$ :

$$\sum_{j \in N} S_N^t(C_j(W_N)) = \frac{1-t^n}{1-t}$$

$$= 1 + t + t^2 + \dots + t^{n-1}$$

14