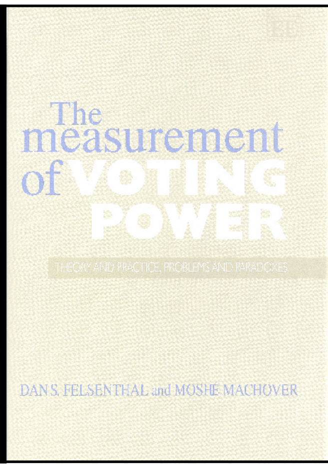
A probabilistic `re-view´on
F&M's *Measurement of Voting Power*

1. The Book
2. Probability models



Contents

vii

viii *Contents*

Achievements of *The Book*:

- Conceptual meaning
- Mathematical theory
- Real applications

Voting weight	Stimmgewicht
⬇	⬇
Voting power	Stimmkraft

1. The Book
2. Probability models

- Assembly
- Profile space
- Regions of growing acceptance

$$N = \{1,2,\ldots,n\}$$

$$\Omega_N = \{-1,0,+1\}^N \ni a = (a_j)_{j \in N}$$

$$[a, 1_N] = \{b \in \Omega_N \mid a \le b \le 1_N\}$$

Voting Power in Practice Symposium
20–22 March 2011, London School of Economics
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$W_N \subseteq \Omega_N$ is a "decision rule" when
(1) $1_N \in W_N$ and (2) $0 \notin W_N$ and
(3) $\forall a \in W_N: \quad [a, 1_N] \subseteq W_N$

Events of interest:

$C_j = \{a \in \Omega_N : j$ is critical in $a\}$

$A_j = \{a \in \Omega_N : j$ agrees with $a\}$

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Probabilities of interest:

$P(W_N)$ efficiency of W_N
$P(C_j)$ influence probability of j
$\sum_{j \in N} P(C_j)$ influence sensitivity of W_N
$P(C_j)/\sum$ power share of j
$P(A_j)$ success probability of j

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Penrose/Banzhaf P_N (no abstentions):

Theorem (4 in Ruff/P. 2010)
If P is a partitioning of N and each bloc B in P has its decision rule W_B then, for all $j \in A \in$ P :

$$P_N(C_j) = P_A(C_j \mid W_A) \cdot Q_{\mathrm{P}}(C_A \mid W_{\mathrm{P}})$$

where $Q_{\mathrm{P}} = \otimes_{B \in \mathrm{P}} \mathrm{Bernoulli}(P_B(W_B))$

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Penrose/Banzhaf P_N^t (with abstentions):

Theorem (5.2 in Birkmeier/Käufl/P. 2011)
With abstention probability $t \in [0,1)$:

$$\sum_{j \in N} P_N^t(C_j(W_N)) = \frac{1}{1-t} \mathrm{E}_{P_N^t}[\sigma_{W_N}]$$

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Shapley/Shubik S_N^t (with abstentions):

Theorem (in Diss. Birkmeier 2011)
With abstention probability $t \in [0,1)$:

$$\sum_{j \in N} S_N^t(C_j(W_N)) = \frac{1-t^n}{1-t}$$

$$= 1 + t + t^2 + \cdots + t^{n-1}$$