Indistinguishable States II: The Imperfect Model Scenario

K Judd & LA Smith

Physica D 196: 224-242. SCI 15

Abstract

Given a perfect model of a chaotic system and a set of noisy observations of arbitrary duration, it is not possible to determine the state of this system precisely, rather one must consider a set of states which are indistinguishable from one another given the observations [K. Judd, L.A. Smith, Indistinguishable states I, Physica D 151 (2001)]. Yet the perfect model scenario is a fiction; in practice all models are imperfect. How do the results from the perfect model scenario change under imperfect models? It is shown to be essential to take even small model imperfections into account: failure to do so can systematically degrade state estimation or prediction of nonlinear systems. With an imperfect model, the system state space and model state space are rarely (if ever) equivalent and so one must consider a projection of the system state into the model state space. Furthermore, it is almost certain that no trajectory of the model is consistent with an infinite series of observations, thus there is no consistent way to estimate the projection of system state using trajectories. There are pseudo-orbits, however, that are consistent with observations and these can be used to estimate the projection of the system state. Using pseudo-orbits one finds that, as in the perfect model scenario, there is a set of states that are indistinguishable from the projection of the system state. Estimation of this set of indistinguishable states and the probability density on these states is discussed. The main conclusions are (i) that there is no state of the model that can be identified with the state of the system; and (ii) that great care must be taken when using an imperfect model to forecast the system, because the initialization of the model state from observations can provide a poor analogue for the system. The forecast may not shadow the future behaviour of the system for very long, even if one were able to obtain a noise-free projection of the system state. The ultimate aims of probability forecasts should be re-examined in light of these results.