How good is an ensemble at capturing truth?

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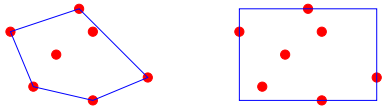
Bounding boxes of ensembles

Ensemble forecasting is used as an attempt to account for the uncertainties of observations and model error. An important minimal property that an ensemble should have is that it *capture truth with a high probability.* But what does capturing truth mean? It could mean that truth lies within the convex hull of the ensemble, but in a d-dimensional state space this requires at least $d+1$ ensemble members. An easily computed alternative is the *bounding box* of the ensemble, which is defined for as few as two members in an ensemble.



Probability of capturing truth

It is easy to compute the probability that a bounding box captures truth. If truth lies at the median of the distribution of the ensemble, then for an ensemble with n members in a d-dimensional state space, $\Pr(\text{BB captures truth}) = \left(1 - \frac{1}{2^{n-1}}\right)^d$.

It follows that to capture truth with probability α requires an ensemble of size

$$n > 1 - \frac{\log\left(1-\alpha^{1/d}\right)}{\log(2)}.$$

The minimum ensemble sizes for different state space dimension d and $\alpha = 0.95$ are

d =	1	10	10^2	10^4	10^6	10^8
n ≥	6	9	12	19	26	32

The ensemble size is surprising small.

Gaussian ensemble with offset

If the ensemble has a Gaussian distribution then the relative offset z of truth from the center of the distribution is important. When $z = 1$ truth is one standard deviation from center of ensemble distribution. The size of the ensemble required to capture truth with 95% probability is

d	z =	0	0.5	1	1.5	2	2.5
1	n >	6	9	18	44	131	481
10	n >	9	15	31	77	230	847
10^2	n >	12	21	44	110	330	1217
10^4	n >	19	34	71	177	530	1956
10^6	n >	26	46	98	243	730	2695
10^8	n >	32	58	124	310	930	3435

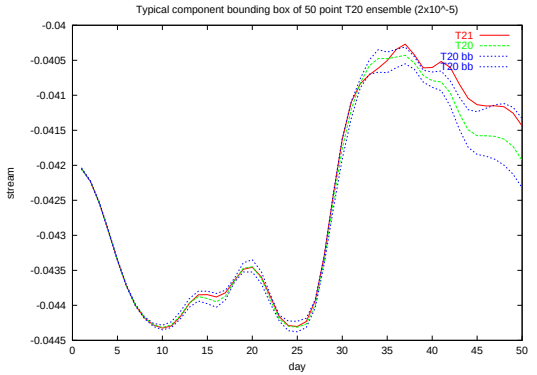
The effect of ensemble offset is very strong. This implies that uncertainty in the observations or model error that bias the ensemble estimates will have a very significant effect on the ability to capture truth.

Example : Model error

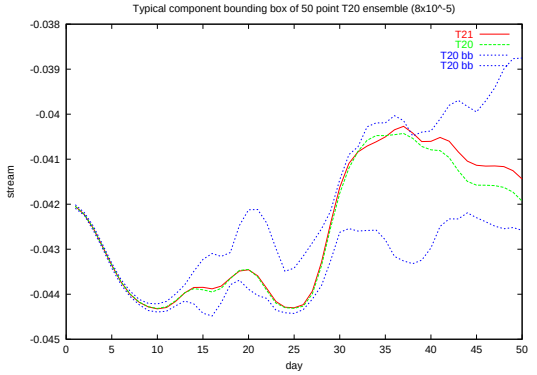
We used bounding boxes to assess strategies for forecasting with imperfect models. We use a global T20 quasi-geostrophic model to make ensemble forecasts a T21 truth run. A realistic initial ensemble spread fails to capture truth after 35 days. Increasing the initial spread captures truth, but ensemble is too wide to be meaningful. Using a stochastic forecast to account for model error captures truth with meaningful ensemble spread.

—, How good is an ensemble : capturing truth.
Physica D, to appear, 2004.
Judd and Smith. Indistinguishable states I
Physica D, 151, 2001.
Judd and Smith. Indistinguishable states II
Physica D, to appear, 2004.

Ensemble fails to capture truth.



Increased spread is meaningless.



Stochastic forecast captures.