PhD Seminar on Combinatorics, Games and Optimisation in 2019

Seminars are listed in reverse chronological order, most recent first

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Friday 13 December - Gal Kronenberg (University of Oxford)
Venue: 32L.B.09 from 12:00 - 13:00

Turán numbers of long cycles in random graphs
For a graph G on n vertices and a graph H, denote by ex(G,H) the maximal number of edges in an H-free subgraph of G. We consider a random graph G~G(n,p) where p=C/n, and study the typical value of ex(G,H), where H is a long cycle.
We determine the asymptotic value of ex(G,C_t), where G~G(n,p), p> C/n and A*log(n)< t< (1- Ɛ)n. The behaviour of ex(G,C_t) can depend substantially on the parity of t. In particular, our results match the classical result of Woodall on the Turán number of long cycles, and can be seen as its random version. In fact, our techniques apply in a more general sparse pseudo-random setting.
We also prove a robustness-type result, showing the likely existence of cycles of prescribed lengths in a random subgraph of a graph with a nearly optimal density.

Joint work with Michael Krivelevich and Adva Mond.

Friday 6 December - Bento Natura (LSE)

A scaling-invariant algorithm for linear programming whose running time depends only on the constraint matrix
Following the breakthrough work of Tardos (Oper. Res. '86) in the bit-complexity model, Vavasis and Ye (Math. Prog. '96) gave the first exact algorithm for linear programming in the real model of computation with running time depending only on the constraint matrix. For solving a linear program (LP), Vavasis and Ye developed a primal-dual interior point method using a layered least squares step, and showed that O(n^3.5 * log chi) iterations suffice to solve linear programs exactly, where chi is a condition measure controlling the size of solutions to linear systems related to the constraint matrix.

Monteiro and Tsuchiya (SIAM J. Optim. '03), noting that the central path is invariant under rescalings of the columns of the constraint matrix and the objective function, asked whether there exists an LP algorithm depending instead on the measure chi*, defined as the minimum chi value achievable by a column rescaling of the constraint matrix, and gave strong evidence that this should be the case. We resolve this open question affirmatively.

We will illustrate the central ideas on how to develop a scaling-invariant algorithm for LP and how to find a near-optimal rescaling.

Friday 22 November - Nora Frankl (LSE)

On the number of discrete chains in the plane
Determining the maximum number of unit distances that can be spanned by n points in the plane is a difficult problem, which is wide open. The following more general question was recently considered by Eyvindur Ari Palsson, Steven Senger, and Adam Sheffer. For given distances t_1,...,t_k a (k+1)-tuple (p_1,...,p_{k+1}) is called a k-chain if ||x_i-x_{i+1}||=t_i for i=1,...,k. What is the maximum possible number of k-chains that can be spanned by a set of n points in the plane? Improving the result of Palsson, Senger and Sheffer, we determine this maximum up to a small error term (which, for k=1 mod 3 involves the maximum number of unit distances). We also consider some generalisations, and the analogous question in R^3. Joint work with Andrey Kupvaskii.

Wednesday 20 November - Franziska Eberle (Universität Bremen)

Commitment in online scheduling made easy
We study a fundamental online job admission problem where jobs with processing times and deadlines arrive online over time at their release dates, and the task is to determine a preemptive single-server schedule which maximizes the number of jobs that complete on time. To circumvent known impossibility results, we make a standard slackness assumption by which the feasible time window for scheduling a job is at least (1+epsilon) times its processing time, for some epsilon>0. We consider a variant of the online scheduling problem where the provider has to satisfy certain commitment requirements. These requirements arise, for example, in modern cloud-services, where customers do not want last-minute rejections of critical tasks and request an early-enough provider-side commitment to completing admitted jobs.

Our main contribution is an optimal algorithm for online job admission with commitment. When a provider must commit upon starting a job, our bound is O(1/epsilon). This is best possible as there is a lower bound of Omega(1/epsilon) for online admission even without commitment. If the commitment decisions must be made before a job's slack becomes less than a delta-fraction of its size, we prove a competitive ratio of O(epsilon/((epsilon - delta) delta)) for 0 <delta<epsilon. This result interpolates between commitment upon starting a job and commitment

upon arrival. For the latter commitment model, it is known that no (randomized) online algorithm does admit any bounded competitive ratio.

Friday 15 November - Cosmin Pohoata (Caltech)

Sets without 4APs but with many 3APs
It is a classical theorem of Roth that every dense subset of $\left\{1,\ldots,N\right\}$ contains a nontrivial three-term arithmetic progression. Quantitatively, results of Sanders, Bloom, and Bloom-Sisask tell us that subsets of relative density at least $1/(\log N)^{1-\epsilon}$ already have this property. In this talk, we will discuss about some sets of N integers which unlike $\left\{1,\ldots,N\right\}$ do not contain nontrivial four-term arithmetic progressions, but which still have the property that all of their subsets of relative density at least $1/(\log N)^{1-\epsilon}$ must contain a three-term arithmetic progression. Perhaps a bit surprisingly, these sets turn out not to have as many three-term progressions as one might be inclined to guess, so we will also address the question of how many three-term progressions can a four-term progression free set may have. Finally, we will also discuss about some related results over $\mathbb{F}_{q}^n$. Based on joint works with Jacob Fox and Oliver Roche-Newton.

Friday 8 November - Olaf Parczyk (LSE)

The size-Ramsey number of tight 3-uniform paths
Given a hypergraph H, the size-Ramsey number is the smallest integer m such that there exists a graph G with m edges with the property that in any colouring of the edges of G with two colours there is a monochromatic copy of H. Extending on results for graphs we prove that the size Ramsey number of the 3-uniform tight path on n vertices is linear in n.

This is joint work with Jie Han, Yoshiharu Kohayakawa, and Guilherme Mota.

Friday 18 October - Oliver Janzer (University of Cambridge)

The extremal number of subdivisions
For a graph H, the extremal number ex(n,H) is defined to be the maximal number of edges in an H-free graph on n vertices. For bipartite graphs H, determining the order of magnitude of ex(n,H) is notoriously difficult. In this talk I present recent progress on this problem.
The k-subdivision of a graph F is obtained by replacing the edges of F with internally vertex-disjoint paths of length k+1. Most of our results concern the extremal number of various subdivided graphs, especially the subdivisions of the complete graph and the complete bipartite graph.

Partially joint work with David Conlon and Joonkyung Lee.

Friday 4 October - Natalie Behague (QMUL)

Semi-perfect 1-factorizations of the Hypercube
A 1-factorization of a graph H is a partition of the edges of H into disjoint perfect matchings {M_1 , M_2 , . . . , M_n}, also known as 1-factors. A 1-factorization M = {M_1 , M_2 , . . . , M_n} of a graph G is called perfect if the union of any pair of distinct 1-factors M_i , M_j is a Hamilton cycle. The existence or non-existence of perfect 1-factorizations has been studied for various families of graphs. Perhaps the most famous open problem in the area is Kotzig's conjecture, which states that the complete graph K_2n has a perfect 1-factorization. In this talk we shall focus on another well-studied family of graphs: the hypercubes Q_d in d dimensions. There is no perfect 1-factorization of Q_d for d > 2. As a result, we need to consider a weaker concept.
A 1-factorization M is called k-semi-perfect if the union of any pair of 1-factors M_i , M_j with 1 ≤ i ≤ k and k + 1 ≤ j ≤ n is a Hamilton cycle. It was proved that there is a 1-semi-perfect 1-factorization of Q_d for every integer d ≥ 2 by Gochev and Gotchev, Královič and Královič, and Chitra and Muthusamy, in answer to a conjecture of Craft. My main result is a proof that there is a k-semi-perfect 1-factorization of Q_d for all k and all d, except for one possible exception when k = 3 and d = 6. I will sketch the proof and explain why this is, in some sense, best possible. I will conclude with some questions concerning other generalisations of perfect 1-factorizations.

Summer Term 2019

Friday 31 May - Christoph Spiegel (UPC)
Venue: 32L.B.09 from 12:00 - 13:00

Intervals in the Hales-Jewett Theorem
The Hales–Jewett Theorem states that any r–colouring of [m]^n contains a monochromatic combinatorial line if n is large enough. Shelah's proof of the theorem implies that for m = 3 there always exists a monochromatic combinatorial line whose set of active coordinates is the union of at most r intervals. I will present some recent findings relating to this observation. This is joint work with Nina Kamcev.

Friday 10 May - Gwen McKinley (MIT)

Super-logarithmic cliques in dense inhomogeneous random graphs

In the theory of dense graph limits, a graphon is a symmetric measurable function $W\colon[0,1]^2\to [0,1]$. Each graphon gives rise naturally to a random graph distribution, denoted $\mathbb{G}(n,W)$, that can be viewed as a generalization of the Erd\H{o}s-R\'enyi random graph. Recently, Dole{\v{z}}al, Hladk{\'y}, and M{\'a}th{\'e} gave an asymptotic formula of order $\log n$ for the clique number of $\mathbb{G}(n,W)$ when W is bounded away from 0 and 1. We show that if W is allowed to approach 1 at a finite number of points, and displays a moderate rate of growth near these points, then the clique number of $\mathbb{G}(n,W)$ will be $\Theta(\sqrt{n})$ almost surely. We also give a family of examples with clique number $\Theta(n^\alpha)$ for any $\alpha\in(0,1)$, and some conditions under which the clique number of $\mathbb{G}(n,W)$ will be $o(\sqrt{n})$, $\omega(\sqrt{n}),$ or $\Omega(n^\alpha)$ for $\alpha\in(0,1)$.

Lent Term 2019

Friday 22 March - Alberto Espuny Diaz (University of Birmingham)

Resilient degree sequences with respect to Hamiltonicity in random graphs
The local resilience of a graph with respect to a property P can be defined as the maximum number of edges incident to each vertex that an adversary can delete without destroying P. The resilience of random graphs with respect to various properties has received much attention in recent years, with a special emphasis on Hamiltonicity. Based on different sufficient degree conditions for Hamiltonicity, we investigate a notion of local resilience in which the adversary is allowed to delete a different number of edges at each vertex, and obtain some results which improve on previous results. This is joint work with P. Condon, J. Kim, D. Kühn and D. Osthus.

Friday 22 February - Bernhard von Stengel (LSE)

Computational progress on the Catch-Up game
We report on computational progress on the game "Catch-Up" analyzed by Isaksen, Ismail, Brams, and Nealen in 2015. Two players pick and remove numbers from the set {1,...,n} according to the "catch-up" rule that says you pick the next number as long as the sum of your numbers is less than that of the other player. After that (having reached an equal or higher sum), players switch. The player with the higher sum at the end wins. For n=5, for example, the first player wins by first picking 3. With old-fashioned memory-saving coding but large computer memory of a

standard modern laptop (8 GByte RAM, 37 GByte virtual memory, 28 hours computation time), we extend the computational analysis for optimal play from maximally n=20 to n=34. The following pattern emerges: When there can be a draw (when n or n+1 is divisible by 4), there will be a draw, and when there cannot be a draw, then for n=21 or larger the first player wins with the first picked number no larger than 3. (For n = 9,10,14,18 the first player loses, which seem to be the exceptions.) We prove that if there is a first winning move then it is unique. Further computational analysis may help in eventually finding an induction proof to prove this in general, which is challenging.

Friday 15 February - Dennis Clemens (TUHH)

On minimal Ramsey graphs and Ramsey equivalence for multiple colours.
For an integer $q\geq 2$, we call G a q-Ramsey-minimal graph for H if every q-edge-colouring of G contains a monochromatic copy of H, but no proper subgraph of G has this property. Utilising the method of signal senders and generalising so-called indicators to multiple colours, we prove that if $2\leq r< q$ and G is an r-Ramsey-minimal graph for H then G is contained in an infinite number of q-Ramsey-minimal graphs for H, provided that H is 3-connected. As a consequence we can show that the collection $\{M_q(H) : H \text{ is 3-connected}\}$ forms an antichain, where $M_q(H)$ denotes the set of all graphs that are q-Ramsey-minimal for H. Moreover, call H_1 and H_2 q-equivalent if $M_q(H_1)=M_q(H_2)$ holds. We generalize previous results by Bloom and Liebenau as well as Fox, Grinshpun, Liebenau, Person and Szab\'o regarding the class of graphs that are q-equivalent to the complete graph K_k. Joint work with Damian Reding and Anita Liebenau.

Friday 8 February - Attila Dankovics (LSE)

Low independence number and Hamiltonicity implies pancyclicity
Given a Hamiltonian graph G with independence number at most k we are looking for the minimum number of vertices $f(k)$ that guarantees that G is pancyclic. The problem of finding $f(k)$ was raised by Erdős who showed that $f(k)\le 4k^4$, and conjectured that $f(k)=\Theta(k^2)$. Formerly the best known upper bound was $f(k)=O(k^{7/3})$ by Lee and Sudakov. We improve this bound and show that $f(k)=O(k^{11/5})$.

Friday 25 January - Cedric Koh (LSE)

An Efficient Characterization of Submodular Spanning Tree Games

Cooperative games are an important class of problems in game theory, where the goal is to distribute a value among a set of players who are allowed to cooperate by forming coalitions. An outcome of the game is given by an allocation vector that assigns a value share to each player. A crucial aspect of such games is submodularity (or convexity). Indeed, convex instances of cooperative games exhibit several nice properties, e.g. regarding the existence and computation of allocations realizing some of the most important solution concepts proposed in the literature. For this reason, a relevant question is whether one can give a polynomial time characterization of submodular instances, for prominent cooperative games that are in general non-convex.

In this talk, we focus on a fundamental and widely studied cooperative game, namely the spanning tree game. An efficient recognition of submodular instances of this game was not known so far, and explicitly mentioned as an open question in the literature. We here settle this open problem by giving a polynomial time characterization of submodular spanning tree games.

This is joint work with Laura Sanità.