

Joint Risk & Stochastics and Financial Mathematics Seminar in 2015/16

Seminars are listed in reverse chronological order, most recent first

30 June - Johannes Muhle-Karbe (Michigan) Information and Inventories in High-Frequency Trading

We study how short-term informational advantages can be monetized in a high-frequency setting, when large inventories are explicitly penalized. We find that if most of the additional information is revealed regardless of the high-frequency traders' actions, then optimized inventory management allows to minimize positions with only second-order losses to expected returns. (Joint work with Kevin Webster)

12 May - Remi Peyre (École des Mines de Nancy) Fractional Brownian motion, financial mathematics and stopping times

Fractional Brownian motion (fBm) is a Gaussian continuous stochastic process with stationary increments, which generalises ordinary Brownian motion by allowing for any Hölder regularity between 0 and 1. Despite being a very natural model for stochastic evolutions, fBm a priori irrelevant in financial mathematics, as it is not a semimartingale. However if we restrict the authorised trading strategies, or if we introduce transaction costs, it becomes a nontrivial natural question whether the price of a financial asset may be modelled by a (geometric) fBm while satisfying the non-arbitrage condition.

This kind of questions lead to look at what may or may not happen to the trajectory of a fBm just after an arbitrarily chosen stopping time. Obviously the behaviour of the trajectory may be very different from what it is typically, but to which extent?

For example, can one find a stopping time after which the fBm would have nonzero probability to go on upwards or downwards? If not, fBm would be said to have the two-way crossing property, which has been beautifully shown to imply non-arbitrage results.

In this talk I will present a work of mine in which I proved the two-way crossing property for fBm, by studying sharp properties of its behaviour after a stopping time, in particular at the order of the local law of the iterated logarithm. Emphasis will be set of the ideas and techniques involved, which seem to me interesting as such.

5 May - Andreea Minca (Cornell) Dynamics and Stability of Debt Capacity

We propose a dynamic model that explains the build-up of short term debt when the creditors are strategic and have different beliefs about the prospects of the borrowers' fundamentals. We define a dynamic game among creditors, whose outcome is the short term debt. As common in the literature, this game features multiple Nash equilibria. We give a refinement of the Nash equilibrium concept that leads to a unique equilibrium.

For the resulting debt-to-asset process of the borrower we define a notion of stability and find the debt ceiling which marks the point when the borrower becomes illiquid. We show existence of early warning signals of bank runs: a bank run begins when the debt-to-asset process leaves the stability region and becomes a mean-fleeing sub-martingale with tendency to reach the debt ceiling.

Our results are robust across a wide variety of specifications for the distribution of the capital across creditors' beliefs. This is joint work Johannes Wissel.

24 March - Teemu Pennanen (King's College London)

Convex duality in optimal investment and contingent claim valuation in illiquid markets

Abstract: This paper studies convex duality in optimal investment and contingent claim valuation in markets where traded assets may be subject to nonlinear trading costs and portfolio constraints. Under fairly general conditions, the dual expressions decompose into three terms, corresponding to the agent's risk preferences, trading costs and portfolio constraints, respectively. The dual representations are shown to be valid when the market model satisfies an appropriate generalization of the no-arbitrage condition and the agent's utility function satisfies an appropriate generalization of asymptotic elasticity conditions. When applied to classical liquid market models or models with bid-ask spreads, we recover well-known pricing formulas in terms of

martingale measures and consistent price systems. Building on the general theory of convex stochastic optimization, we also derive optimality conditions in terms of an extended notion of a shadow price.

17 March - Jérôme Renault (University Toulouse 1 Capitole) Optimal Dynamic Information Provision

We study a dynamic model of information provision. A state of nature evolves according to a Markov chain. An advisor with commitment power decides how much information to provide to an uninformed decision maker, so as to influence his short-term decisions. We deal with a stylized class of situations, in which the decision maker has a risky action and a safe action, and the payoff to the advisor only depends on the action chosen by the decision maker. The greedy disclosure policy is the policy which, at each round, minimizes the amount of information being disclosed in that round, under the constraint that it maximizes the current payoff of the advisor. We prove that the greedy policy is optimal (whatever the discount factor) in many cases - but not always.

J. Renault (TSE) , E. Solan (U Tel-Aviv) and N. Vieille (HEC Paris)

10 March - Zhenjie Ren (CMAP, École Polytechnique) Viscosity Solution of Path-dependent PDEs

This talk will give an overview of the recent study on the path-dependent PDEs. We will show the general motivations of this new topic, and explain why the viscosity- type solutions are interesting. By comparing with the classical notion of viscosity solutions to PDEs, we try to understand the advantage of the new definition of viscosity solutions in the path-dependent context. Also, in this talk we will see what important role the comparison principle plays in the theory of viscosity solutions, and will show the recent progress in proving the comparison result. At the end of the talk, we will see a collection of references on the theory of path-dependent PDEs including both the theoretical improvements and the applications.

25 February - Christa Cuchiero (Vienna)

Polynomial Processes in Stochastic Portfolio Theory

Inspired by volatility stabilized market models introduced by Robert Fernholz and

Ioannis Karatzas [2], we introduce a class of processes which are polynomial in the sense of [1] to model both, asset prices (or market capitalizations of companies in an equity market) and their corresponding market weights. More precisely, we characterize the class of polynomial diffusion models for the asset price process whose market weights process is again a polynomial diffusion process on the unit simplex. Explicit parameter conditions assuring the existence of relative arbitrages with respect to the market portfolio are given and the connection to non-attainment of the boundary is discussed. We also consider extensions to models with jumps and the computation of optimal relative arbitrage strategies.

References

- [1] C. Cuchiero, M. Keller-Ressel, and J. Teichmann. Polynomial processes and their applications to mathematical finance. *Finance and Stochastics*, 16(4):711–740, 2012. [2] R. Fernholz and I. Karatzas. Relative arbitrage in volatility-stabilized markets. *Annals of Finance*, 1(2):149–177, 2005.

11 February - Xiaolu Tan (Paris-Dauphine)

Tightness and duality of martingale transport on the Skorokhod space

The martingale optimal transport aims to optimally transfer a probability measure to another along the class of martingales. This problem is mainly motivated by the robust superhedging of exotic derivatives in financial mathematics, which turns out to be the corresponding Kantorovich dual. In this paper we consider the continuous-time martingale transport on the Skorokhod space of cadlag paths. Similar to the classical setting of optimal transport, we introduce different dual problems and establish the corresponding dualities by a crucial use of the S -topology and the dynamic programming principle.

28 January - Alexandru Hening (Oxford)

Killed Brownian motion with a prescribed lifetime distribution and models of default

Abstract: In finance, the default time of a counterparty is sometimes modeled as the first passage time of a credit index process below a barrier. It is therefore relevant to consider the following question: If we know the distribution of the default time, can we find a unique barrier which gives this distribution? This is known as the Inverse First Passage Time (IFPT) problem in the literature. We consider a more general 'smoothed' version of the inverse first passage time problem in which the first passage time is replaced by the first instant that the time spent below the barrier exceeds an independent exponential random variable. We show that any smooth distribution results from some unique continuously differentiable barrier. In current work with B. Ettinger and T. K. Wong, we use PDE methods to show the uniqueness and existence of solutions to a discontinuous version of the IFPT problem.

25 January - Julio Bakhoff (Technical University of Vienna) On optimal transport under the causality constraint

In this talk we shall examine causal transports and the associated optimal transportation problem under the causality constraint (P_c) introduced by Rémi Lasalle. Loosely speaking, causal transports are a relaxation of adapted processes in the same sense as Kantorovich transport plans are the extension of Monge-type transport maps. We will establish a simple primal-dual picture of both (P_c) and the so-called bicausal transportation problem (whereby causality runs in both directions) in euclidean space or equiv. for discrete-time processes, and discuss the limit to continuous-time.

14 January - Blanka Horvath (ETH) Robust methods for the SABR model

During its decade and a half of existence, the SABR model has changed the daily routine of interest rate modelling. The tractability of the SABR formula—derived by Hagan et al— has led to the formula rather than the model itself becoming an industry standard. While this formula is prone to yield inconsistent prices around zero interest rates, thereby inducing distortions and arbitrage into modelling, the model itself yields more consistent prices. In the historically low interest-rate environments of the past years this has become increasingly relevant and spurred research both among practitioners and academics. From an academic perspective, the concrete but delicate example of SABR-type models exhibit several advantageous but also certain

challenging properties. In this talk we review some approaches from this perspective and propose new effective solutions to the arbitrage problem.

Thursday 3 December - Kristoffer Glover (University of Technology, Sydney) The optimal time to close an open-ended mutual fund

In this talk I will attempt to shed some light onto two intimately linked questions about empirically observed mutual fund behaviour: (1) Why do open-ended mutual funds decide to close their doors to new investors? and (2) Why do these funds underperform after closing? A theoretical model for the optimal closure of an open-ended mutual fund is developed in which the fund is subject to performance sensitive fund flows and a decreasing return-to-scale on its investment portfolio.

Such funds are found to optimally close 'too late' from the perspective of the fund investors, since the optimal fund size that maximises the fund manager's expected fee income is larger than the size at which the fund's 'alpha' is expected to become negative. In other words, the fund closes at a point where the decreasing returns-to-scale have already started to negatively affect fund performance; thus explaining the empirically observed underperformance after closure. Additional empirical predictions generated from the model will also be discussed.

19 November - Rémy Praz (Copenhagen Business School) Equilibrium asset pricing with both liquid and illiquid markets

I study a general equilibrium model in which investors face endowment risk and trade two correlated assets; one asset is traded on a liquid market whereas the other is traded on an illiquid over-the-counter (OTC) market. Endowment shocks not only make prices drop, they also make the OTC asset more difficult to sell, creating an endogenous liquidity risk. This liquidity risk increases the risk premium of both the OTC asset and liquid asset. Furthermore, the OTC market frictions increase the trading volume and the cross-sectional dispersion of ownership in the liquid market. Finally, if the economy starts with only the OTC market, then I explain how opening a correlated liquid market can increase or decrease the OTC price depending on the illiquidity level. The model's predictions can help explain several empirical findings.

5 November - Yaroslav Melnyk (Swiss Financial Institute @ École polytechnique fédérale de Lausanne)

Portfolio Optimization with Recursive Utility under Small Transaction Costs

In this article we investigate the portfolio problem of an investor with Epstein-Zin recursive utility under proportional transaction costs. We characterize the solution via variational inequalities and prove existence of classical solutions for small cost parameters. We also provide a suitable verification theorem. This allows us to derive rigorous asymptotic expansions for optimal no-trading regions and consumption strategies and to investigate the effects of the investor's relative risk aversion and the elasticity of intertemporal substitution (EIS) ψ on the optimal strategies. Our main findings are: (a) At the leading order, the no-trading region is the same as with expected additive utility; in particular, it is determined solely by the relative risk aversion. The no-trading region depends on the investor's EIS only at the next-to-leading order, and only indirectly (via optimal consumption). (b) The investor's optimal consumption depends on his EIS also at the leading order. The optimal consumption rate is higher, as a percentage of current wealth, than in the frictionless case if and only if $\psi > 1$.

Based on joint work with Johannes Muhle-Karbe and Frank Thomas Seifried.

22 October - Johannes Ruf (University College London and an Associate Member at the Oxford-Man Institute of Quantitative Finance)

Föllmer's Measure and Novikov/Kazamaki-Type Conditions

In the first part of the talk, I will discuss the construction of Föllmer's measure on the canonical path space. In the second part, I will discuss sharpened Novikov/Kazamaki-type conditions that provide sufficient and necessary conditions for the martingale property of a nonnegative local martingale.

This talk is based on joined papers with Nicolas Perkowski and Martin Larsson.

8 October - Sebastian Herrmann (ETH Zürich) Hedging with Small Uncertainty Aversion

We study the pricing and hedging of derivative securities with uncertainty about the volatility of the underlying asset. Rather than taking all models from a prespecified class equally seriously, we penalise less plausible ones based on their "distance" to a reference local volatility model. In the limit for small uncertainty aversion, this leads to

explicit formulas for prices and hedging strategies in terms of the security's cash gamma.

6 October - Nevroz Sen (McGill)

Estimation theory for non-linear major-minor mean field games

In the Mean Field Games (MFG) framework where there is an agent (so-called Major) which has asymptotically non vanishing influence on any other Minor agent, the best response control process of each Minor agent depends upon its own state, the Major agent's state and the conditional distribution of the generic minor agent, namely the system's stochastic mean field; this is in contrast to the basic MFG setup where the mean field is deterministic. The theory of MFG with a Major agent (MM-MFG) is well understood when the observations of the Minor agents are complete.

In this talk we analyze the non-linear MM-MFG problem where each Minor agent partially observes the Major agent's state. We employ non-linear filtering theory derived for McKean-Vlasov type state equations and the Separation Principle in order to analyze the game in the infinite population limit. The main results are the existence and uniqueness of the solutions to the stochastic MFG system equations and the epsilon-Nash equilibrium property where the best response control process of each Minor agent depends upon the conditional density generated by that agent's non-linear filter together with the system's mean field and its own state.

Joint work with Peter E. Caines