

## PROFESSOR AILSA LAND

I graduated in Economics at LSE in 1950 and joined the Economics Research Division as a Research Assistant. Linear Programming (LP) was just beginning to be of interest to academic economists. The Primal and Dual Conditions of Optimality of an LP are, of course, equivalent to the conditions of economic equilibrium. (I have to confess that in later years I failed to inspire OR students with much interest in this equivalence, but to me the Dual values of an LP always had some sort of interpretation as a virtual price mechanism.) As it happened, my first research supervisor was Helen Makower, who had just published a book on the relationship of LP to economic theory.

At the time it was all very well to formulate an LP model representing a real problem to which one would very much like to obtain an optimum solution (for instance, in planning the refining strategy to apply to a given quantity of crude oil to maximise the total sale value of the refined products). But determining the optimum solution of that model LP was not so easy. The Simplex Algorithm by Dantzig has the advantage of eventually reaching the solution. But at LSE in 1950 we had no access to an electronic computer, nor would have for several years. What the Research Division had were a sufficient supply of good quality electric desk calculators, particularly designed for the use by statisticians for the calculations of correlations or line fitting. Solving small LP models could be done by paper and pencil using such calculators.

I was transferred to a new supervisor, George Morton, who had used an LP model to compute the minimum cost of selecting a diet from the foods available in British shops, which would satisfy a standard list of about a dozen requirements of the calories, vitamins, proteins, etc. necessary to sustain a healthy life. In the post-war conditions when even many European citizens were not in fact receiving an adequate diet, George (who had worked for an aid organisation) considered that it would be of interest to apply this LP model to a variety of other countries to help in comparing the plight of different countries.

Lists of data of the available foods with their nutritional properties and prices in each of the countries had already been obtained. These lists were varied and extensive, but fortunately it turned out that the number of different foods entering the optimum solution for each country never exceeded 4 or 5 -- thus the so-called reduced basic solution to the LP was never more than 4 or 5 food variables. Once we had solutions for two or three different countries, it proved easy to find the best one to use as a starting basis for a new country. George, at that time, became involved in other projects and our work did not reach publication. On the other hand, I became adept at moving from one basis to another to achieve feasibility and optimality of an LP.

In 1957 I met Alison Doig (later Harcourt), an Australian graduate from Melbourne University, at the first international Operational Research(OR) conference held at Oxford. She joined the Research Division and we worked together for some years. The extension of LP to Integer Linear Programming (ILP) was a lively area of interest and publication at the time. It was realised that this apparently simple extension (of relaxing the condition of continuity of solution values in LP) enabled an enormous extension of the class of problems which could be modelled as an ILP. Algorithms to solve ILPs were vigorously sought. Indeed Alison's MSc thesis in Melbourne had concerned this problem.

British Petroleum by then had a computer working on the 'Refinery Problem'. When they wanted to extend their LP models to cover choices of locations for operations, they obviously recognised that the fundamental continuity of solution variables in LP leads to fractional values which can be meaningless in real decision making. I seem to remember that it was an Australian member of the British Petroleum's team that was responsible for their decision to pay the Economics Research Division a year's salary for Alison and me to 'solve' their ILP problem! We were still in the pre-computer situation, and rapidly convinced British Petroleum that we were not able to use their actual model data, but would work on developing a general ILP algorithm and test it on smaller, generally available known problems.

We developed what later became known as the branch and bound method (though we didn't apply that name which had already been applied to a specific sub-class of ILP models). It was a simple extension of our algorithm to solve more general Discrete LP models rather than just Zero/One values of the variables, so we built the discrete option into our algorithm. We felt that it was essential to demonstrate that our method was capable of solving some known problems that had not by then been solved by a totally automatic algorithm. So we embarked on a large program of desk computing (with some help from other members of the Research Division) to offer British Petroleum and our algorithm (with their permission, but without mention of their sponsorship) to offer the details of this algorithm to the journal *Econometrica* for publication.

*Econometrica* accepted our paper, but apologised that we would not go into the first available (June) issue because they had already planned to publish in the June issue another ILP algorithm, by R E Gomory (a cutting plane method). Our paper was published as 'An Automatic Method of Solving Discrete Programming Problems', *Econometrica*, 28, No.3 497-520 (July 1960).

In time, both Alison and I both became LSE teaching staff, and Alison eventually returned (as she had always intended) to Melbourne. I was sorry that she did not continue to work in the Mathematical Programming area but continued working in Statistics at Melbourne.

Later, when we had access to London University computers, I met a research colleague (John Murchland) who was a competent Fortran user but had not tackled LP by Simplex. I taught him Simplex and he taught me Fortran. This was very lucky because although Fortran has improved enormously over the years, its updates have always been 'downward compatible' and old programs like ours will still compile and run on the latest standard Fortran version. I transferred Simplex and our Discrete ILP algorithm to Fortran, carrying packages of punched cards to the London University Atlas Computer in Guildford Street. This was not an ideal way to develop code—waiting for the results of an overnight run only to discover that it had failed to compile because of a silly little misplaced hole in one punched card! But it was a definite improvement on paper and desk calculator.

As well as teaching I had students and colleagues and visiting academics interested in using these basic Fortran programs as part of their own research. Susan Powell was an MSc and PhD student and then colleague who worked using these computer programs. These became basic packages which a user could modify by adding additional constraints and variables to solve their particular research model. We made the programs freely available to anyone who wanted to make use of them, and over the years they became the basic test beds for many PhDs. Eventually Susan and I decided it was worth collecting the whole set of Fortran codes and the explanatory use that could be made of them into a book, 'Fortran Codes for Mathematical Programming' John Wiley & Sons. 1973. This sold well, and was reprinted. It was certainly used by some of its purchasers to develop their own programs. Susan and I continued to develop variations, including the cutting plane Method of Integer Forms by Gomory. Indeed Branch and Cut became a useful tool for solving many problems, and much effort has been devoted to finding good branching rules and ways of developing good cutting planes for varying problems, including the TSP, Traveling Salesman Problem.