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Testing Explosive Bubbles with Time-Varying Volatility: The Case of the Spanish Public Debt, 1850-2021

Vicente Esteve* and María A. Prats**

Abstract

This paper analyzes the dynamics of the Spanish public debt-GDP ratio during the period 1850-2021. We use recent procedures to test for explosive bubbles under the presence of time-varying volatility (Harvey, Leybourne, Sollis and Taylor, 2016; Harvey, Leybourne and Zu, 2019, 2020; Kurozumi, Skorobotov and Tsarev, 2022) to test for explosive behavior of the Spanish public debt over this long period. We extend the previous analysis of Esteve and Prats (2022) where constant unconditional volatility in the underlying error process was assumed.

Keywords: Public debt; Rational bubble; Explosive autoregression; Time-varying volatility; Right-tailed unit root testing

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Table of Contents

Acknowledgments

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Testing Explosive Bubbles with Time-Varying Volatility: The Case of the Spanish Public Debt, 1850-2021

1. Introduction

Issues such as the balancing of budget deficits, the connections between monetary and fiscal policies, and the fiscal discipline required in a monetary union, have been widely debated in the last decades. Specifically, one of the central problems affecting fiscal authorities is the sustainability of government deficits, which is related to the issue of long-run solvency.

Fiscal policy is considered sustainable when, if maintained in the indefinite future, it does not violate the solvency constraint, and a government is said to be solvent if the present value budget constraint, i.e., its intertemporal budget constraint (IBC), is satisfied. In other words, the public deficit is sustainable if the government can borrow. However, if the interest rate on the government debt exceeds the growth rate of the economy, debt dynamics would lead to an ever-increasing ratio of debt to GDP. The dynamics of debt accumulation could only be stopped if the ratio of the primary budget deficit to GDP would change to a surplus, or if seigniorage were allowed for (Esteve and Prats, 2022).

The condition for fiscal sustainability implies that initial public debt equals the expected present value of the future primary public surpluses, commonly known as the Intertemporal Budget Constraint (IBC), if and only if the discounted future public debt converges to zero (the Transversality Condition (TC) of the government's intertemporal decision problem). The TC rules out a Ponzi scheme (whereby debt is perpetually rolled over) as the necessary condition for lenders to hold government bonds.[1] There is a large literature on this topic, although empirical tests of solvency (or fiscal sustainability) have gone through different stages.[2] Several methods have been used in empirical applications to test whether this TC is fulfilled.[3]

The Spanish case can be of interest given the permanent difficulties experienced in balancing the government budget throughout those years. Furthermore, the Spanish economy seems to be an interesting case study because it has been characterized by chronic government deficits and episodes with high levels of public debt.

In a recent paper, Esteve and Prats (2022) analyzed the dynamics of the Spanish public debt-GDP ratio during the period 1850-2020. They used different tests for recurrent explosive bubbles proposed by Phillips, Wu and Yu (2011) and Phillips, Shi and Yu (2015a, 2015b) to identify episodes of explosive dynamics of public debt, which can be attributed to active budget policies (unsustainable) that were carried out in the past. These tests assume constant unconditional volatility in the underlying error process.

However, Harvey, Leybourne, Sollis, and Taylor (2016) and Harvey, Leybourne and Zu (2019) demonstrated that in the presence of heteroskedasticity, the asymptotic null distribution of the tests of Phillips, Wu and Yu (2011) and Phillips, Shi and Yu (2015a, 2015b) depends on the nature of the volatility through the variance profile. In this case, tests for bubbles derived under the assumption of a homokedastic error might suffer from size distortion because the limiting distributions depend on the volatility structure. This lack of size control typically leads to severe oversizing, and consequently frequent spurious identification of a bubble. This implies that we cannot be confident that the application of standard critical values for the tests of Phillips, Wu and Yu (2011) and Phillips, Shi and Yu (2015a, 2015b) will produce a size-controlled procedure in the presence of non-stationary volatility.

[1] On the arithmetic of deficit and debt sustainability, see Esteve and Prats (2022) for details.
[2] A very good, updated, and clarifying study of the different approaches to evaluate this question is D'Erasmo and Zhang (2016), which identifies the more important defaults in the traditional approach to evaluate debt sustainability, and examines three alternative approaches that provide useful econometric and model-simulation tools to analyze debt sustainability.
[3] For a recent review of empirical applications, see Beqiraj et al. (2018) and its references.

A general decline in the unconditional volatility of the shocks that drive the macroeconomic series has been a commonly observed phenomenon. Furthermore, time-varying volatility is a well-known stylized fact observed in economic time series and especially in financial data (see, for example, Rapach, Strauss, and Wohar, 2008).

To overcome the problem identified in Harvey, Leybourne, Sollis, and Taylor (2016) and Harvey, Leybourne and Zu (2019), in this article we extend the previous analysis of Esteve and Prats (2022) using recently developed procedures to test for the existence of explosive bubbles in the presence of time-varying volatility (Harvey, Leybourne, Sollis, and Taylor, 2016; Harvey, Leybourne and Zu, 2019, 2020; Kurozumi, Skorobotov and Tsarev, 2022) in order to identify the explosive behavior of the Spanish public debt over the period 1850-2021.

The scheme of this paper is as follows. Section 2 introduces the econometric methodology. Section 3 presents and discusses the main empirical results. Section 4 draws the main conclusions.

2. Econometric methodology

2.1 The heteroskedastic bubble model

Kurozumi, Skorobotov and Tsarev (2022) consider the time series process $\{y_t\}$ generated according to the following DGP that allows one explosive regime with a subsequent collapsing regime,

$$y_t = \eta + u_t \tag{1}$$

$$u_t = \left\{ \begin{array}{ll} u_{t-1} + \varepsilon_t, & t = 1, \ldots, \lfloor \tau_{1,0} T \rfloor, \\ (1+\delta_1) u_{t-1} + \varepsilon_t, & t = \lfloor \tau_{1,0} T \rfloor + 1, \ldots, \lfloor \tau_{2,0} T \rfloor, \\ (1-\delta_2) u_{t-1} + \varepsilon_t, & t = \lfloor \tau_{2,0} T \rfloor + 1, \ldots, \lfloor \tau_{3,0} T \rfloor, \\ u_{t-1} + \varepsilon_t, & t = \lfloor \tau_{3,0} T \rfloor + 1, \ldots, T, \end{array} \right\} \tag{2}$$

$$\varepsilon_t = \sigma_t e_t \tag{3}$$

where $\delta_1 \geq 0$, $\delta_2 \geq 0$, $0 \leq \tau_{1,0} < \tau_{2,0} \leq \tau_{3,0} \leq 1$. The process $\{y_t\}$ evolves as a unit root process, but a possible bubble emerges at $\lfloor \tau_{1,0} T \rfloor + 1$ with the explosive AR(1) coefficient given by $1 + \delta_1$ followed by the collapsing regime from $\lfloor \tau_{2,0} T \rfloor + 1$ to $\lfloor \tau_{3,0} T \rfloor$ generated as a stationary process, which is interpreted as the return to the normal behavior of the time series. The magnitude of δ_2 specifies the extent of the collapse of the bubble, with a duration between $\lfloor \tau_{2,0} T \rfloor + 1$ to $\lfloor \tau_{3,0} T \rfloor$.

In the presence of heteroskedasticity, the volatility of the innovations is given by σ_t in (3) and it can be non-stationary, while the conventional homoskedasticity assumption, as employed in PWY and PSY and other papers, implies that $\sigma_t = \sigma$ for all t.

On the other hand, the time series process $\{y_t\}$ can be simply rewritten as

$$y_t = (1 + \delta_t) y_{t-1} + \varepsilon_t \qquad (4)$$

or

$$\Delta y_t = \delta_t y_{t-1} + \varepsilon_t \qquad (5)$$

The null hypothesis, H_0, is that no bubble is present in the series and y_t follows a unit root process throughout the sample period, i.e., $\delta_t = 0$ in expression (4).[4] The alternative hypothesis H_1 is that a bubble is present in the series, which corresponds to the case where δ_t in (4) is not stable at 1 and the model is given by (1){(3) with $\delta_1 > 0$.

[4] The null hypothesis can be expressed using (2) in several ways such that $\tau_{1,0} = 1$, $\delta_1 = 0$, $\tau_{2,0} = 1$, or $\delta_1 = \delta_2 = 0$.

2.2 Test for explosive bubbles under stationary volatility

Phillips, Wu and Yu (2011) and Phillips, Shi and Yu (2015a, 2015b) proposed test statistics of explosive bubbles based on recursive right-tailed Dickey-Fuller-type unit root tests which can detect evidence of the explosive behavior of a time series $\{y_t\}$.

Phillips, Wu and Yu (2011) proposed the maximum of the ADF test statistics constructed using subsamples. The testing procedure was developed from a regression model of the form

$$\Delta y_t = \mu + \delta y_{t-1} + \varepsilon_t \qquad (6)$$

for $t = \lfloor \tau_1 T \rfloor + 1$ to $\lfloor \tau_2 T \rfloor$.

The key parameter of interest is δ. We want to test the null hypothesis of a unit root, $H_0 : \delta = 1$, against the right-tailed alternative, $H_1 : \delta > 1$, at least in some subsample. The model is estimated by Ordinary Least Squares (OLS) and the t-statistics associated with the estimated δ is referred to as ADF statistic.

The $SADF$ test is then a supremun statistic based on the forward recursive regression and is simply defined as

$$SADF(r_0) = \sup_{r_2 \in [r_0, 1]} ADF_0^{r_2} \qquad (7)$$

where the right-tail is the rejection region. This test can be used for testing for a unit root against explosive behavior in some subsample.

Second, Phillips, Shi and Yu (2015a, 2015b) proposed a generalized version of the *sup ADF (SADF)* test of Phillips, Wu and Yu (2011). Their Generalized Supremum *ADF (GSADF)*

$$GSADF(r_0) = \sup_{r_2 \in [r_0, 1], r_1 \in [0, r_2 - r_0]} ADF_0^{r_2} \qquad (8)$$

The statistic (8) is used to test the null of a unit root against the alternative of recurrent explosive behavior, as the statistic (7).

Note that the $SADF$ test previously proposed by Phillips, Wu and Yu (2011) is a special case of the $GSADF$ test, obtained by setting $r_1 = 0$ and $r_2 = r_\omega \in [r_0, 1]$.[5]

$SADF$ and $GSADF$ assume constant unconditional volatility in the underlying error process, and recently Harvey, Leybourne, Sollis, and Taylor (2016) and Harvey, Leybourne and Zu (2019) demonstrated that the asymptotic null distribution of the test of Phillips, Wu and Yu (2011) and Phillips, Shi and Yu (2015a, 2015b) depends, in the presence of heteroskedasticity, on the nature of the volatility, through the variance profile $\eta(s)$. Therefore, if the test is compared to critical values derived under the assumption of homoskedastic error, its size is not controlled under time-varying volatility. This lack of size control typically leads to serious oversizing and, consequently, frequent spurious identification of a bubble.[6]

2.3 Test for explosive bubbles under time-varying volatility

To take this issue into account, several tests for explosive bubbles, under the assumption of time-varying volatility, have been recently proposed:

a) Harvey, Leybourne, Sollis, and Taylor (2016), Harvey, Leybourne and Zu (2019) and Kurozumi, Skorobotov and Tsarev (2022) developed a wild bootstrap algorithm for the $SADF$ test and $GSADF$ test. They propose to use this bootstrap scheme, applied to the first differences of the data, to replicate in the bootstrap data the pattern of non-stationary volatility present in the original innovations. We call these tests $SADF_b$ and $GSADF_b$.

b) Harvey, Leybourne and Zu (2019) proposed two tests:
   - A weighted least squares (WLS) modification of Phillips, Wu and Yu (2011) test. Their supremum-based test is

[5] Phillips and Shi (2018) showed that although the $GSADF$ procedure is designed to detect bubble behavior, it can also detect crisis periods (see also Phillips and Shi, 2019, and Phillips and Shi, 2020), which are often observed in empirical applications, for example, Esteve and Prats (2022).

[6] Some classical unit root tests are severely oversized because their limiting distributions depend on a particular function, the so-called variance profile, of the underlying volatility process (see Cavaliere, 2004; Cavaliere and Taylor, 2007a, 2007b, 2008, 2009 and references therein).

$$SBZ(r_0) = \sup_{r \in [r_0, 1]} BZ_r \tag{9}$$

- o A union U test of rejections testing strategy because neither the SBZ nor $SADF$ test dominate each other across all volatility specifications. We call these tests SBZ_u .

c) Harvey, Leybourne and Zu (2020) proposed another method which controls the size under time-varying volatility. They proposed two tests:

- o A sign-based variant of Phillips, Shi and Yu (2015a, 2015b) test for explosive behavior. This supremum sign-based test is

$$sGSADF(r_0) = \sup_{r_2 \in [r_0, 1], r_1 \in [0, r_2 - r_0]} sADF_{r_1}^{r_2} \tag{10}$$

- o A union U test of rejections testing strategy with wild bootstrap implementation with $GSADF$ and $sGSADF$ tests, the same approach as that of Harvey, Leybourne and Zu (2019). We call this test $sGSADF_u$ (and $sSADF_u$).

d) Kurozumi, Skorobotov and Tsarev (2022) proposed a test based on the sup-type t -statistics expanded under the null hypothesis, using the time transformed data based on the variance profile, $\eta(s)$. They consider the $SADF$ and $GSADF$ test statistics with a version of the GLS-type demeaning. Their test statistics based on the time-transformed ADF test statistics are

$$SADF = \sup_{r_2 \in [r_0, 1]} TADF_{r_0}^{r_2} \tag{11}$$

and

$$GSTADF = \sup_{r_2 \in [r_0, 1], r_1 \in [0, r_2 - r_0]} TADF_{r_1}^{r_2} \tag{12}$$

3. Empirical application

We consider a long historical time series in which many fiscal crisis events are known to have occurred. The length of this database (172 years) makes it particularly suitable for the econometric approach adopted in this paper.

The data and sources are:

- 1850-2000: a) public debt, total outstanding liabilities, B_t, from Carreras and Tafunell, X. (2005), Table 12.34, series 2895; b) nominal GDP, Y_t, from Carreras and Tafunell, X. (2005), Table 17.7, series 4744; c) the public debt to GDP ratio, $b_t = B_t/Y_t$.

- 2001-2021: d) public debt, general government, debt compiled according to the Excessive Deficit Procedure (EDP), from Banco de España (2021), Table, 2.15.a, and Banco de España (2022), Table 11.B; e) nominal GDP, Y_t, from Banco de España (2022), Table 23.a; f) the public debt to GDP ratio (EDP), $b_t = B_t/Y_t$. Some descriptive statistics for both series are shown in Table 1.[7]

In our empirical analysis, we use annual data on the Spanish public debt GDP ratio, b_t, for the period 1850-2021. We can broadly follow the dynamics of the path of the Spanish public debt, in % of GDP, between 1850 and 2021 in Figure 1, where the expansions of public debt are markedly visible. A more detailed account of the evolution of Spanish public finances and the historical public debt cycle during this period can be found in Esteve and Prats (2022).

Figure 2 contains the plot of the first differences of b_t. A simple visual analysis of this plot suggests that the assumption of the stationary unconditional volatility of Esteve and Prats (2022) could be unrealistic for this time series, with volatility appearing

[7] Data available on request from the authors.

during the sample period in some subperiod. Figure 3 shows the estimated variance profile of b_t, which is defined as $\hat{\eta}(s)$. We construct an estimator of the variance profile using the approach suggested by Cavaliere and Taylor (2007b), Harvey, Leybourne and Zu (2022) and Kurozumi, Skorobotov and Tsarev (2022). They used the kernel-type local least squares method to estimate the time varying parameter δ_t in (4) and (5).

From Figure 3 one notes that the variance profile of this time series shows that there are three regimes in which volatility moves from high (1874{1880) to low (1881{2003) and finally to high (2004-2021). The two regimes in which volatility is high are the same subperiods identified with an explosive bubble in the ratio of Spanish public debt to GDP by Esteve and Prats (2022). However, it is important to highlight that these two regimes are of short duration and only constitute 28% of the total sample.

Table 2 presents the results of the tests for explosive bubbles under the assumption of stationary volatility and the tests for explosive bubbles when allowing for time-varying volatility, presented in the previous section: the standard $SADF$ and $GSADF$ tests, the wild bootstrap $SADF$ and $GSADF$ tests ($SADF_b$ and $GSADF_b$), a union of rejections of the $SADF_b$ and SBZ tests, and $GSADF_b$ and SBZ tests (SBZ_u), a union of rejections of the $SADF_b$ and sign-based tests $sSADF$, and $SGADF_b$ and sign-based tests $sGSADF$ ($sSADF_u$ and $sGSADF_u$) and the $STADF$ and $GSTADF$ tests. We show the bootstrap p-values associated with the different tests, $SADF$ tests (Panel A) and $GSADF$ tests (Panel B).[8]

First, we observe that the standard $SADF$ statistic does not reject the null in favor of

[8] Following Kurozumi, Skorobotov and Tsarev (2022) for the wild bootstrap p-values, B = 999 bootstrap replications were used. For the standard $SADF$ and $GSADF$ tests and the time-transformed tests $STADF$ and $GSTADF$, the p-values are obtained by simulations of the asymptotic distributions of the test statistics under homoskedasticity. We use $r_0 = \lfloor 0.01+1.8/\sqrt{T} \rfloor$ for calculations of the p-values.

explosive behavior at conventional significance levels.[9] This is the same result obtained in Esteve and Prats (2022). This non-explosive behavior is preserved when we use all $SADF$ tests for explosive bubbles under time-varying volatility. Second, we find evidence of explosive behavior with the standard $GSADF$ statistic at the 0.01-level in this period, as in Esteve and Prats (2022). Finally, this pattern of result is also obtained when considering the wild bootstrap $GSADF_b$ test at the 0.10-level, providing evidence of the presence of a speculative bubble, under the new assumption of time-varying volatility of the time series.

Overall, these findings corroborate those presented in Esteve and Prats (2022) but show also that the behavior explosive of the Spanish public debt GDP ratio over the period 1850{2021 can also be partly explained by volatility changes.

4. Conclusions

This paper analyzes the dynamics of the Spanish public debt GDP ratio over the period 1850--2021. We use recently developed procedures to test for explosive bubbles under time-varying volatility (Harvey, Leybourne, Sollis, and Taylor, 2016; Harvey, Leybourne, Sollis and Taylor, 2016; Harvey, Leybourne and Zu, 2019, 2020; Kurozumi, Skorobotov and Tsarev, 2022) in order to test for explosive behavior of the Spanish public debt during this long period. We extend the previous analysis of Esteve and Prats (2022) where evidence of explosive behavior of the ratio of the Spanish public debt GDP ratio during the period 1850-2020 is presented under the assumption of constant unconditional volatility in the underlying error process. Now, our empirical application demonstrates that is important to take non-stationary volatility into account when testing for a bubble in a time series with unstable volatility. The methods used in our work are significantly more robust because they allow for episodes of explosivity and heteroskedasticity.

[9] Note that the original SADF test suffers from severe size distortion under nonstationary volatility, as is observed in the existing literature.

In general, the results corroborate those presented in Esteve and Prats (2022) identifying the same four periods: the first between 1874–1880 related to the first and second Cuban wars and the budget efforts made in this period; the second episode, occurred in 1917–1920, related to the fiscal adjustment that occurred after an explosive debt path associated with Cuban war; the third episode, dated between 1951 and 1981, is associated to another fiscal adjustment during the period of Franco's regime until the arrival of democracy in 1979; and the fourth episode, between 1982 and 2002, was the result of chronic government deficits.

But also, the results demonstrate that the explosive behavior of the Spanish public debt GDP ratio during the period 1850-2021 can also be partly explained by changes in volatility.

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Appendix

Table 1
Descriptive Statistics, Spanish public-debt-GDP ratio

Statistics	1850-2021
Mean	67.3
Minimum	11.2
Maximum	169
Standard deviation	35.289
Variance	1245
Skewness	0.636
Kurtosis	0.293

Table 2
Test for explosive bubbles under time-varying volatility in Spanish public debt-GDP ratio p-values

Panel (a) $SADF$ tests				
$SADF$	$SADF_b$	SBZ_u	$sSADF_u$	$STADF$
0.4695	0.4034	0.2565	0.6649	0.1071

Panel (b) $GSADF$ tests				
$GSADF$	$GSADF_b$	$GSBZ_u$	$sGSADF_u$	$STADF$
0.000	0.0788	0.1239	0.1514	0.3083

Note: *, **, and *** denote significance at the 1%, 5%, and 10% levels, respectively.

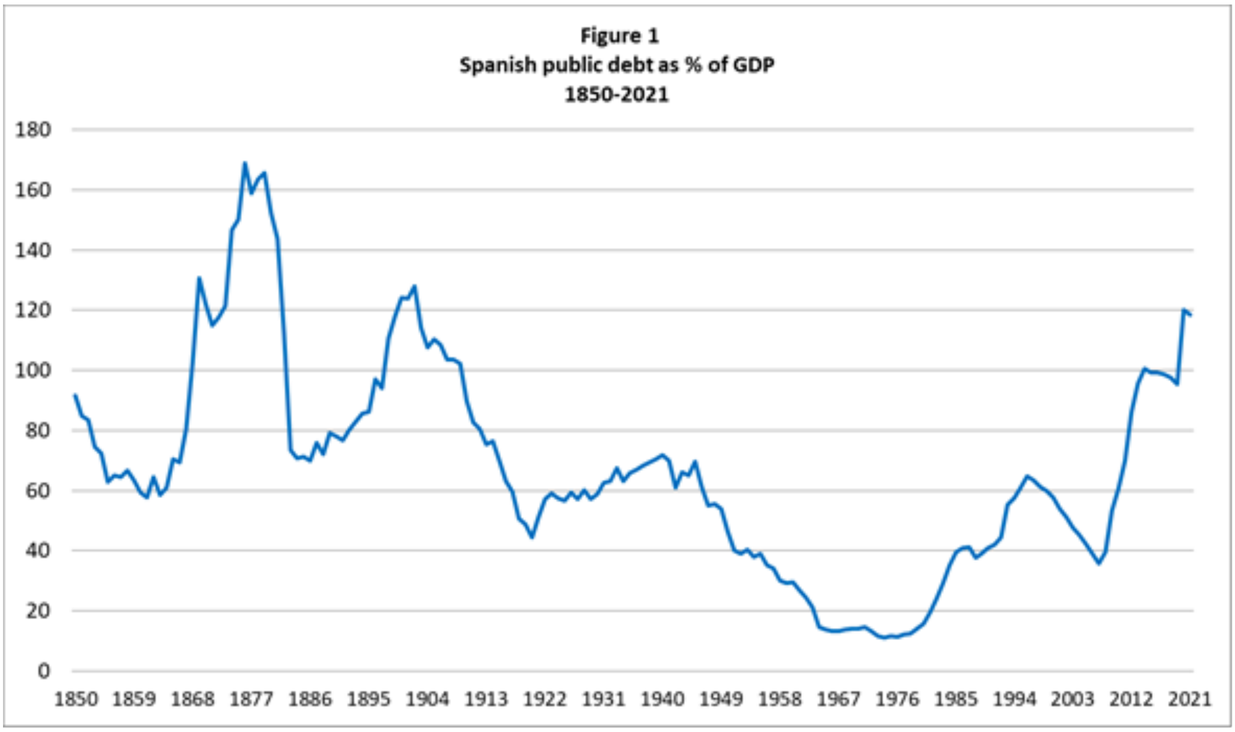
Figure 1
Spanish public debt as % of GDP
1850-2021
180
160
140
120
100
80
60
40
20
0
1850
1859
1868
1877
1886
1895
1904
1913
1922
1931
1940
1949
1958
1967
1976
1985
1994
2003
2012
2021

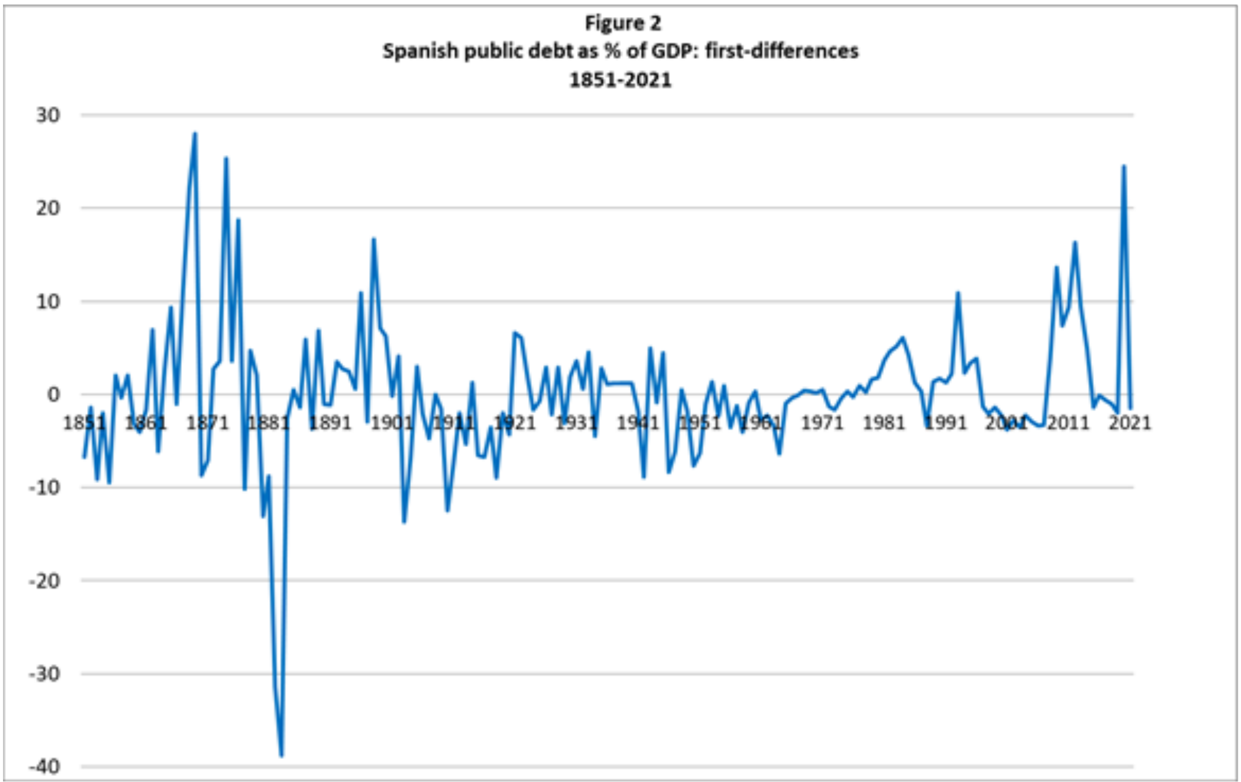
Figure 2
Spanish public debt as % of GDP: first-differences
1851-2021
30
20
10
0
-10
-20
-30
-40
1851 1861 1871 1881 1891 1901 1911 1921 1931 1941 1951 1961 1971 1981 1991 2001 2011 2021

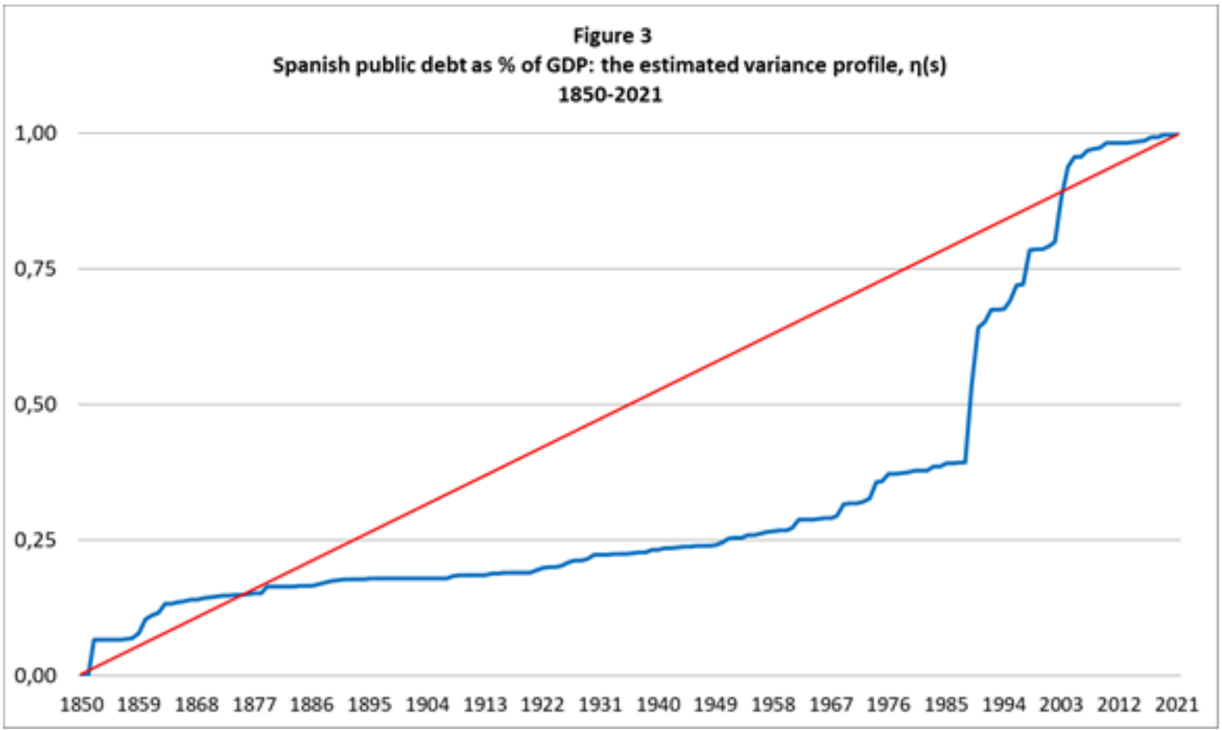
Figure 3
Spanish public debt as % of GDP: the estimated variance profile, η(s)
1850-2021
1,00
0,75
0,50
0,25
0,00
1850 1859 1868 1877 1886 1895 1904 1913 1922 1931 1940 1949 1958 1967 1976 1985 1994 2003 2012 2021

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