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Abstract

This paper offers a theoretical framework to understand the coevolution of social interactions and long-term economic growth. It begins by considering that most traditional societies did not have educational markets. Thus, access to the required knowledge for transiting to a modern economy had to be transmitted through social interactions, in particular, through the interaction between heterogeneous groups of people–i.e. distant interactions. Once immersed in a modern economy, the productive system should have increased the demand for knowledge, promoting more distant interactions. Simultaneously, the emergence of distant interactions should have affected the connectivity of society, reducing its heterogeneity, making cheaper posterior interactions but reducing their profitability. Moreover, social interactions competed and benefited from other non-market activities, child rearing specifically. The model arrives at four basic predictions. First, modern economic growth brings a more cohesive society. Second, modern economic growth brings long-term reductions in fertility with potential short-term increases. Third, initial barriers to social interactions could explain the timing of modern economic growth arrival. Forth, the timing of modern economic growth arrival could explain current output levels. I exploit different data sources to offer evidence in support of these predictions.

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1 Introduction

The arrival of modern economic growth has been one of the most drastic transformations in social history. In less than three centuries, modern economic growth has brought about 97% of the population and 99% of the market-value wealth that has ever existed (The Maddison Project, 2018). It has also been related to gigantic changes in living conditions. In England, for instance, low income groups had, for thousands of years, a life expectancy of 33 years, an average stature of 168.5 cm., and a literacy rate of 30%. After a couple of centuries of modern economic growth, these groups have a life expectancy of 74.3 years, a stature of 176 cm., and a literacy rate of 88% (Clark, 2008).

The massive impact of modern economic growth has motivated a complete field in economics that looks for its causes. From this field, we have learned the importance of the interaction between technology, preferences, institutions, and geography on the economic performance of societies in the long-run. An agenda in this field has noticed some empirical regularities, which suggest that the way people interact in non-market contexts plays an important role in long-term performance as well. Classical studies such as Easterly and Levine (1997), Alesina et al. (2003), and Alesina and Ferrara (2005) find, for cross-country-level data, negative correlations between ethnic diversity/heterogeneity and economic growth. This negative correlation is usually explained as a result of the propensity to political conflict, and the constraints to the diffusion of technology attached to people's heterogeneity. Putterman and Weil (2010), Ager and Brückner (2013), and Ashraf and Galor (2013), exploiting different episodes of migration flows at different levels of analysis, find certain circumstances in which population diversity/heterogeneity correlates positively with economic growth. The explanation in these cases is that certain levels of heterogeneity promote the emergence of new ideas and technological progress through social learning. Recent studies by Desmet et al. (2012, 2016) and Spolaore et al. (2016) offer additional evidence in favor of these results. Despite the success of this agenda, its results have been poorly articulated with long-term economic growth theory. Thus, we ignore how these social interaction aspects coevolve with essential elements of long-term growth, such as the changes in consumption level, fertility, and sectoral composition.

This paper intends to fill that gap, offering a mainstream economic growth model that incorporates social interactions in non-market environments. The model describes households that have the same productivity and preferences over consumption and fertility, but which differ in a non-market dimension–e.g. creed, worldview, habits. They live in a society that starts at a Malthusian regime, with zero per capita economic growth. The society has two economic sectors and several production technologies. Human capital markets do not exist. Therefore, the knowledge required for using the most advanced technologies comes from social interactions–i.e. spending resources with others in non-market activities. This leads households to invest in social interactions. The social-interaction decisions affect the structure of the social network, which modifies social-interaction cost. Therefore, social interactions are an endogenous process that feeds back from economic growth.

The model has four predictions. First, societies with fewer restrictions to social interactions during their pre-industrial period should have adopted the most advanced technologies more quickly, industrializing and transiting to a sustained-growth path earlier. Second, the model predicts that the initial differences in the timing of industrialization should have generated persistent effects that can explain some of the current income disparities across societies. Third, the model implies that economic growth brings a non monotonic reduction of fertility rates. Forth, the model predicts that, once a society industrializes, the amount and intensity of social interactions between different types of people will increase with time, and the social distance will decrease. I exploit different data sources to offer evidence in support of these predictions

Therefore, I offer a theoretical framework, which helps to understand that besides the causal effect of diversity on current output levels, there is a set of dynamic processes–most of which involve bidirectional causality–that connect social interactions and economic activity in the long-run.

The paper is organized as follows. First, as a conceptual framework, I reflect on the definition of modern economic growth in Section 2. Then, I present the related literature in Section 3. In Section 4, I present the model, dividing the presentation into a static and a dynamic component. The main results from the model are tested empirically in Section 5. Finally, I discuss the results in light of the current literature on long-run economic growth in Section 6.

2 Defining Modern Economic Growth

There are two definitions of modern economic growth. On the one hand, Simon Kuznets defines modern economic growth as a path characterized by five elements: first, high rates of growth of population and per capita income; second, an increasing productivity; third, a high rate of structural transformation of the economy, mainly referring to the shift away from agriculture to non-agricultural pursuits; fourth, modernization in sociological terms, referring to aspects like urbanization and secularization (see Inglehart and Welzel, 2005); and fifth, a more interconnected society (Kuznets, 1966, 1973). On the other hand, we have the definition of the unified growth theory (hereafter UGT) (Galor and Weil, 1999; Galor, 2005, 2007, 2011). This theory considers that economic history can be categorized in three different regimes: a Malthusian, a post-Malthusian, and a modern growth regime. Thus, under this tradition, modern economic growth refers to a situation in which population growth no longer counterbalances the rise in aggregate income triggered by rapid technological progress and factor accumulation.

Kuznets' and UGT's definitions have several points in common. The most fundamental is that they embrace the "incremental" conception of growth, typical of the postwar economic development thought (see Rostow, 1992; Easterly, 2006). This conception, well identified in Rostow's idea of "stages of growth", argues that societies go through a sequence of stages, each one of more economic sophistication and higher living standards (Rostow, 1990, 1960, 1956). Within such conception, both Kuznets' and UFT's definitions of modern economic growth refer to similar historical episodes. For instance, both Europe and Latin America experienced an industrial take-off, Europe in the 19th century and Latin America in in the early 20th century. However, their concepts differ in the particular social changes they emphasize. For instance, demographic transition is a minor concern in Kuznets' view, but it is the essence of UFT's definition. Meanwhile, structural change might be considered a secondary element in UFT, but an indispensable one in Kuznets' idea.

Skipping the conceptual discussion, most current studies define modern economic growth in a fairly pragmatic manner (e.g. Prak, 2005; Mokyr, 2005; Baten and Van Zanden, 2008; Strulik, 2014). They focus on the general features of the idea: the drastic transformation of the productive structure and the arrival of sustained economic growth–leaving the ambiguities of the term to be solved as particularities of the historical episodes analyzed. This paper follows such pragmatic approach. Thus, the focus on the evolution of social interactions and structural change is quite close to the Kuznets' ideas. However, the analytic tools used are completely inspired by UGT.

3 Related Literature

Four sets of literature motivate this study.

In the first place, the literature on diversity and economic growth offers the most immediate reason to expect a relationship between social interactions and modern economic growth. This literature considers that the composition of cultural/genetic attributes of a country's population determines its long-term economic performance (see Easterly and Levine, 1997; Alesina and Ferrara, 2005; Spolaore and Wacziarg, 2009; Desmet et al., 2012; Ashraf and Galor, 2013; Spolaore et al., 2016). This literature is mostly empirical and lacks a theoretical framework coherent with regular growth facts. This paper offers such theoretical framework.

In the second place, there is an extensive theoretical literature in macroeconomics on ideas/knowledge diffusion via social interactions. Jovanovic and Rob (1989), Alvarez et al. (2013), and Buera and Oberfield (2016) describe the emergence of new ideas and technological progress as a result of the interaction between agents with different prior knowledge. They model social interactions as reduced-form matches between individuals that meet with certain probability. My model follows a similar approach. The knowledge required for using modern technology will come–with certain probability–from social interactions. However, social interactions will not be stochastic, they will be optimal choices, determined by the fundamentals of the social network[1].

In the third place, a well-established agenda in social network analysis and organizational studies support empirically the social-interaction abstractions of my model. They relate creativity and innovation to social network positions that connect different components of the network. For instance, Burt (2004) offers evidence from managerial networks in organizations that shows how nodes that link groups fairly disconnected in a network–i.e. *brokers*–have a larger proportion of "good ideas". In addition, these nodes are more likely to express their ideas, less likely to have ideas dismissed, and are faster innovation adopters (Burt, 1987). Another example is the work of Cattani and Ferriani (2008), who examine the role of social networks in shaping individuals' ability to generate creative outcomes. They test their hypotheses within the context of the Hollywood motion picture industry. They find that intermediate core/periphery positions are highly correlated with more creative outcomes. Similar results are found in several different contexts by Cross and Cummings (2004), Rodan and Galunic (2004), Fleming et al. (2007), Baer (2010), Dahlander and Frederiksen (2012), Reagans and Zuckerman (2008), Gilsing et al. (2008), and Vasudeva et al. (2013)[2].

Finally, an increasing number of papers in economic history are interested in social interactions that promoted the technological innovations related to industrial activities[3]. For

[1] In that sense, this paper is related to Fogli and Veldkamp (2012). They explore how different social network structures affect technology diffusion, thereby a society's rate of technological progress, and, implicitly, its rate of economic growth. They argue that certain societies, as a result of their efforts to reduce the risk of an infection entering the collective, might develop stable network structures that inhibit technology diffusion, growing less than other societies with different social network structures.

[2] In terms of group performance, Hong and Page (2001, 2004); Lazer and Friedman (2007); Page (2008) show under what theoretical conditions groups of diverse problem solvers can outperform groups of high-ability problem solvers. Thus, they find results for complete groups consistent with the idea that networks that favor non-redundant information flows have better performance. However, the empirical literature in organizations is not conclusive (Cummings and Cross, 2003; Cummings, 2004; Mehra et al., 2006; Balkundi et al., 2007; Shore et al., 2009).

[3] Another group of studies in economic history have directed their attention to social networks for interpreting modern growth transformations. Rose (2000), Musacchio and Read (2007), and Schisani and Caiazzo (2016) explore the origins of industrialization, while Greif (2006), Lopez-Morell and O'Kean (2008), Rubin (2010), and James and Weiman (2010) do it on banking. Most of these studies focus on the role of trust and personal ties, following a similar argument to the one presented in the seminal papers of Greif (1989) and Greif (1993)

instance, Squicciarini and Voigtländer (2014) show the importance of exceptional channels of information like the *Encyclopédie* in human capital accumulation and city growth after the onset of French industrialization. Mejia (2018) shows that people better located, in terms of their capacity to connect different components of the social network, had a higher probability of becoming industrial entrepreneurs in the first stages of the industrialization of Colombia. Dudley (2012); de la Croix et al. (2015) offer similar arguments for the British Industrial Revolution[4].

4 The Model

4.1 General Assumptions

I develop a non-overlapping generations model inspired by the UGT tradition. Its structure is quite similar to Galor and Mountford (2008) autarkic model. As Galor and Mountford (2008), it has two sectors with different technologies that become available in certain stages of development. Individuals take decisions on consumption and fertility. All this happens in a world where aggregate knowledge increases inertially, there is a feedback between technology and human capital accumulation, and a quantity-quality children trade-off exists.

My main innovation with regard to Galor and Mountford (2008) is to incorporate non-market social interactions. In this model, individuals make choices on how much resources to use interacting with others. Social interaction is supported by the fact that individuals have the same productivity and preferences over consumption and fertility, but they differ in a non-market dimension–e.g. creed, worldview, habits. This opens the door to learning processes from social interaction–i.e. human capital accumulation coming from social interaction[5]. The differences between individuals determine the cost–i.e. opportunity cost in terms of

on the Maghribi traders of the 11th-century Mediterranean. Those papers support the idea that in absence of formal institutions that support market-based exchange, closely-knit and multi-stranded social networks might generate social capital, norms, information and sanctions that provide an alternative framework within which exchange and complex productive activity can develop.

[4]Hardly cataloged as part of one of the mentioned sets of literature, Lindner and Strulik (2014) relates closely to this paper. They also explore the coevolution of social networks and economic growth, with a particular emphasis on modernization. The essence of their model is an informational mismatch between investors and managers. I consider that such mismatch does not capture the real role of social interactions in long-term economic activity–i.e. the non-market channels of knowledge diffusion. This is specially relevant for the historical contexts where the question of modernity arrival makes sense. Therefore, our papers are similar in purposes but different in approaches.

[5]This is an idea that can be traced back to John Stuart Mill, who argued that ”it is hardly possible to overrate the value... of placing human beings in contact with persons dissimilar to themselves, and with modes of thought and action unlike those with which they are familiar... Such communication has always been, and is peculiarly in the present age, one of the primary sources of progress.” (Mill, 1848, p.581).

consumption–and benefits–i.e. higher wages from human capital accumulation–of social interaction. Eventually, the socialization process affects the growth path.

Two assumptions characterize the incorporation of social interactions in the model.

On the one hand, human capital accumulation is an indirect result of social interaction. In particular, forefathers' socialization determines the likelihood that a particular individual becomes a skilled worker[6]. This assumption intends to capture two aspects of knowledge transmission in societies in transit to modern economic growth. First, the type of knowledge relevant for industrial take-off is not easily transmitted through markets. It is a mix of scientific, marketing, and organizational skills, which usually results from personal experience and the interaction with people from different backgrounds[7]. Second, in most cases, pre-modern societies were characterized by the absence of education markets–i.e. there was no supply of higher education. Knowledge was transmitted from person to person in a process conditional on the institutional environment where family was essential (de la Croix et al., 2015).

On the other hand, I assume that the costs and profits from social interaction are endogenous, resulted from how different individuals are and how does the economy perform. Thus, this model not only shades lights on the effects of social networks on economic growth, but also on the effects of economic growth on social networks.

4.2 Basic Structure

Consider an economy that extends over infinite discrete time. There are two factors of production: skilled labor–or human capital, H_t, and unskilled labor, L_t. The supply of both types of labor evolves over time in an endogenous process. There are two goods, a manufactured good, Y_t^m, and an agricultural good, Y_t^a, each of which is produce by a different firm.

4.2.1 Production

The manufactured good can be produced with two different technologies, a traditional and a modern technology. Just one of the technologies is active in period t. In Section 4.4.1 I discuss the conditions under which each technology is viable. The introduction of the new

[6]The stochastic aspect of this assumption–i.e. parents cannot choose the precise number of skilled children they have–does not eliminate the quantity-quality children trade-off, but do restrict the capacity of individuals to exploit it.

[7]Squicciarini and Voigtländer (2014) call it "upper-tail knowledge" and consider to access information to the *Encyclopédie* as a mechanism for its acquisition.

technology in manufacturing represents the appearance of industrial production, and the arrival of modern economic growth. Meanwhile, there is a unique technology for producing the agricultural good[8].

Agriculture The agricultural good is produced using a constant returns to scale function that employs only unskilled labor. Thus, the output of the agricultural good in period t, Y_t^a, is

$$Y_t^a = A_t^a L_t^a \tag{1}$$

where L_t^a and A_t^a are the unskilled labor employed and the level of productivity of the agricultural technology in period t, respectively.

Manufacturing The manufacturing firm can produce with either a traditional technology or a modern one. If the firm uses the traditional technology, its output at period t, $Y_t^{m,O}$, follows a constant returns to scale production function, similar to the technology used in the agricultural sector.

$$Y_t^{m,O} = a_t^m L_t^{m,O} \tag{2}$$

where $L_t^{m,O}$ and a_t^m are the unskilled labor employed and the level of productivity of the traditional industrial technology in period t, respectively.

If the firm uses the modern technology, they need to employ an additional productive factor, H_t. As I argued above, this is a particular kind of skilled labor required in industrial production. It might be interpreted as the entrepreneurial initiative of offering a new product, the technological abilities to structure a production based on modern science, or the managerial capacity to coordinate patterns of high division of labor. In any case, it should not be confused with the regular conception of human capital as education level–as this economy does not have an education market that offers homogeneous units of human capital.

The manufactured output resulted from the modern technology, $Y_t^{m,N}$, follows a neoclassical constant returns to scale production function.

$$Y_t^{m,N} = A_t^m F(H_t L_t^{m,N}) = A_t^m f(h_t) L_t^{m,N} \tag{3}$$

[8] Incorporating an additional technology with decreasing returns would generate a Malthusian regime. This would not change the results with regard industrial take-off.

where $h_t \equiv \frac{H_t}{L_t^{m,N}}$. $L_t^{m,N}$, A_t^m are the unskilled labor employed and the level of productivity of the modern manufacturing technology in period t, respectively.

Prices From 1 we have that the inverse demand for unskilled labor in agriculture is

$$w_t^u = p_t A_t^a \tag{4}$$

where w_t^u is the nominal wage of unskilled labor, and p_t the relative price of the agricultural good in period t. The manufactured good is used as numerary, thus, all the nominal variables are measured in number of manufactured goods.

Similarly, 2 and 3 give the inverse demand for skilled and unskilled labor in manufacturing, which is

$$w_t^u = \begin{cases} a_t^m & \text{if } Y_t^{m,0} > 0 \\ A_t^m[f(h_t) - h_t f'(h_t)] \equiv A_t^m w^u(h_t) & \text{if } Y_t^{m,N} > 0 \end{cases} \tag{5}$$

and

$$w_t^h = A_t^m f'(h_t) \equiv A_t^m w^h(h_t) \qquad \text{if } Y_t^{m,N} > 0 \tag{6}$$

where w_t^h is the wage of skilled labor.

The ratio of skilled and unskilled wages will determine the factor allocation between sectors at equilibrium. Let me define it as

$$\frac{w_t^h}{w_t^u} = \frac{f'(h_t)}{f(h_t) - h_t f'(h_t)} \equiv w(h_t) \qquad \text{if } Y_t^{m,N} > 0 \tag{7}$$

As a result of the neoclassical properties of $f(h_t)$ we have that $w'(h_t) < 0$, $\lim_{h\to 0} w(h_t) \to \infty$, and $\lim_{h\to\infty} w(h_t) = 0$.

Finally, the agricultural and manufacturing firms share a common unskilled labor market– i.e. there is perfect labor mobility. Thus, if both goods are produced, the wages of unskilled workers in agriculture and manufacturing will equalize. Based on this and from 4 and 5 we have that p_t, the relative price of the agricultural good in period t, is

$$p_t = \begin{cases} \frac{a_t^m}{A_t^a} & \text{if } Y_t^{m,0} > 0 \\ \frac{A_t^m w^u(h_t)}{A_t^a} & \text{if } Y_t^{m,N} > 0 \end{cases} \tag{8}$$

4.2.2 Individuals

Individuals are passive agents during their first period of life. They limit to consume a fraction of their parental earnings in the form of child rearing. In the transition from their first to their second period of life, some of them become skilled, h, or unskilled workers, u; depending on the socialization choices of their forefathers.

In their second period they are endowed with one unit of time that they will offer inelastically in the labor market. The price of their unit of time would depend on whether they became skilled or unskilled workers. During that period, individuals optimally allocate their earnings between consumption, child rearing, and social interaction. After the second period, individuals die.

Social interaction A key novelty of this model is that it incorporates social interaction choices as part of the optimal behavior of individuals. These choices are conscious decisions regarding the amount of resources spent interacting with people in non-market contexts–e.g. meeting new people at parties or rituals, having a meal with colleagues, giving presents to friends and relatives. Individuals' decisions on social interactions configure the structure of the social network, which defines how costly these interactions are for each individual in the future. Therefore, social interaction is an endogenous process in this model.

To begin with, based on the literature on social network analysis, it is possible to categorize social interactions into two types. On the one hand, there are *embedded interactions*, which are those that are developed among people that share a common social circle. For instance, time spent with family is an embedded interaction, as far as most family members share ties among them. On the other hand, there are *distant interactions*, which are those that take place between people that do not share a common social circle. In that sense, ties with people from different casts or tribes would be distant interactions. Embedded interactions are closely related to the idea of strong ties and network closure, while distant interactions relate to weak ties and structural holes (see Granovetter, 1973; Burt, 2005; Aral and Van Alstyne, 2011; Brashears and Quitane, 2016).

As the literature that inspires this paper (see Section 1) emphasizes the role of non-redundant information on innovation and modernity, this model will explore exclusively distant interactions [9]. Thus, from now on, the term social interactions will refer uniquely to distant interactions.

Formally, in period t, Individual i invests τ_t^i units of social interaction[10]. Social interaction

[9] However, it is possible to incorporate also embedded interactions in an identical set up and have the same qualitative results.

[10] This is an abstraction of the intensity of social interaction. It might include an extensive margin–e.g.

choices accumulate through time in stocks of social capital. The stock of *social capital* of Individual i in period $t+1$ is defined as S^i_{t+1}. Without loss of generality I will assume that the stock of social capital does not depreciate. In summary, the law of motion for social capital is

$$S^i_{t+1} = S^i_t + \tau^i_t \tag{9}$$

Assuming completely reciprocal interactions, the *aggregate stock of social capital* at time t is $S_t = N_t S^i_t$, where N_t is the adult population at time t. This stock can be interpreted as the number–and strength–of connections formed over history in the whole society. The aggregate stock of social capital is one of the two aspects that characterize the network structure in this model. The other aspect is *social distance*, which refers to the general idea of how “far away” are people in society. In this context, “far away” means how easily people can be accessed by others for establishing a particular kind of social interaction, for instance, getting married[11]. Formally, social distance at period $t+1$, D_{t+1}, is a function d of the contemporary aggregate stock of social capital and the previous social distance.

$$D_{t+1} = d(S_{t+1}, D_t) \tag{10}$$

The particular shape of d depends on cultural and institutional aspects, but, in general, it would be true that $d_S < 0$, $d_{SS} > 0$, $d_D > 0$, $d_{DD} < 0$. Moreover, social distance has a lower bound: $D_t \geq \bar{D}\ \forall\, t$, with $\bar{D} \in \mathbb{R}^+$, and $\lim_{S_t \to \infty} d_S = 0$.

Social distance determines how costly social interactions are. In particular, in societies with higher social distances it costs more to socialize. I consider a convex cost function. More precisely, if Individual i invests τ^i_t units of social capital it would cost him $\frac{\varphi(D_t)}{2}\frac{{\tau^i_t}^2}{S^i_t}$ units of salary, with $\varphi(D_t) > 0$, $\varphi'(D_t) > 0$, and $\varphi''(D_t) < 0$, $\forall\, D_t$ that satisfies the properties of 10. This implies that an additional unit of social capital invested is more costly *per se*, and increasing social distances increment the costs of investing in social capital.

The particular shape of $\varphi(D_t)$ depends on cultural and institutional factors. Hence, the model describes a permanent feedback between social network structure and cultural/institutional conditions. Consider, for instance, a society with a large social distance. The high distance must be supported by cultural/institutional constraints that inhibit the interaction between people with certain types of attributes–e.g. segregated spaces by race, restrictions for inter-ethnic marriages–but the infrequent interaction between the groups with these attributes

number of meetings with new people, number of ties created–and an intensive one–e.g. amount of information shared, efforts building trust for every existing tie.

[11] The closest concept in network topology for social distance is the average path length, which is defined as the average number of steps along the shortest paths for any pairs of network nodes. However, other measures such as the diameter of the network also capture the spirit of social distance concept.

must support those cultural/institutional constraints as well[12].

Preferences and budget constraints Individuals value consumption and the future welfare of their children. The model captures children's welfare as a warm-glow giving, in which parents receive utility directly from the expected wages of their children. Individuals become skilled or unskilled in the interim period as a result of their ancestry's social interaction choices.

In particular, consider $g(\eta_t^i = \eta_t^{i,h}|S_{t+1}^i)$ as the probability that each child of i becomes a skilled worker conditional to his stock of social capital. It follows that $g(\eta_t^i = \eta_t^{i,u}|S_{t+1}^i) \equiv 1 - g(\eta_t^i = \eta_t^{i,h}|S_{t+1}^i)$ is the probability that each child of i becomes an unskilled worker conditional to his stock of social capital. With $\frac{\partial g(\eta_t^i=\eta_t^{i,h}|S_{t+1}^i)}{\partial S_{t+1}^i} > 0$, and $\frac{\partial^2 g(\eta_t^i=\eta_t^{i,h}|S_{t+1}^i)}{\partial {S_{t+1}^i}^2} < 0$.[13] The intuition here is that individuals that inherited a higher social capital must know more people (or must have known them better), having access to larger amounts of information and ideas, and being more likely to develop those abilities that the new productive sector requires.

As n_t^i is the number of i's children, the expected wage of i's children in period t is $w_{t+1}^h n_t^i g(\eta_t^i = \eta_t^{i,h}|S_{t+1}^i) + w_{t+1}^u n_t^i \left(1 - g(\eta_t^i = \eta_t^{i,h}|S_{t+1}^i)\right)$.

In summary, the utility function of a member i, of generation t–i.e. someone born in $t-1$–is

$$u_t = (c_t^{i,a})^\alpha (c_t^{i,m})^\beta \left[w_{t+1}^h n_t^i g(\eta_t^i = \eta_t^{i,h}|S_{t+1}^i) + w_{t+1}^u n_t^i \left(1 - g(\eta_t^i = \eta_t^{i,h}|S_{t+1}^i)\right)\right]^{1-\alpha-\beta} \quad (11)$$

where $c_t^{i,a}$ is i's consumption of the agricultural good and $c_t^{i,m}$ her consumption of the manufactured good.

I will assume preferences that satisfy Engel's Law. In order to do this, I incorporate an additional consumption constraint[14], whereby individuals must consume a subsistence level of the agricultural good, $\bar{c}$.

In addition to the subsistence consumption restriction, individuals have a regular budget

[12] As you can see, social distance is a quite general and abstract concept that can describe several kinds of network structures. Moreover, it might not have a clear correlation between other aspects of the social network, such as the degree or the clustering coefficient distribution, that are important for individual performance. However, my interest focuses on the capacity of individuals to access new information, ideas, and knowledge. Thus, social distance do capture the basic role of network structure in this process: the farther the individuals are in society, the harder it is for a particular individual to access the knowledge of the others. In any case, this can only be interpreted as a reduced form of a complex social network structure.

[13] Notice that the probability of acquiring human capital depends on the ancestry's social capital investment. Individuals do not take any social interaction choices before they get educated.

[14] This assumption generates identical qualitative results to the use of Stone-Geary functions, the usual way of having preferences that follow Engel's Law.

constraint. It includes the cost of consumption, social interaction, and child rearing. Child rearing has a constant cost of θw_t^i per child. With $0 < \theta < 1$. Thus, the budget constraint of individual i at period t is:

$$p_t c_t^{i,a} + c_t^{i,m} + w_t^i \left[\theta n_t + \frac{\varphi(D_t)}{2} \frac{{\tau_t^i}^2}{S_t^i} \right] \leq w_t^i \tag{12}$$

Optimization Individuals allocate their income between consumption, social interactions, and child rearing. They choose the number of children, the amount invested in social capital, and the amount of consumption of manufactured and agricultural goods.

Hence, a member i of generation t chooses the set $\{c_t^{i,a}, c_t^{i,m}, n_t^i, \tau_t^i\}$ that maximizes her utility. Formally, the individual's problem is

$$\max_{\{c_t^{i,a}, c_t^{i,m}, n_t^i, \tau_t^i\}} \quad u_t = (c_t^{i,a})^{\alpha} (c_t^{i,m})^{\beta} \left[w_{t+1}^h n_t^i g(\eta_t^i = \eta_t^{i,h} | S_{t+1}^i) + w_{t+1}^u n_t^i \left(1 - g(\eta_t^i = \eta_t^{i,h} | S_{t+1}^i) \right) \right]^{1-\alpha-\beta}$$

such that,

$$\begin{gathered} p_t c_t^{i,a} + c_t^{i,m} + w_t^i \left[\theta n_t + \tfrac{\varphi(D_t)}{2} \tfrac{{\tau_t^i}^2}{S_t^i} \right] \leq w_t^i \\ S_{t+1}^i = S_t^i + \tau_t^i \\ D_{t+1} = f(S_{t+1}, D_t) \\ N_{t+1} = N_t n_t \\ c_t^{i,a} > \bar{c} \end{gathered}$$

Given $\{S_0^i, D_0, N_0\}$

The characteristics of the utility function guarantee positive consumption of both goods. However, the individual's optimal choice depends on the income level with regard to the subsistence consumption constraint. If income is low enough, the constraint will bind and a large, but decreasing, share of the income will be devoted to agricultural consumption. Meanwhile, if the subsistence consumption constraint does not bind, the log-linearity of the utility function generates fixed shares of income assigned to non-market interactions–child rearing and socialization–and to the consumption of agricultural and manufactured goods.

In summary, the consumption of the agricultural good by Individual i during period t, $c_t^{i,a}$, is

$$c_t^{i,a} = \begin{cases} \bar{c} & \text{if } \alpha \dfrac{w_t^i}{p_t} < \bar{c} \\ \alpha \dfrac{w_t^i}{p_t} & \text{if } \alpha \dfrac{w_t^i}{p_t} \geq \bar{c} \end{cases} \tag{13}$$

The consumption of the manufactured good by Individual i during period t, $c_t^{i,m}$, is

$$c_t^{i,m} = \begin{cases} 0 & \text{if } \frac{w_t^i}{p_t} < \bar{c} \\ \frac{\beta}{1-\alpha}(w_t^i - p_t\bar{c}) & \text{if } \alpha\frac{w_t^i}{p_t} < \bar{c} < \frac{w_t^i}{p_t} \\ \beta w_t^i & \text{if } \alpha\frac{w_t^i}{p_t} \geq \bar{c} \end{cases} \tag{14}$$

The aggregate income spent by Individual i during period t in non-market activities–which determines her number of children and her amount of social interaction–is

$$\left[\theta n_t + \frac{\varphi(D_t)}{2}\frac{{\tau_t^i}^2}{S_t^i}\right] = \begin{cases} 0 & \text{if } \frac{w_t^i}{p_t} < \bar{c} \\ \frac{1-\alpha-\beta}{1-\alpha}\frac{w_t^i - p_t\bar{c}}{w_t^i} & \text{if } \alpha\frac{w_t^i}{p_t} < \bar{c} < \frac{w_t^i}{p_t} \\ 1-\alpha-\beta & \text{if } \alpha\frac{w_t^i}{p_t} \geq \bar{c} \end{cases} \tag{15}$$

where,

$$\frac{w_{t+1}^h}{w_{t+1}^u} = 1 + \frac{\varphi(D_t)\tau_t^i}{\theta S_t n_t^i \frac{\partial g(\eta_t^i = \eta_t^{i,h} | S_{t+1}^i)}{\partial S_{t+1}^i} - \varphi(D_t)\tau_t^i P(\eta_t^i = \eta_t^{i,h} | S_{t+1}^i)} \tag{16}$$

4.2.3 Social Interaction and Fertility Decisions

From equations 15 and 16 we have the following optimal relations between n_t^i and τ_t^i

$$n_t^i = \begin{cases} 0 & \text{if } \frac{w_t^i}{p_t} < \bar{c} \\ \frac{1}{\theta}\left[\frac{1-\alpha-\beta}{1-\alpha}\frac{w_t^i - p_t\bar{c}}{w_t^i} - \frac{\varphi(D_t)}{2}\frac{{\tau_t^i}^2}{S_t^i}\right] & \text{if } \alpha\frac{w_t^i}{p_t} < \bar{c} < \frac{w_t^i}{p_t} \\ \frac{1}{\theta}\left[(1-\alpha-\beta) - \frac{\varphi(D_t)}{2}\frac{{\tau_t^i}^2}{S_t^i}\right] & \text{if } \alpha\frac{w_t^i}{p_t} \geq \bar{c} \end{cases} \tag{17}$$

and

$$n_t^i = \frac{\varphi(D_t)\tau_t^i}{\theta S_t \frac{\partial g(\eta_t^i = \eta_t^{i,h} | S_{t+1}^i)}{\partial S_{t+1}^i}}\left[\frac{w_{t+1}^u}{w_{t+1}^h - w_{t+1}^u} + g(\eta_t^i = \eta_t^{i,h} | S_{t+1}^i)\right] \tag{18}$$

Equation (17) describes the force that relates negatively the number of children and the social capital investment. This is the result of the inherent constraint of time using. As individuals need to use a fraction θ of their income rearing each child, the more children they have, the less money will remain to socialize.

On the other hand, equation (18) presents the force that relates positively the number of children and the social capital investment (see Lemma 9.1). The profits of social capital investment, which increases with the number of children, is the phenomenon behind this force. In other words, as socializing increases the expected wage of children, the more children the individual has, the larger the investment in social capital that will be optimal.

Eventually, the balance of these two forces will determine the optimal combination of number of children and social interaction for an individual i at period t, $\{n_t^{i*}, \tau_t^{i*}\}$.

The second force depends on how profitable social capital investment is. In that sense, it is independent from current income. However, the first force does depend on the individual's current income. Thus, the model predicts three scenarios:

Scenario I (absolute consumption constraint): In this case, characterized by $\frac{w_t^i}{p_t} \leq \bar{c}$, current income is so low that individuals use all their resources to survive. Therefore, $\{n_t^{i*}, \tau_t^{i*}\} = \{0, 0\}$. As they do not reproduce, the economy collapses after one period.

Scenario II (relative consumption constraint): In this case, characterized by $\alpha\frac{w_t^i}{p_t} < \bar{c} < \frac{w_t^i}{p_t}$, current income is high enough for guaranteeing the individuals' survival. There is a negative relation between n and τ. However, set of number of children and social capital investment is constrained.

Scenario III (non-consumption constraint): In this case, characterized by $\alpha\frac{w_t^i}{p_t} \geq \bar{c}$, current income is high enough for guaranteeing the individuals' survival and their unrestricted election of $\{n_t^{i*}, \tau_t^{i*}\}$.

As the interest of the paper is not economic collapses, I will ignore the absolute-constraint scenario. Therefore, there will be only two types of societies: relatively constrained societies and non-constrained societies. In both of these scenarios the household's optimization problem will have a non-trivial solution (see Figure 1).

[Figure 1 here]

Lemma 4.1. *If $Y_t^{m,N} > 0$, then, $\forall$ t there is a unique optimal combination of number of children and the amount of social interaction, $\{n_t^{i*}, \tau_t^{i*}\} > 0$*

Proof. It follows from Figure 1, noting that one curve starts at the origin and it is monotonically increasing, while the other starts at a positive level and is monotonically decreasing. □

Lemma 4.1 marks that when society values the skills produced by social interaction–i.e. when the modern sector is active and demands human capital–individuals will always have children and invest positive amounts of social capital. Moreover, there will be a unique solution.

In addition to the result from Lemma 4.1, there is a corner solution resulted from a context in which modern manufacturing technology is not viable. Under these circumstances, skilled work will not be demanded. Therefore, the optimal choice of individuals will be to have children, but to assign no income to social interactions (see Lemma 9.2).

4.3 Comparative Statics

Examining the static equilibrium sheds light on how the network structure affects the social interaction and fertility decisions of individuals. Let me present the most relevant cases.

4.3.1 Constrained vs Unconstrained Societies

Consider two societies A and B, which are identical in every aspect except by the fact that A is below the consumption constraint while B is above it. Crossing the consumption constraint represents an income increase. Hence, *ceteris paribus*, an individual in B will socialize more and will have more children than an individual in A.

This result, which is formalized in Lemma 9.3, does not imply a monotonic and positive relationship between income and number of children and socialization. It only means that an individual, after crossing the income restriction, will have access to a higher set of children and social interactions. Thus, if the other society's fundamentals remain constant, individuals will prefer that new set of consumption. Sections 4.4.3 and 4.4.4 explore what happens with social capital and demographics along the growth path.

4.3.2 Differences in Technological Conditions

Now, consider differences in technological conditions–i.e. the parameters A_t^a, a_t^m, A_t^m differ across societies. First, notice that in equilibrium these differences will only affect constrained societies, because a variation in technological parameters will change the relative prices in the goods market, not in the labor market. This, because of perfect labor mobility. Let me describe this more deeply.

First, when income is low enough to put a society under a constrained situation, household's decisions on fertility and socialization will depend of the agricultural good price. As this price depends of the relative productivity of both sectors, if there are differences in the agricultural technology across societies, the relative price of the agricultural good will differ. In particular, consider two constrained societies, A and B, which are identical in every aspect except their total factor productivity level in the agricultural sector. More precisely, $(A_t^a)^A < (A_t^a)^B$. Under these circumstances, the number of children at A will be lower than those at B. The reason is that the agricultural good is cheaper in B. This frees a larger share of the individual's income to devote to child rearing and social interactions–see Figure 2.

[Figure 2 here]

If A and B differ only in the productivity of the manufacturing sector–consider, for instance, $(A_t^m)^A < (A_t^m)^B$). There will be no impact in fertility and socialization decisions. In this case, the technological difference will have an effect in wages–B will have higher wages than A. However, these differences will be exactly compensated by differences in goods' prices.

Under unconstrained circumstances the characteristics of household's preferences make fertility and socializing choices independent from current income and good prices. Therefore, differences in technological conditions will not impact those choices either.

Lemma 9.4 formalizes these ideas.

4.3.3 Differences in Social Distance

Now, consider two societies, A and B, which are identical in every aspect except their social distance. In particular, $D_t^A < D_t^B$. As the cost of social interaction increases monotonically with D_t, it follows that $\varphi(D_t^A) < \varphi(D_t^B)$. In other words, it is cheaper to form ties in A than in B. This drives simultaneous effects on the two forces that characterize the optimal decision of individuals. On the one hand, as social interaction costs less in A than in B, for any given level of social interaction, individuals in A can have more children with the same income. To put it differently, the negative relation between number of children and social interaction relaxes. Graphically, the downward curve has a lower slop in A than in B–see Figure 3.

On the other hand, as social interaction costs less, it is more profitable to invest in it. Therefore, for any given number of children, individuals in A will prefer to invest a larger amount of resources on social interaction. In graphical terms, the upwards curve has a lower slop in A than in B.

[Figure 3 here]

As a result, in equilibrium individuals in A invest more in social interactions than individuals in B. This is formalized in Lemma 9.5. In other words, reductions in social distance–ceteris paribus–generate higher levels of social interactions. The effects on fertility are ambiguous and depend on the relative effects over the two curves.

4.3.4 Differences in the Stock of Social Capital

Continuing with these analytic exercise, consider identical societies, which only differ in their social capital accumulation history. More precisely, A has a larger stock of social capital than B in period t. A larger stock of social capital reduces social interaction cost. Therefore, higher stocks of social capital generates identical qualitative results to the ones generated by shorter social distance. That is, a reduction in the slope of both curves (see Figure 3). Thus, in equilibrium, individuals in society A will invest more in social interaction than in B. This is formally presented in Lemma 9.6.

4.3.5 Differences in Population Size

Finally, what is the effect of having a larger population on fertility and social interaction choices? First, population size affects aggregate social capital (S_t) and, through it, it affects social distance. Thus, if A has a larger population than B at period t, it will have smaller cost of social interaction and a larger stock of social capital, leading to identical qualitative results to the ones Figure 3 presents. Lemma 9.7 formalizes this idea.

4.4 Equilibrium Growth Path

4.4.1 Technological Progress

As most UGT developments, this model assumes that economic history can be described as a linear sequence of stages. Societies start at ”early stages” of development, in which production is conducted using the old manufactured technology. The new technology becomes available when the value of the marginal product of the unskilled workers who use it is at least as high as the return to unskilled workers who use the old technology. Formally,

Lemma 4.2. *The new manufacturing technology is economically viable if*

$$\frac{A_t^m}{a_t^m} \geq \frac{1}{w^u(h_t)} \tag{19}$$

Proof. It is a result of establishing from Equation 5 that unskilled labor under the new technology is at least as productive as under the old technology. □

Also following the UGT tradition, I assume that productivity levels in each sector are a function of the aggregate knowledge of society λ_t. Formally,

$$A_t^a = A^a(\lambda_t), \quad a_t^m = a^m(\lambda_t), \quad A_t^m = A^m(\lambda_t) \tag{20}$$

Where $A^{a\prime} > 0$, $a^{m\prime} > 0$, $A^{m\prime} > 0$, and the general knowledge of society evolves as a result of human capital accumulation. More precisely,

$$\frac{\lambda_{t+1} - \lambda_t}{\lambda_t} = l(h_t) \tag{21}$$

With $l'(h_t) > 0$, $l''(h_t) < 0$, and $l(0) > 0$. Therefore, the aggregate knowledge of society increases with human capital accumulation–with decreasing returns. However, that knowledge grows even under the absolute absence of human capital. This is a reasonable assumption, noticing that the human capital considered here (h_t) refers to a particular kind of private knowledge useful in industrial production. Meanwhile, the aggregate knowledge of society (λ_t) refers to a wider idea of knowledge with some sort of non-excludable nature. For instance, popular knowledge of environmental conditions or physical regularities might increase with consuetudinary experimentation, even in a context where there is no improvement in industrial-related skills. However, R&D useful in industrial environments usually contributes to the aggregate knowledge of society.

In order to fit basic historical regularities technological progress must satisfy the following conditions:

1. The new manufacturing technology is not viable in period 0. Formally

$$\frac{A_0^m}{a_0^m} < \frac{1}{w^u(h_0)} \tag{22}$$

2. The productivity progress in manufacturing is larger than that in agriculture, and the new technology advances more rapidly than the old one. Formally,

$$A^{m\prime} > A^{a\prime} > a^{m\prime} > 0; \quad \lim_{h\to\infty} \frac{A^m}{a^m} = \infty \tag{23}$$

These assumptions generate a well-behaved context. Figure 4 describes how Lemma 4.2 develops under that context. On the one hand, $w^u(h_t)$ increases monotonically with h_t because of the relative reduction of unskilled labor. Hence, $\frac{1}{w^u(h_t)}$ is a negative function of h_t. Meanwhile, Equation 23 assures that $\frac{A_0^m}{a_0^m}$ is an increasing function of h_t. New technology will be available if, h_t^* , the human capital resulted from the optimal household choices, $\{n_t^{i*}, \tau_t^{i*}\}$, is at the right side of the point where both curves cut. Otherwise, the new technology will be unavailable.

[Figure 4 here]

Moreover, Equations 22 and 23 guarantee that societies will go through an increasing path towards development. That path will be characterized by growing productivity and, eventually, the perpetual viability of new technology. Lemma 4.3 characterizes the latter feature.

Lemma 4.3. *If Equations 22 and 23 apply, then*

There exists a period t^M in which the new manufacturing technology becomes economically viable, and remains as such onwards. Formally,

$$\frac{A_t^m}{a_t^m} > \frac{1}{w^u(h_t)} \quad \forall t \geq t^M \tag{24}$$

Proof. From Equation 23, and noting that $l(0) > 0$, it follows that $\frac{A_t^m}{a_t^m}$ increases monotonically over time. Meanwhile, as I will describe in Section 4.4.4, h_t is a non-decreasing function of time. □

Figure 5 describes the intuition behind Lemma 4.3. General knowledge increases every period even in absence of human capital accumulation and it impacts more the productivity of the new technology than the one of the old technology. Hence, every period the ratio $\frac{A_t^m}{a_t^m}$, for every level of human capital, increases. In that sense, every period a smaller fraction of skilled work is necessary for making the new technology available. Eventually the required h would be so small that the optimal expected human capital for household will be enough to generate the transition to modern economic growth.

[Figure 5 here]

In conclusion, the model is designed to predict two general stages of development. On the one hand, there is a *traditional stage*, located in $t < t^M$, in which production is conducted using the old manufacturing technology. On the other hand, when $t \geq t^M$, there is a *modern stage*, in which the new technology is used. In that sense, development is an inevitable process in this model. Eventually, every society will pass from the traditional to the modern stage. The interesting aspect of the model comes from the differences in the timing, the magnitude and the long-term effects of the transition to modern economic growth.

4.4.2 Labor Allocation and Structural Change

In this model, economic development comes with a changes in the importance of the different sectors–i.e. there is structural change. Two forces cause the structural change: the consumption constraint, which generates an Engel's Law preferences pattern; and the unbalanced technological change, which brings faster progress in the manufacturing sector.

Traditional societies The usage of the old manufacturing technology defines this stage of development. As skilled work is not demanded, unskilled labor dynamics characterize the full productive system. For that purpose, consider $l_t^a = \frac{L_t^a}{N_t}$ and $l_t^m = \frac{L_t^m}{N_t}$ as the share of unskilled labor employed in agriculture and manufacturing, respectively.

Based on Lemma 9.8 we know that

$$l_t^a = \begin{cases} \dfrac{p_t}{w_t^i}\bar{c} & \text{if } \alpha\dfrac{w_t^u}{p_t} < \bar{c} \\ \alpha & \text{if } \alpha\dfrac{w_t^u}{p_t} \geq \bar{c} \end{cases} \tag{25}$$

$$l_t^{m,0} = \begin{cases} \dfrac{\beta}{1-\alpha}\left(1 - \dfrac{p_t}{w_t^i}\bar{c}\right) & \text{if } \alpha\dfrac{w_t^u}{p_t} < \bar{c} \\ \beta & \text{if } \alpha\dfrac{w_t^u}{p_t} \geq \bar{c} \end{cases} \tag{26}$$

From these equations we can see that the share of labor employed in agriculture will decrease permanently until the society surpasses the consumption constraint. Meanwhile, industrial labor share will increase during the same period. After surpassing the consumption constraint the labor composition will become constant.

Modern societies The usage of the new technology in manufacturing defines this stage of development. Under the new technology, skilled labor is necessary for producing. Therefore, social interaction is profitable for individuals. Based on the results from Section 4.2.3 and the general assumptions of the productive system, we can prove that the labor market is stable. Formally,

Lemma 4.4. *If the new industrial sector is active. There exists a unique and stable ratio of skilled to unskilled labor, h_t^*, such that*

$$\frac{w_t^h}{w_t^u} = w\left(h_t^*\right) > 0 \tag{27}$$

And

$$\tau_{t+1}^i > \tau_t^{i^*};\ n_{t+1}^i < n_t^{i^*} \quad if \quad h_t < h_t^* \tag{28}$$

$$\tau_{t+1}^i < \tau_t^{i^*};\ n_{t+1}^i > n_t^{i^*} \quad if \quad h_t > h_t^* \tag{29}$$

Proof. The uniqueness and stability of h_t^* follows from the properties of $w(h_t)$, noting that $\{n_t^{i^*}, \tau_t^{i^*}\} > 0$, and that $\frac{\partial n_t^{i^*}}{\partial w_t^h} < 0$, $\frac{\partial n_t^{i^*}}{\partial w_t^u} > 0$, $\frac{\partial \tau_t^{i^*}}{\partial w_t^h} > 0$, and $\frac{\partial \tau_t^{i^*}}{\partial w_t^u} < 0$. □

Corollary 4.4.1. *The modern growth equilibrium path is characterized by the following conditions*

$$h_t = h_t^* \quad if \quad Y_t^{m,N} > 0 \tag{30}$$

and accordingly

$$w_t^u = A_t^m w^u(h_t^*) \quad if \quad Y_t^{m,N} > 0 \tag{31}$$

$$p_t = \frac{A_t^m w^u(h_t^*)}{A_t^a} \quad if \quad Y_t^{m,N} > 0 \tag{32}$$

Lemma 4.4 shows that societies at a modern-growth stage will converge to a stable path in which the surpluses and shortages of skilled work will be corrected by individuals' choices on social interaction and fertility over time. Wages will be the sign that guides households. If there is any kind of shock that generates a discrepancy between the supplied and the demanded skilled labor at period t, the relative wage will adjust intermediately and individuals will adjust their socialization and fertility choices for $t+1$.

The labor allocation problem under modern societies does not have an analytic solution[15]. However, similar patterns from those of traditional societies emerge. In particular, Lemma 4.5 shows that once a society surpasses the consumption constraint, the manufacturing sector will increase its participation in the total output.

Lemma 4.5. *If* $\alpha \frac{w_t^u}{p_t} < \bar{c}$, *the modern growth equilibrium path is characterized by the following condition*

$$\frac{Y_t^a}{Y_t^m} < \frac{Y_{t+1}^a}{Y_{t+1}^m} \quad \forall t \tag{33}$$

Proof. It follows from Lemma 9.10 and the technological progress assumptions from Section 4.4.1. □

Nevertheless, the structural change will slow down, as the decreasing returns to skilled work start to take place. Labor market will stabilize. The share of skilled labor will tend to a constant, leading to the ratio of skilled and unskilled wages to a constant too. Formally

Lemma 4.6. *In the long-run labor allocation will be characterized by the following conditions*

$$\lim_{t\to\infty} h_t = \bar{h} \tag{34}$$

[15]See Lemma 9.9 for the implicit expressions

and

$$\lim_{t\to\infty} \frac{w_t^h}{w_t^u} = w\left(\bar{h}\right) \tag{35}$$

Where $\bar{h} < 1$.

Proof. As social capital will tend to accumulate at a constant rate (see Proposition 1), the wage ratio will tend to a constant. With the wage ratio tending to a constant, and considering the decreasing returns property of f, the demand for skilled labor will also tend to a constant. As the consumption of all goods is necessary, due the particular household's preferences, the share of skilled labor must be less than 1. □

4.4.3 Social Network Dynamics

As mentioned above, social interaction is an endogenous process in this model. However, traditional societies lack active social interaction. Only when the new manufacturing technology becomes viable, individuals have incentives to socialize. Therefore, initial values and exogenous shocks will determine the dynamics of social interaction related variables until t^M. After the industrial take-off, social interaction will have positive levels at every period. Therefore, social capital will increase monotonically with time, while social distance will decrease. Formally,

Proposition 1. *The following conditions characterize network dynamics*

1. *The change in aggregate social capital is*

$$\Delta S_t = \begin{cases} 0 & \text{if } t < t^M \\ \tau_t^* & \text{if } t > t^M \end{cases} \tag{36}$$

2. *The change in social distance is*

$$\Delta D_t = \begin{cases} 0 & \text{if } t < t^M \\ k(\tau_t^*) & \text{if } t > t^M \end{cases} \tag{37}$$

Where $\tau_t^* > 0$, $k(\tau_t^*) < 0$, *and* $k'(\tau_t^*) > 0\ \forall D_t \geq \bar{D}$. *Moreover,* $\lim_{t\to\infty} \tau_t = \bar{\tau}$ *and* $\lim_{t\to\infty} D_t = \bar{D}$

Proof. It follows from Equations 9 and 10. The asymptotic behavior results from the assumptions of □

Prop 1 proposes that modern economic growth implies an increasing amount of social interaction and a better interconnected society–i.e. lower social distance. However, it also establishes a natural limit to the connectivity of society. In other words, the feedback between social interaction and social distance reduction does not last forever. Having in mind that socialization in this model refers to distant interactions, these results are consistent with historical trends and with the ideas of several scholars on modernity (Kuznets, 1966; Inglehart and Welzel, 2005). In Section 5, I will show some robust evidence on this.

4.4.4 Demographic Dynamics

Four regimes characterize population dynamics in this model. They result from the combination of development stage and whether or not consumption constraint binds. Thus, there are pre-modern-restricted societies, pre-modern non-restricted societies, modern restricted societies, and modern non-restricted societies. Formally,

Proposition 2. *The following conditions characterizes demographic dynamics*

$$N_{t+1} = \begin{cases} \frac{N_t}{\theta}\left[\frac{1-\alpha-\beta}{1-\alpha}\frac{A_t^a-\bar{c}}{A_t^a}\right] & \text{if } t < t^M \wedge \alpha\frac{w_t^u}{p_t} < \bar{c} < \frac{w_t^u}{p_t} \\ \frac{N_t}{\theta}(1-\alpha-\beta) & \text{if } t < t^M \wedge \alpha\frac{w_t^u}{p_t} \geq \bar{c} \\ N_t\chi^1(D_t, S_t, \tau_t^*) & \text{if } t > t^M \wedge \alpha\frac{w_t^u}{p_t} < \bar{c} < \frac{w_t^u}{p_t} \\ N_t\chi^2(D_t, S_t, \tau_t^*) & \text{if } t > t^M \wedge \alpha\frac{w_t^u}{p_t} \geq \bar{c} \end{cases} \tag{38}$$

and

$$\frac{N_t}{\theta}\left[\frac{1-\alpha-\beta}{1-\alpha}\frac{A_t^a-\bar{c}}{A_t^a}\right] < \frac{N_t}{\theta}(1-\alpha-\beta) \tag{39}$$

$$N_t\chi^1(D_t, S_t, \tau_t^*) < N_t\chi^2(D_t, S_t, \tau_t^*) \tag{40}$$

$$\frac{N_t}{\theta}\left[\frac{1-\alpha-\beta}{1-\alpha}\frac{A_t^a-\bar{c}}{A_t^a}\right] > N_t\chi^1(D_t, S_t, \tau_t^*) \tag{41}$$

$$\frac{N_t}{\theta}(1-\alpha-\beta) > N_t\chi^2(D_t, S_t, \tau_t^*) \tag{42}$$

Where $\chi^j : \mathbb{R}_+^3 \to \mathbb{N} \quad \forall j = 1, 2$ *and* $\lim_{t\to\infty}\chi^2(D_t, S_t, \tau_t^*) = \bar{\chi}$.

Proof. It follows from Equations 17 and 18, noticing that $N_{t+1} = n_t N_t$. The asymptotic behavior follows from noticing that τ_t^* tends to a constant, as well as n_t^*. □

Corollary 4.6.1. *Consider three societies at different stages of development; A a traditional society below the consumption constraint, B a traditional society above the consumption constraint, C a modern society below the consumption constraint, and D a modern society above the consumption constraint. The four societies have identical social conditions and the same technological level in the agricultural sector. The fertility in this societies will satisfy the following condition*

$$n_t^B > n_t^A \lesseqgtr n_t^D > n_t^C \tag{43}$$

Proposition 2 and its corollary describe a pattern that is consistent with several demographic-history facts. Overall, it is consistent with the long-term reduction of fertility all over the world–i.e. modern societies have lower fertility rates than traditional ones. At the same time, the model predicts a non-monotonic relation between fertility and development, which enables it to explain several historical episodes, including the recent increase of fertility in high-income countries (see Guinnane, 2011; Myrskylä et al., 2009; Esping-Andersen and Billari, 2015). I offer robust evidence on this in Section 5.

4.4.5 Industrial Take-Off

The viability of the new manufacturing technology represents the arrival of industrial production and the dawn of modern economic growth. This model predicts that every society will arrive at that stage of development. However, the timing and intensity of this process will differ across societies, depending on the fundamentals of their social network and their productive system.

To begin with, every society starts to accumulate social capital at least one period before the new manufacturing technology becomes viable (see Lemma 4.7). Intuitively, as the inertia in the growth of aggregate knowledge will lead to the inevitable viability of the new manufacturing technology at period t, and w_t^h is always larger than w_t^u; thus, every individual at period $t^M - 1$ will have incentives to socialize, because it will augment the expected wage of her children. Formally,

Lemma 4.7. *If individuals have rational expectations, then, t^M is such that $\tau^*_{t^M-1} > 0$*

Proof. From Lemma 4.1 it follows that $\tau_t^* > 0$ if $Y_t^{m,N} > 0$. From Lemma 4.3 it follows that $Y_{t^M}^{m,N} > 0$ and that, if $\tau^*_{t^M-1} = 0$, it will be true that:

$$\frac{A^m\left(\lambda(\bar{h}_{t^M})\right)}{a^m\left(\lambda(\bar{h}_{t^M})\right)} > \frac{1}{w^u\left(\bar{h}_{t^M}\right)} \tag{44}$$

where $\bar{h}_{t^M} = 0$ is the optimal share of skilled workers if $\tau^*_{t^M-1} = 0$.

Moreover, from Equation 21 and from noticing that $l'(h_t)$, and $\lambda_{t+1} = \lambda_t[1 + l(h_t)]$, it follows that if $\tau^*_{t^M-1} > 0$ it will be true that:

$$\frac{A\left(\lambda(h_{t^M})\right)^m}{a\left(\lambda(h_{t^M})\right)^m} > \frac{A\left(\lambda(\bar{h}_{t^M})\right)^m}{a\left(\lambda(\bar{h}_{t^M})\right)^m} > \frac{1}{w^u\left(\bar{h}_{t^M}\right)} > \frac{1}{w^u\left(h_{t^M}\right)} \tag{45}$$

Where $h_{t^M} > 0$ is the human capital of equilibrium if $\tau^*_{t^M-1} > 0$. Therefore, $\tau^*_{t^M-1} > 0 \iff Y_{t^M}^{m,N} > 0$. As $Y_{t^M}^{m,N} > 0$, it is true that $\tau^*_{t^M-1} > 0$.

□

Hence, every society will start investing in social capital at least one period before the arrival of modern economic growth. But what determines the precise moment in which modern growth arrives? Why do certain societies industrialize earlier than others?

For answering these questions, consider a situation in which a particular society industrializes prior to the rest. I will call this situation an *early industrial take-off*. Formally,

Definition 4.1. *Early industrial take-off. Situation in which* $\exists\, t \,\wedge\, j$ *such that* $(Y_t^{m,N})^j > 0$ *and* $(Y_t^{m,N})^k = 0$, $\forall k \neq j$.

Early industrial take-off will depend on several aspects of the productive system. However, the social network structure may play an essential role in this phenomenon. Proposition 3 describes how.

Proposition 3. *Consider identical endowments, preferences, and technological conditions in* A *and* B*, if Equations 22 and 23 apply, then,* $\exists\, \tilde{t} \neq 0$, $\epsilon = (D_{\tilde{t}-1})^B - (D_{\tilde{t}-1})^A > 0$*, such that*

$$\frac{A^m(\lambda(h_{\tilde{t}}^A))}{a^m(\lambda(h_{\tilde{t}}^A))} > \frac{1}{w^u(h_{\tilde{t}})} > \frac{A^m(\lambda(h_{\tilde{t}}^B))}{a^m(\lambda(h_{\tilde{t}}^B))} \tag{46}$$

where $h_{\tilde{t}}^A$ *is the optimal share of skilled if* $(\tau_{\tilde{t}-1}^*)^A = \tau((D_{\tilde{t}-1})^A)$*, and* $h_{\tilde{t}}^B$ *is the human capital of equilibrium if* $(\tau_{\tilde{t}-1}^*)^B = \tau((D_{\tilde{t}-1})^B)$.

Proof. Consider a proof by contradiction. Concretely, consider that Proposition 3 is false. Thus, $\forall\, t \neq 0$ and $\forall\, \epsilon > 0$ one of the following statements must be true

1. $$\frac{A^m(\lambda(h_t^A))}{a^m(\lambda(h_t^A))} \leq \frac{1}{w^u(h_t)} \quad \wedge \quad \frac{A^m(\lambda(h_t^B))}{a^m(\lambda(h_t^B))} \geq \frac{1}{w^u(h_t)}$$

2. $$\frac{A^m(\lambda(h_t^A))}{a^m(\lambda(h_t^A))} \leq \frac{1}{w^u(h_t)} \quad \wedge \quad \frac{A^m(\lambda(h_t^B))}{a^m(\lambda(h_t^B))} < \frac{1}{w^u(h_t)}$$

3. $$\frac{A^m(\lambda(h_t^A))}{a^m(\lambda(h_t^A))} > \frac{1}{w^u(h_t)} \quad \wedge \quad \frac{A^m(\lambda(h_t^B))}{a^m(\lambda(h_t^B))} \geq \frac{1}{w^u(h_t)}$$

From Lemma 4.2 it follows that the first two statements are false.

With regard to the third statement, from Equation 22 it follows that

$$\frac{A^m(\lambda(h_0^A))}{a^m(\lambda(h_0^A))} < \frac{1}{w^u(h_0)} \quad \wedge \quad \frac{A^m(\lambda(h_0^B))}{a^m(\lambda(h_0^B))} < \frac{1}{w^u(h_0)} \tag{47}$$

In order for the third statement to be true, the inequalities from Equation 47 must switch at $t = 1$. However, consider $(D_0)^B = \frac{1}{a}$, $(D_0)^A = a$, which implies that $\lim_{a\to 0} \epsilon = \infty$. Under those circumstances $\lim_{a\to 0}(\tau_0^*)^B = 0$ and $\lim_{a\to 0} h_1^B = 0$. Therefore, $\lim_{a\to 0} \frac{1}{w^u(h_1)} = \infty$, and $\frac{A^m(\lambda(h_1^B))}{a^m(\lambda(h_1^B))} \geq \frac{1}{w^u(h_1)}$. Thus, the third statement is also false.

As all the above statements are false, we can conclude that Proposition 3 is true. □

Corollary 4.7.1. *Consider identical technological and social conditions in A and B, if the assumptions from Proposition 3 hold, and* $(D_{\tilde{t}-1})^B - (D_{\tilde{t}-1})^A \geq \epsilon$*, then, A will experience an early industrial take-off at* $\tilde{t} - 1$*.*

Proposition 3 and its corollary prove that under quite general conditions, differences in social distance could explain early take-offs. Intuitively, as technology advances more rapidly with larger shares of skilled workers, it is reasonable to think that societies with larger incentives to socialize–therefore, to accumulate more human capital–might reach the threshold of industrialization earlier. Figure 6 shows this intuition. Following the set-up of Proposition 3, A will have a larger optimal share of skilled labor (h_t^A) than B at the prelude of modernity transition. Under the new general knowledge conditions, at period $t + 1$, h_t^A will suffice to make available the new production technology. Meanwhile, h_t^B will not be high enough to do it at $t + 1$.

[Figure 6 here]

As I discuss in Section 4.4.6, early take-offs generate permanent disparities in economic performance.

4.4.6 Modern Economic Growth

In the previous sections, I showed that the transition to modern economic growth brings changes in the social network and the population dynamics. With regard to population, equations 41 and 42 describe a fertility transition related to modern economic growth. Meanwhile, Proposition 1 indicates that modern economic growth implies an increasing amount of social interaction and a better connected society–i.e. lower social distance.

Nevertheless, the social network and the population dynamics also affect the features of modern economic growth. Focusing on the social network side, Proposition 1 claims that traditional societies are uninterested in socialization–remember that socialization here refers to frequent distant social interactions. Nevertheless, as Proposition 3 indicates, when the technological conditions improve and modern economic growth approaches, the latent social network structure affects the economic performance. Moreover, Proposition 4 shows that the effects of social network structure are not temporary but persistent.

Proposition 4. *Consider identical technological and social conditions in A and B. If both societies are over the consumption constraint and A experienced an early industrial take-off in period $\tilde{t}-1$, then*

$$Y^A_{\tilde{t}+1} > Y^B_{\tilde{t}+1} \;\wedge\; Y^A_t \geq Y^B_t \quad \forall t > \tilde{t}+1 \tag{48}$$

With $Y^j_t = (Y^a_t)^j + (Y^m_t)^j \quad \forall j \in \{A, B\}$

Proof. It follows from propositions 4.5 and 9.11, noticing that agriculture output is non-decreasing with wages. □

Therefore, the model predicts that under identical circumstances, societies with lower social distance are more likely to industrialize first and to grow more after that. In other words, current economic disparities might be generated by initial differences in social distance across societies. Thus, Propositions 3 and 4 prove that the social network structure might be a fundamental cause of long-term economic growth.

The persistent effect of network structure on economic growth results from two types of feedback. On the one hand, the classical feedback from unified growth literature between technological progress and skilled work. Technology advances faster if there is a larger share of skilled workers, and the demand for skilled work increases faster with technological progress. On the other hand, there is the feedback between social capital accumulation and social distance. As soon as social capital starts to increase, social distance reduces, making it cheaper to accumulate social capital.

Notice that the social network structure is completely irrelevant in traditional societies. The effect of the network on economic performance appears once modern technology is available. This is a similar argument to the one that Acemoglu et al. (2002) offer regarding the effects of institutions on long-term economic growth. As institutions, social network structure might support economic growth only under certain technological conditions.

5 Supportive Evidence

The model offers four propositions on the relationship between economic growth, structural change, social interactions, and fertility. In this section I offer evidence on their support.

5.1 Long-Term Effects of Early Industrial Take-Off

Proposition 4 and its corollary claim that, *ceteris paribus*, the timing of industrialization determines long-term differences in output levels. In particular, a society that experienced an early industrial take-off should not be surpassed by another one with similar conditions but which industrialized later.

The persistent ranking of output levels is a well-known feature of modern growth when looking at country-level data (Acemoglu, 2008). However, due to the lack of homogeneous information on industrial history, we do not know much about the relationship between the timing of industrialization and the output levels at an international scale. For this reason, I build an industrialization take-off variable, which offers an approximate measure of the year when modern technology was widely available in a country (see Section 9.2 for details). Based on the new data–and as predicted by the model–Figure 7 shows a negative correlation between current output levels and the timing of industrial take-off. In other words, societies that industrialized first, are still ahead in economic outcomes.

[Figure 7 here]

Table 1 shows that this correlation is robust to consider a large set of controls. The table indicates that one year difference in the timing of industrialization relates to a 6% difference in current income per capita. This, for countries with similar observable attributes.

[Table 1 here]

5.2 Social Distance and Early Industrial Take-Off

Proposition 3 and its corollary claim that, *ceteris paribus*, social distance can determine the timing of industrialization. More precisely, societies with a lower social distance should have been more likely to experience an early industrial take-off.

In order to test this, I will consider the linguistic distance index (GI) proposed by Greenberg (1956), in particular, the estimates made by Desmet et al. (2009). The GI has been widely used in the diversity literature (see Fearon, 2003; Bossert et al., 2011). It can be interpreted as the expected linguistic distance between two randomly chosen individuals. For our purpose, this approximates the idea of how far away two random individuals are in the society. We should expect that the barriers to interact between individuals with similar linguistic backgrounds would be lower than those for individuals with quite dissimilar linguistic backgrounds. As predicted by the model, Figure 8 shows a positive correlation at country-level between the linguistic distance and the timing of industrial take-off. In other words, societies with shorter social distance industrialized first.

[Figure 8 here]

The GI index refers to current linguistic distances, while Proposition 3 refers to pre-modern distances. For this reason, I consider also the ethnolinguistic fractionalization (ELF) index at the highest level of aggregation estimated by Desmet et al. (2012). The ELF has been widely used in the diversity literature (see Easterly and Levine, 1997; Alesina et al., 2003; Desmet et al., 2009). It measures the probability of two randomly selected individuals being from different linguistic groups. The measure considered here defines groups at the root of the linguistic tree. For our purpose, this means that it accounts for the existing linguistic differences in pre-modernity[16]. Table 9 presents an almost identical pattern to the one presented in Table 8. This suggests a slow transformation rate of linguistic attributes.

[Figure 9 here]

Table 2 shows that the positive between the social distance and the timing of industrial take-off is robust to consider a large set of controls. The table indicates that one standard deviation increase of the linguistic distance relates to a 8,15 years lag in the timing of industrial take-off. This, for countries with similar observable attributes.

[Table 2 here]

5.3 Fertility Dynamics

First and foremost, Proposition 2 and its corollary describe a long-term reduction in fertility resulted from economic development. *Ceteris paribus*, traditional societies should have larger fertility rates than modern ones. This is consistent with the most widely studied subject in demographic history; the fertility transition (see Guinnane, 2011; Galor, 2012).

However, Proposition 2 does not imply a monotonic reduction of fertility, in that sense, the model is capable of predicting more complex patterns. For instance, it is also consistent with fertility expansions in pre-modern societies that faced economic growth. This was the case of the late 17th and early 18th century pre-industrial United Kingdom, where the gross birth rate passed, in a century, from almost 25 births per 1,000 people, to 35 (see Figure 10).

[16]Unfortunately, the ELF does not account the distance between groups. Therefore, it should be interpreted as a a discrete measure of the barriers, rather than a continuous one as the GI.

Similarly, the model is consistent with fertility expansions under industrialized contexts, like the baby boom in the United States during the postwar period. In a couple of decades, the gross birth rate broke the long-term downward trend and passed from less than 20 births per 1,000 people to more than 25.

[Figure 10 here]

Both of these cases have been well-identified in the economic history literature (see Clark, 2005; Greenwood et al., 2005; Klemp, 2012; Møller and Sharp, 2014) and they have as a common feature the income expansion without a technological breakthrough, which is precisely the scenario that the model predicts as likely to experience fertility expansions.

5.4 Economic Growth and Social Distance Reduction

Finally, Proposition 1 indicates that, once a society industrializes, the quantity/quality of distant social interactions increases monotonically with time. Simultaneously, the distance between individuals reduces.

The major challenge for testing these claims is the lack of detailed information on social interactions for complete societies during long periods of time. A good source of information comes from the U.S. Census, which has been tracking race and marriage data for decades. This enables the reconstruction of the frequency of interracial marriages, which is a good proxy of the quantity of distant social interactions. First, a marriage is not a trivial link. It represents an active interaction between the two individuals married, as well as between each of their social circles. Second, racial differences have been clear boundaries between social groups in the U.S. history. Thus, interracial marriages do capture the important interactions between distant individuals.

Figure 11 shows a clear increase in time of interracial marriages, particularly intense in the second half of the 20th century, reaching up to almost 0.02 marriages per person. This trend seems to relate to the Civil Right Movement and more specifically, to the abolition of the miscegenation laws, which prohibited the sexual and marital relations between individuals of different races in several states of the Union. The miscegenation laws were repealed progressively since the 1940s, until ruled unconstitutional in 1967 by the U.S. Supreme Court. In terms of my model, the abolition of these laws is equivalent to a reduction in the cost of interaction–i.e. a shift downwards in the $\varphi(D_t)$.

[Figure 11 here]

However, Table 3 shows that the increasing trend in interracial marriages was not an exclusive result of the Civil Right Movement, but a long-term phenomenon consistent with the implications of Proposition 1. Table 3 shows panel regressions at county-level. They show that between 1850 and 1880, a period in which industrial activity was not yet dominant in the whole country, the number of interracial marriages did not increase. Nevertheless, in the posterior period, 1900-1940, when industrial technology was spread over the country but before the Civil Right Movement, the positive trend in interracial marriages already appears.

[Table 3 here]

Even though the increase of distant social interactions imply the connection of different groups of the society, it is not a sufficient condition for the reduction of the aggregate social distance–for example, because of the arrival of new members to the society. To the best of my knowledge, the only available data for a society in transition to modernity, that offers information on the evolution of the social network structure comes from Mejia (2018). He explores the elite of a region in Colombia while it passed from a mining to an industrial economy. Because of the cultural and institutional conditions of the region, this elite can be interpreted as a fairly isolated society. Based on these data, we can see that social distance–measured as the diameter and the average path length of the network–reduced in time, as industrial activity increased (see Figure 12).

[Figure 12 here]

Additional evidence supporting this claim comes from Kaplanis et al. (2018). They collected 86 million profiles from publicly-available online data shared by genealogy enthusiasts in Western societies. With these data, they explore the evolution of marital relations in the long-run. In particular, they look at the median distance of spouses' birth places and their genetic relatedness (IBD). They find quite stable patterns in both variables between 1650 and 1800. During this period, the median geographical distance between spouses moved between 7 km and 10 km and the IBD index stayed between -8 and -8.5. Modernity drastically changed this pattern. The geographical distance increased, reaching up to 100 km in the first half of the 20th century, while the IBD index felt to -11. However, there is a 50-year lag between the advent of increased familial dispersion and the decline of genetic relatedness between couples. From this result, Kaplanis et al. (2018) hypothesize that changes in 19th century transportation were not the primary cause for decreased consanguinity. Instead, shifting in cultural factors played a more important role. This is completely consistent to the logic of my model.

6 Final Remarks

This paper offers a theoretical framework to understand the coevolution of social interactions and long-term economic growth. It begins by considering that most traditional societies did not have educational markets. Thus, access to the required knowledge for transiting to a modern economy had to be transmitted through social interactions, in particular, through the interaction between heterogeneous groups of people–i.e. distant interactions. Once immersed in a modern economy, the productive system should have increased the demand for knowledge, promoting more distant interactions. Simultaneously, the emergence of distant interactions should have affected the connectivity of society, reducing its heterogeneity, making posterior interactions cheaper but reducing their profitability. Moreover, social interactions competed and benefited from other non-market activities, child rearing specifically.

In this way, the model arrives at four basic predictions. First, modern economic growth brings a more cohesive society. Second, modern economic growth brings long-term reductions in fertility with potential short-term increases. Third, initial barriers to social interactions are likely to explain the timing of modern economic growth arrival. Forth, the timing of modern economic growth arrival is likely to explain current output levels.

In the empirical analysis, I show that the model is consistent with historical data and widely accepted results in several fields. First, I describe that the expansion of the modern economy coincided with an increase in distant interactions in the U.S., and the reduction of social distance in Colombia. Second, I present the fertility expansion in the U.K. between 1650 and 1750, and the U.S. between 1940 and 1960, as episodes that defy the demographic transition pattern in the short term but which are consistent with my model predictions. Third, I show that countries with a population more distant in linguistic terms took longer to industrialize. Finally, I show that current income disparities at country level correlate with the timing of industrialization.

Nevertheless, a more detailed empirical test of the model is desirable if new information on social interactions over history is gathered. For now, the data on this regard is fairly fragmented, incomplete, and non-comparable. The field requires collective efforts in archival research to have a more promising future.

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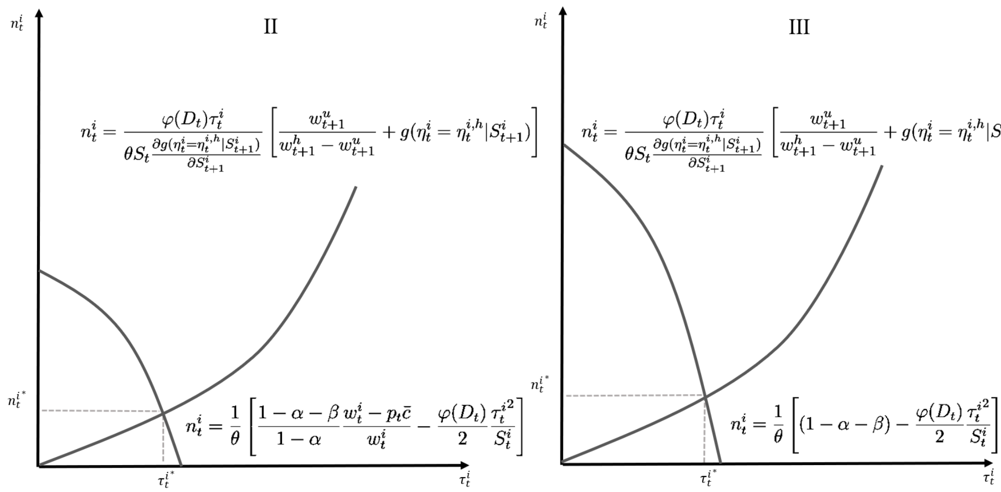
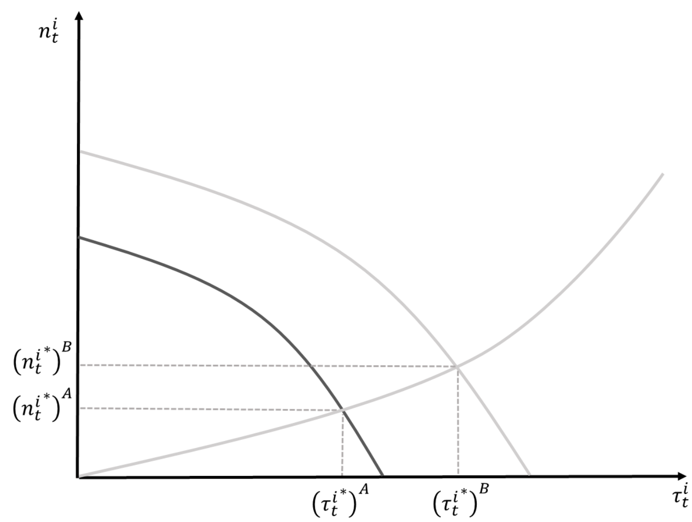
7 Figures

Figure 1: Optimal fertility and socializing decisions



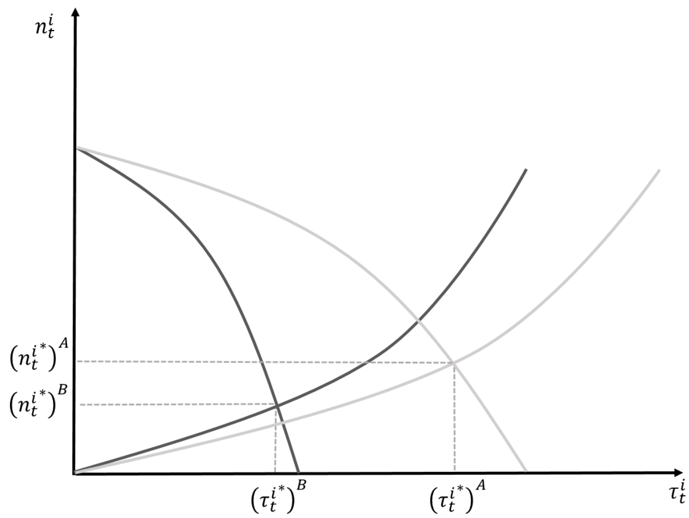
Note: This figure presents equations 17 and 18, which capture the optimal conditions of fertility and socialization. Scenario II is conditioned to $\alpha \frac{w_t^i}{p_t} < \bar{c} < \frac{w_t^i}{p_t}$. Scenario III is conditioned to $\alpha \frac{w_t^i}{p_t} \geq \bar{c}$.

Figure 2: Optimal fertility and socializing decisions. Differences in technological conditions



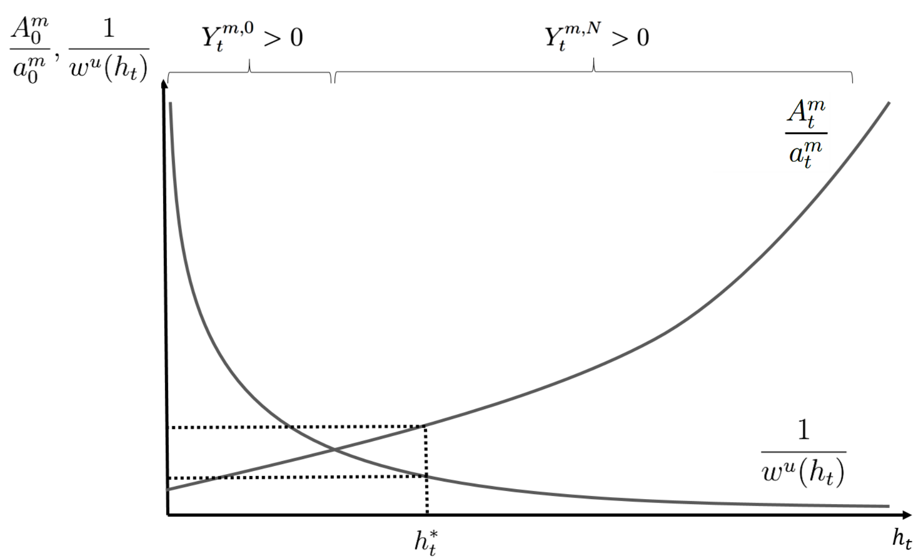
Note: This figure presents equations 17 and 18 for societies A and B under Scenario II–relative consumption constraint. There is only one parameter that distinguishes A from B, $(A_t^a)^A < (A_t^a)^B$. Both societies share the same upward curve (Equation 18) because–from a static perspective–current technology does not affect the benefits of non-market interaction. However, as B is more productive in the agricultural sector, the relative price of the agricultural good is lower, which frees a larger share of the individual's income to have children or invest in social capital. Hence, the downward curve (Equation 17) for B is above and to the right of the A's one. Notice that the slop of the curve does not change, only the intercepts.

Figure 3: Optimal fertility and socializing decisions. Differences in social distance, stocks of social capital, and population size



Note: This figure presents equations 17 and 18 for societies A and B. It represents a situation in which A and B are identical except for one of the following conditions: $D_t^A < D_t^B$, $S_t^A > S_t^B$, $N_t^A > N_t^B$. All of these conditions make social interaction cheaper in A. Having a twofold effect on the optimal fertility-socialization decision. On the one hand, it makes more profitable to invest in social interactions than to have a large number of children–i.e. the slop of the upward curve reduces. On the other hand, there is an income effect that allows to have more children for every unit of social interaction–i.e. the slop of the downward curve reduces.

Figure 4: Technology viability



Note: This figure presents the ratio of technological parameters in manufacturing and agriculture as a function of the share of skilled workers in the economy. It also presents the reciprocal of $w^u(h_t)$. The cutting point of the two curves is where the modern technology becomes viable. The optimal share of skilled workers is located at the right of this point.

Figure 5: General knowledge dynamics

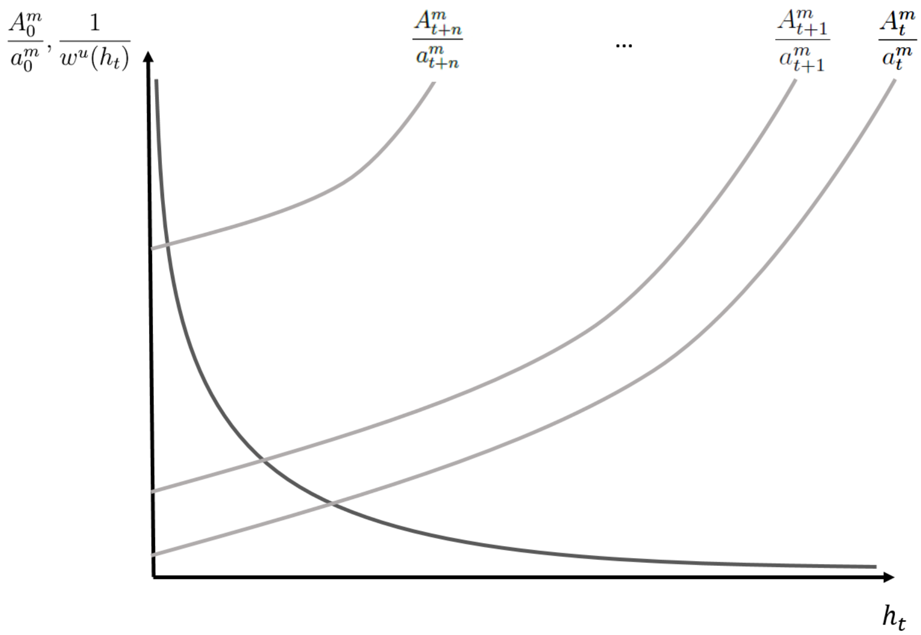


Figure 6: Industrial take-off

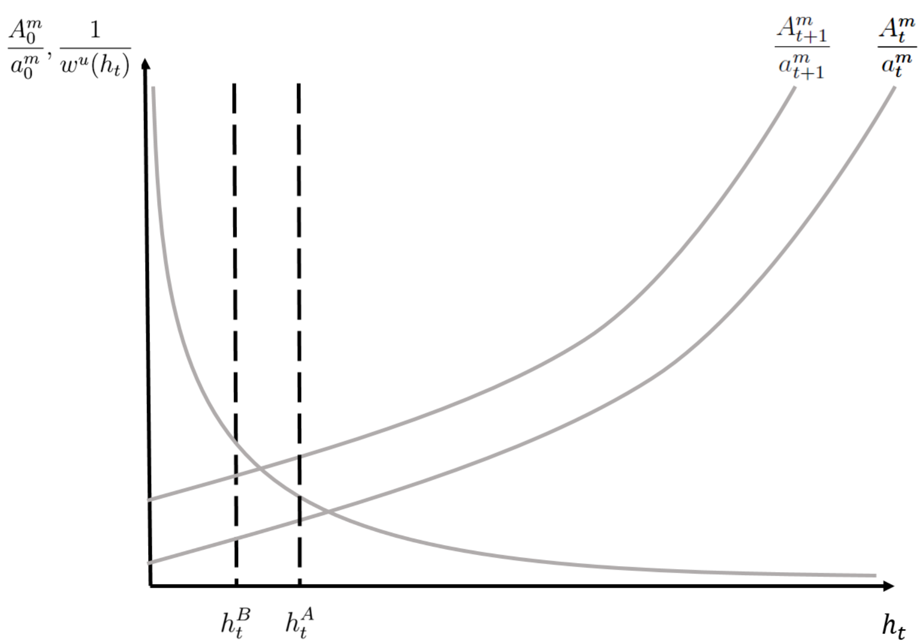
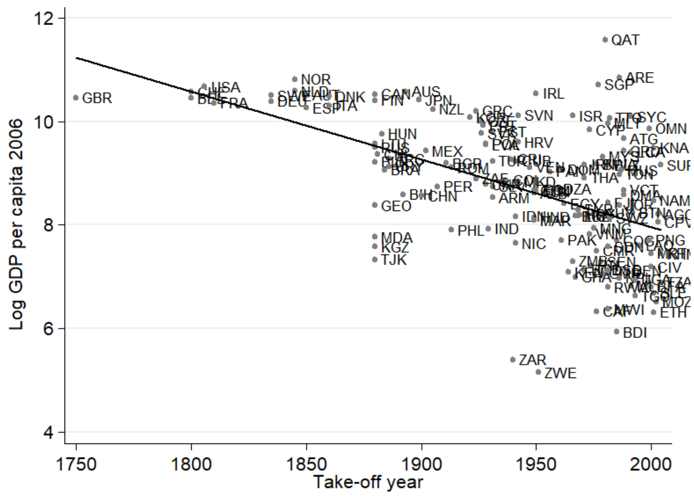
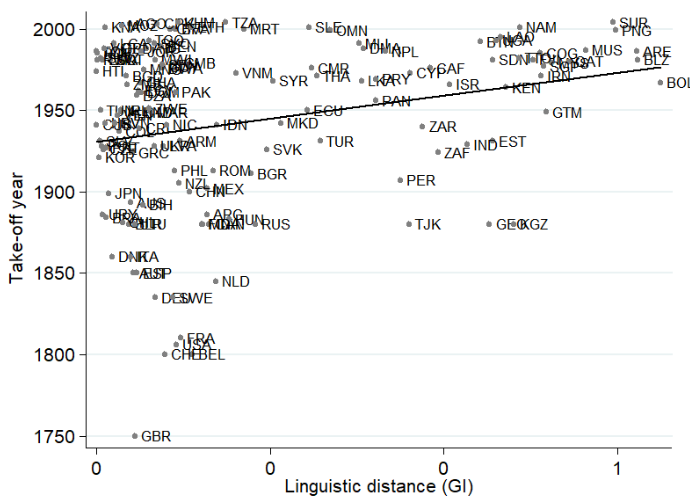


Figure 7: Log GDP per capita vs industrial take-off timing



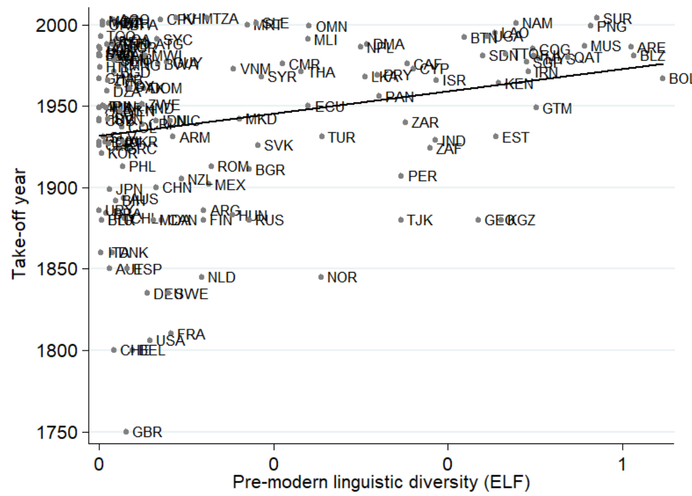
Note: This figure presents the negative correlation between the timing of industrial take-off and current GDP per capita.

Figure 8: Industrial take-off timing vs linguistic distance



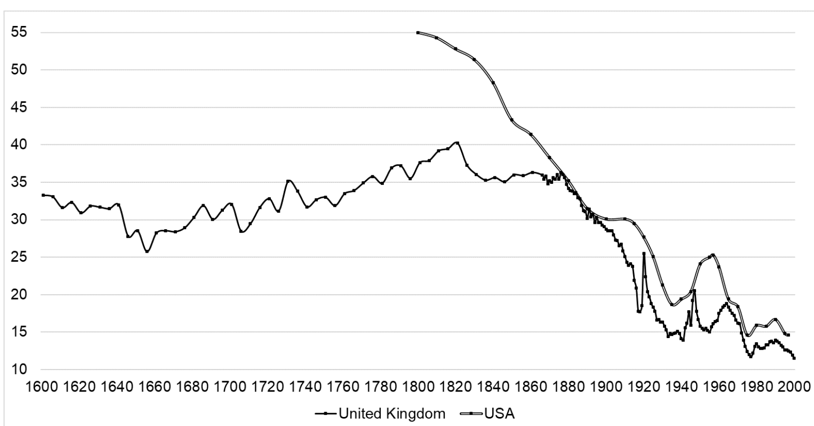
Note: This figure presents the positive correlation between linguistic distance (GI) and the timing of industrial take-off.

Figure 9: Industrial take-off timing vs pre-modern linguistic diversity



Note: This figure presents the positive correlation between ancient linguistic diversity (ELF) and the timing of industrial take-off.

Figure 10: Fertility in the United Kingdom and the United States. 1800-2000

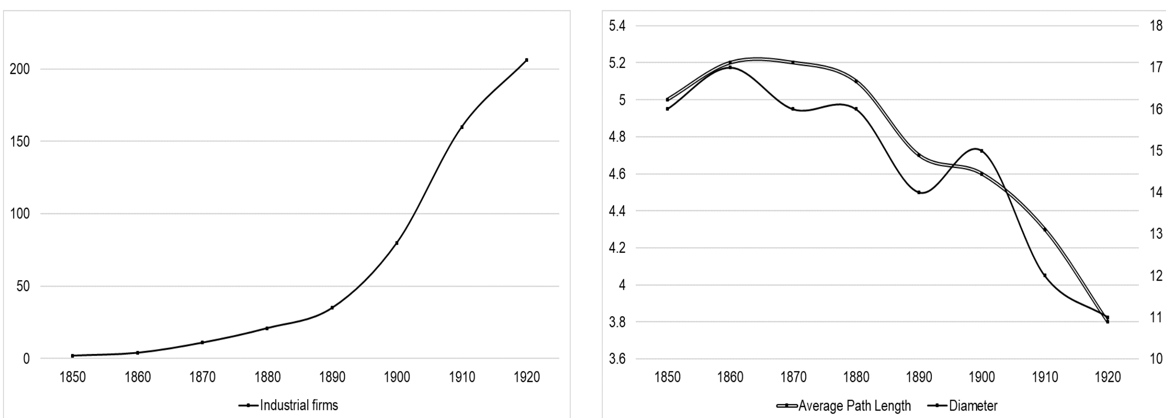


Note: This figure presents the evolution of U.S. gross birth rate–i.e. births per 1000 population per annum.

Figure 11: Interracial marriages in the United States. 1850-2000

Note: This figure presents the evolution of interracial marriages per capita in the U.S.

Figure 12: Industrialization and social distance in the network of Antioquia's elite. 1850-1930



Note: The right-side figure presents the cumulative number of industrial firms created in Antioquia. The left-side figure presents the average path length of the network (left axis) and its diameter (right axis).

8 Tables

Table 1: Industrial take-off timing and current per capita GDP. Cross section

	Log real GDP per capita in 2006		
	(1)	(2)	(3)
Take-Off	-0.013***	-0.007***	-0.006***
	(0.001)	(0.002)	(0.002)
East Asia and Pacific		-0.226	-0.304
		(0.359)	(0.328)
Sub-Saharan African		-1.622***	-1.763***
		(0.344)	(0.347)
Latin America and Caribbean		-0.215	-0.279
		(0.306)	(0.306)
Latitude		0.001	0.003
		(0.005)	(0.005)
British legal origin			-0.518**
			(0.237)
French legal origin			-0.737***
			(0.203)
Socialist legal origin			-1.213***
			(0.209)
German legal origin			0.044
			(0.183)
Observations	141	141	141
R-squared	0.280	0.481	0.534

Note: This table establishes the statistically and economically significant correlation between the timing of industrial take-off and current income per capita. The unit of observation is the country. The sample is the world. Real GDP per capita comes from World Penn Tables. Robust standard error estimates in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Table 2: Linguistic distance and industrial take-off timing. Cross section

	Timing of industrial take-off					
	(1)	(2)	(3)	(1b)	(2b)	(3b)
Linguistic distance (GI)	12.924***	10.718***	8.155**			
	(3.173)	(3.052)	(3.153)			
Pre-modern linguistic diversity (ELF)				12.536***	10.429***	8.435***
				(3.261)	(3.131)	(3.099)
East Asia and Pacific		29.385**	41.011***		29.321**	41.422***
		(13.373)	(12.549)		(13.447)	(12.369)
Sub-Saharan African		58.454***	63.400***		58.509***	64.222***
		(13.522)	(12.660)		(13.867)	(12.686)
Latin America and Caribbean		28.948**	32.961***		27.744**	32.493***
		(12.234)	(11.806)		(12.362)	(11.770)
Latitude		-0.240	0.106		-0.274	0.123
		(0.261)	(0.321)		(0.266)	(0.318)
British legal origin			68.464***			75.127***
			(22.249)			(21.267)
French legal origin			59.294***			66.199***
			(19.665)			(18.702)
Socialist legal origin			53.816***			60.275***
			(13.273)			(12.199)
German legal origin			-10.364			-3.886
			(17.761)			(16.714)
Observations	141	141	141	141	141	141
R-squared	0.063	0.350	0.429	0.063	0.350	0.429

Note: This table establishes the statistically and economically significant correlation between linguistic distance and industrial take-off timing. The unit of observation is the country. The sample is the world. Real GDP per capita comes from World Penn Tables. Robust standard error estimates in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Table 3: Incremental Trend in Interracial Marriages. Panel

	Interracial marriages							
	1850-1880		1900-1940		1950-1990		1850-1990	
Decade	0.013	0.010	0.180***	0.161***	117.7***	208.9***	1.435***	0.969***
	(0.019)	(0.020)	(0.022)	(0.028)	(8.823)	(32.830)	(0.203)	(0.288)
Population	149.5***	149.7***	196.0***	196.2***	809.8***	847.7***	1,510.4***	1,510.9***
	(12.205)	(12.203)	(4.140)	(4.142)	(99.436)	(97.916)	(15.010)	(15.025)
Sex ratio		0.461**		0.980		-2,959.7**		27.5***
		(0.192)		(0.616)		(1,190.1)		(7.343)
Average age		-0.264		0.565		880.3		14.579*
		(0.240)		(0.555)		(727.2)		(8.456)
Fertility		-0.023		-0.180		2,919.4***		-7.670
		(0.232)		(0.677)		(854.9)		(8.974)
Individual FEs	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	8,036	8,036	14,816	14,816	1,063	1,063	23,915	23,915
Number of Counties	2,448	2,448	3,090	3,090	527	527	3,192	3,192
Number of Periods	4	4	5	5	5	5	14	14

Note: This table establishes the trend in the number of interracial marriages over the years in the U.S. The unit of observation is the county-decade. For 1890 there is no information available for race composition of the household. Robust standard error estimates in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

9 Appendixes

9.1 Lemmas

Lemma 9.1. *Let n_t^i and τ_t^i be such that Equation (18) is satisfied. Then,* $\frac{\partial n_t^i}{\partial \tau_t^i} > 0$

Proof. Considering the following function.

$$F(n_t^i, \tau_t^i) = n_t^i - \frac{\varphi(D_t)\tau_t^i}{\theta S_t \frac{\partial g(\eta_t^i = \eta_t^{i,h} | S_{t+1}^i)}{\partial S_{t+1}^i}} \left[\frac{w_{t+1}^u}{w_{t+1}^h - w_{t+1}^u} + g(\eta_t^i = \eta_t^{i,h} | S_{t+1}^i) \right] \tag{49}$$

By the implicit function theorem we know that:

$$\frac{\partial n_t^i}{\partial \tau_t^i} = - \frac{\frac{\partial F(n_t^i, \tau_t^i)}{\partial \tau_t^i}}{\frac{\partial F(n_t^i, \tau_t^i)}{\partial n_t^i}} \tag{50}$$

Considering that $\frac{\partial F(n_t^i, \tau_t^i)}{\partial \tau_t^i} < 0$ and $\frac{\partial F(n_t^i, \tau_t^i)}{\partial n_t^i} = 1$, it follows from (50) that $\frac{\partial n_t^i}{\partial \tau_t^i} > 0$. □

Lemma 9.2. *If $Y_t^{m,N} < 0$, then, $\forall$ t there is a unique optimal combination of number of children and time spent socializing:*

$$n_t^i = \begin{cases} 0 & \text{if } \frac{w_t^i}{p_t} < \bar{c} \\ \frac{1}{\theta}\left[\frac{1-\alpha-\beta}{1-\alpha} \frac{w_t^i - p_t \bar{c}}{w_t^i} \right] & \text{if } \alpha \frac{w_t^i}{p_t} < \bar{c} < \frac{w_t^i}{p_t} \\ \frac{1}{\theta}(1-\alpha-\beta) & \text{if } \alpha \frac{w_t^i}{p_t} \geq \bar{c} \end{cases} \tag{51}$$

$$\tau_t^{i^*} = 0 \tag{52}$$

Proof. It follows from Equations 17 and 18 noting that $\tau_t^{i^*} = 0$ □

Lemma 9.3. *Consider identical technological and social conditions in A and B, if $\alpha \left(\frac{w_t^i}{p_t}\right)^A < \bar{c} < \left(\frac{w_t^i}{p_t}\right)^A$ and $\alpha \left(\frac{w_t^i}{p_t}\right)^B \geq \bar{c}$, then, $\forall$ t it must be true that*

1. *If $Y_t^{m,N} = 0 \Rightarrow \{n_t^{i^*}\}^A < \{n_t^{i^*}\}^B$ and $\{\tau_t^{i^*}\}^A = \{\tau_t^{i^*}\}^B = 0$*
2. *If $Y_t^{m,N} > 0 \Rightarrow \{n_t^{i^*}, \tau_t^{i^*}\}^A < \{n_t^{i^*}, \tau_t^{i^*}\}^B$*

Proof. 2 follows from noticing in Figure 1 that under scenario III, the downwards curve keeps the same slop but cuts y axis at a higher level, that is, $\frac{1}{\theta}\left[\frac{1-\alpha-\beta}{1-\alpha}\frac{(w_t^i)^A-(p_t)^A\bar{c}}{(w_t^i)^A}\right] < \frac{1}{\theta}(1-\alpha-\beta)$.

This can be shown through contradiction. Consider that

$$\begin{aligned}\frac{1}{\theta}\left[\frac{1-\alpha-\beta}{1-\alpha}\frac{(w_t^i)^A-(p_t)^A\bar{c}}{(w_t^i)^A}\right] &\geq \frac{1}{\theta}(1-\alpha-\beta)\\ \frac{1-\alpha-\beta}{1-\alpha}\frac{(w_t^i)^A-(p_t)^A\bar{c}}{(w_t^i)^A} &\geq 1-\alpha-\beta\\ (w_t^i)^A-(p_t)^A\bar{c} &\geq w_t^i(1-\alpha)\\ \alpha\left(\frac{w_t^i}{p_t}\right)^A &\geq \bar{c}\end{aligned}$$

As $\alpha\left(\frac{w_t^i}{p_t}\right)^A < \bar{c}$, we have found our contradiction. Therefore, it must be true that $\{n_t^{i*},\tau_t^{i*}\}^A < \{n_t^{i*},\tau_t^{i*}\}^B$.

Notice that this is also a proof for 1. □

Lemma 9.4. *Consider identical social conditions in A and B, if $(A_t^a)^A < (A_t^a)^B$ and $(A_t^m)^A \neq (A_t^m)^B$, then, $\forall\, t$ it must be true that*

1. *If $\alpha\left(\frac{w_t^i}{p_t}\right)^j < \bar{c} < \left(\frac{w_t^i}{p_t}\right)^j$, $\forall\, j=\{A,B\} \Rightarrow \{n_t^{i*},\tau_t^{i*}\}^A < \{n_t^{i*},\tau_t^{i*}\}^B$*
2. *If $\alpha\left(\frac{w_t^i}{p_t}\right)^j \geq \bar{c}$, $\forall\, j=\{A,B\} \Rightarrow \{n_t^{i*},\tau_t^{i*}\}^A = \{n_t^{i*},\tau_t^{i*}\}^B$*

Proof. Both statements follow from noticing that variations in technological parameters change the relative prices in the goods' market, not in the labor market. Therefore, replacing Equations 5, 6, and 8 in Equations 17 and 18 we have that

$$n_t^i = \begin{cases} 0 & \text{if } \frac{w_t^i}{p_t} < \bar{c} \\ \frac{1}{\theta}\left[\frac{1-\alpha-\beta}{1-\alpha}\frac{A_t^m w^x(h_t) - \frac{A_t^m w^u(h_t)}{A_t^a}\bar{c}}{A_t^m w^x(h_t^i)} - \frac{\varphi(D_t)}{2}\frac{\tau_t^{i2}}{S_t^i}\right] & \text{if } \alpha\frac{w_t^i}{p_t} < \bar{c} < \frac{w_t^i}{p_t} \\ \frac{1}{\theta}\left[(1-\alpha-\beta) - \frac{\varphi(D_t)}{2}\frac{\tau_t^{i2}}{S_t^i}\right] & \text{if } \alpha\frac{w_t^i}{p_t} \geq \bar{c} \end{cases} \tag{53}$$

and

$$n_t^i = \frac{\varphi(D_t)\tau_t^i}{\theta S_t \frac{\partial g(\eta_t^i = \eta_t^{i,h}|S_{t+1}^i)}{\partial S_{t+1}^i}} \left\{ \frac{A_{t+1}^m w^u(h_{t+1})}{A_{t+1}^m \left[w^h(h_{t+1}) - w^u(h_{t+1})\right]} + g(\eta_t^i = \eta_t^{i,h}|S_{t+1}^i) \right\} \tag{54}$$

Where $x = \{h, u\}$.

From this two equations it is clear that differences in A_t^m have no impact on fertility and socializing choices. Meanwhile, A_t^a has an inverse effect. □

Lemma 9.5. *Consider identical technological and social conditions in A and B, if $(Y_t^{m,N})^j > 0$, and $(D_t)^A < (D_t)^B$, then, $\forall$ t, and $\forall$ $j = \{A, B\}$, it must be true that $(\tau_t^{i*})^A > (\tau_t^{i*})^B$*

Proof. Equations (17) and (18) characterize τ_t^{i*}. Considering that $\alpha \frac{w_t^i}{p_t}$, from their equalization we have that

$$F(\tau_t^{i*}, \varphi(D_t)) = (1-\alpha-\beta) - \frac{\varphi(D_t)}{2} \frac{\tau_t^{i*2}}{S_t^i} - \frac{\varphi(D_t)\tau_t^{i*}}{S_t \frac{\partial g(\eta_t^i = \eta_t^{i,h}|S_{t+1}^i)}{\partial S_{t+1}^i}} \left[\frac{w_{t+1}^u}{w_{t+1}^h - w_{t+1}^u} + g(\eta_t^i = \eta_t^{i,h}|S_{t+1}^i) \right] = 0$$

Considering the assumptions about $\varphi(D_t)$, and $g(\eta_t^i = \eta_t^{i,h}|S_{t+1}^i)$, it follows that

$$\frac{\partial F(\tau_t^{i*}, \varphi(D_t))}{\partial \tau_t^{i*}} < 0$$

$$\frac{\partial F(\tau_t^{i*}, \varphi(D_t))}{\partial \varphi(D_t)^*} < 0$$

Applying the implicit function theorem we have that

$$\frac{\partial \tau_t^{i*}}{\partial \varphi(D_t)} = -\frac{\frac{\partial F(\tau_t^{i*}, \varphi(D_t))}{\partial \varphi(D_t)}}{\frac{\partial F(\tau_t^{i*}, \varphi(D_t))}{\partial \tau_t^{i*}}} < 0$$

Therefore, considering that $\varphi'(D_t) > 0$ it follows that $\frac{\partial \tau_t^{i*}}{\partial D_t} < 0$.

When $\alpha \frac{w_t^i}{p_t} < \bar{c} < \frac{w_t^i}{p_t}$ the proof follows in the same manner.

□

Lemma 9.6. *Consider identical technological and social conditions in A and B, if $(Y_t^{m,N})^j > 0$ and $(S_t)^A > (S_t)^B$, then, $\forall$ t, and $\forall$ $j = \{A, B\}$, it must be that $(\tau_t^{i*})^A > (\tau_t^{i*})^B$*

Proof. Equations (17) and (18) characterize τ_t^{i*}. Considering that $\alpha \frac{w_t^i}{p_t}$, from their equalization we have that

$$F(\tau_t^{i*}, S_t^i) = (1-\alpha-\beta) - \frac{\varphi(D_t)}{2} \frac{\tau_t^{i*2}}{S_t^i} - \frac{\varphi(D_t)\tau_t^{i*}}{S_t \frac{\partial g(\eta_t^i = \eta_t^{i,h}|S_{t+1}^i)}{\partial S_{t+1}^i}} \left[\frac{w_{t+1}^u}{w_{t+1}^h - w_{t+1}^u} + g(\eta_t^i = \eta_t^{i,h}|S_{t+1}^i) \right] = 0$$

Considering the assumptions about $\varphi(D_t)$, and $g(\eta_t^i = \eta_t^{i,h}|S_{t+1}^i)$, it follows that

$$\frac{\partial F(\tau_t^{i*}, S_t^i)}{\partial \tau_t^{i*}} < 0$$

$$\frac{\partial F(\tau_t^{i*}, S_t^i)}{\partial S_t^{i*}} > 0$$

Applying the implicit function theorem we have that

$$\frac{\partial \tau_t^{i*}}{\partial S_t^i} = -\frac{\frac{\partial F(\tau_t^{i*}, S_t^i)}{\partial S_t^{i*}}}{\frac{\partial F(\tau_t^{i*}, S_t^i)}{\partial \tau_t^{i*}}} > 0$$

When $\alpha \frac{w_t^i}{p_t} < \bar{c} < \frac{w_t^i}{p_t}$ the proof follows in the same manner. □

Lemma 9.7. *Consider identical technological and social conditions in A and B, if* $(Y_t^{m,N})^j > 0$ *and* $(N_t)^A > (N_t)^B$*, then,* $\forall\, t$*, and* $\forall\, j = \{A, B\}$*, it must be that* $(\tau_t^{i*})^A > (\tau_t^{i*})^B$

Proof. Equations (17) and (18) characterize τ_t^{i*}. Considering that $\alpha \frac{w_t^i}{p_t}$, from their equalization we have that

$$F(\tau_t^{i*}, S_t^i) = (1-\alpha-\beta) - \frac{\varphi(D_t)}{2}\frac{\tau_t^{i*2}}{S_t^i} - \frac{\varphi(D_t)\tau_t^{i*}}{S_t \frac{\partial g(\eta_t^i = \eta_t^{i,h}|S_{t+1}^i)}{\partial S_{t+1}^i}} \left[\frac{w_{t+1}^u}{w_{t+1}^h - w_{t+1}^u} + g(\eta_t^i = \eta_t^{i,h}|S_{t+1}^i)\right] = 0$$

Considering the assumptions about $\varphi(D_t)$, and $g(\eta_t^i = \eta_t^{i,h}|S_{t+1}^i)$, it follows that

$$\frac{\partial F(\tau_t^{i*}, S_t^i)}{\partial \tau_t^{i*}} < 0$$

$$\frac{\partial F(\tau_t^{i*}, S_t^i)}{\partial N_t^i} > 0$$

Applying the implicit function theorem we have that

$$\frac{\partial \tau_t^{i*}}{\partial N_t^i} = -\frac{\frac{\partial F(\tau_t^{i*}, S_t^i)}{\partial N_t^i}}{\frac{\partial F(\tau_t^{i*}, S_t^i)}{\partial \tau_t^{i*}}} > 0$$

When $\alpha \frac{w_t^i}{p_t} < \bar{c} < \frac{w_t^i}{p_t}$ the proof follows in the same manner. □

Lemma 9.8. *In traditional societies resource allocation follows this conditions*

1. *The employment of labor in agriculture is*

$$L_t^a = \begin{cases} \frac{p_t}{w_t^i}\bar{c}N_t & \text{if } \alpha \frac{w_t^u}{p_t} < \bar{c} \\ \alpha N_t & \text{if } \alpha \frac{w_t^u}{p_t} \geq \bar{c} \end{cases} \tag{55}$$

2. *The employment of labor in manufacturing is*

$$L_t^{m,0} = \begin{cases} \frac{\beta N_t}{1-\alpha}\left(1 - \frac{p_t}{w_t^i}\bar{c}\right) & \text{if } \alpha \frac{w_t^u}{p_t} < \bar{c} \\ \beta N_t & \text{if } \alpha \frac{w_t^u}{p_t} \geq \bar{c} \end{cases} \tag{56}$$

3. *The aggregate spending in child rearing is*

$$\sum \theta n_t^{i,u} = \begin{cases} \frac{1-\alpha-\beta}{1-\alpha}\left(1 - \frac{p_t}{w_t^i}\bar{c}\right) N_t & \text{if } \alpha \frac{w_t^u}{p_t} < \bar{c} \\ (1-\alpha-\beta) N_t & \text{if } \alpha \frac{w_t^u}{p_t} \geq \bar{c} \end{cases} \tag{57}$$

Proof. It follows from Equations 1, 2, and 13 - 15, considering that $w_t^u = p_t A_t^a$. □

Lemma 9.9. *In modern societies resource allocation follows this conditions*

1. *The employment of labor in agriculture is*

$$L_t^a = \begin{cases} \phi^1(w(h_t), \lambda_t) & \text{if } \alpha \frac{w_t^i}{p_t} < \bar{c} \\ \phi^2(w(h_t), \lambda_t) & \text{if } \alpha \frac{w_t^u}{p_t} < \bar{c} < \alpha \frac{w_t^h}{p_t} \\ \frac{\alpha w(h_t) N_t (\alpha+\beta)[-\beta + w(h_t) + \beta ln'(f(h_t))]}{\beta ln'(f(h_t))[\alpha w(h_t) + w(h_t) - \alpha] + [w(h_t) - \beta][\alpha w(h_t) - \alpha + 1]} & \text{if } \alpha \frac{w_t^h}{p_t} \geq \bar{c} \end{cases} \tag{58}$$

2. *The employment of unskillled labor in manufacturing is*

$$L_t^{m,0} = \begin{cases} \phi^3(w(h_t), \lambda_t) & \text{if } \alpha \frac{w_t^i}{p_t} < \bar{c} \\ \phi^4(w(h_t), \lambda_t) & \text{if } \alpha \frac{w_t^u}{p_t} < \bar{c} < \alpha \frac{w_t^h}{p_t} \\ \frac{\beta w(h_t) ln'(f(h_t)) N_t (\alpha+\beta)}{\beta ln'(f(h_t))(\alpha w(h_t) + w(h_t) - \alpha) + (w(h_t) - \beta)(\alpha w(h_t) - \alpha + 1)} & \text{if } \alpha \frac{w_t^h}{p_t} \geq \bar{c} \end{cases} \tag{59}$$

3. *The employment of skilled labor in manufacturing is*

$$H_t = \begin{cases} \phi^5(w(h_t), \lambda_t) & \text{if } \alpha \frac{w_t^i}{p_t} < \bar{c} \\ \phi^6(w(h_t), \lambda_t) & \text{if } \alpha \frac{w_t^u}{p_t} < \bar{c} < \alpha \frac{w_t^h}{p_t} \\ -\frac{N_t(\alpha+\beta)((\alpha-1)(w(h_t) - \beta) + \alpha\beta ln'(f(h_t)))}{\beta ln'(f(h_t))(\alpha w(h_t) + w(h_t) - \alpha) + (w(h_t) - \beta)(\alpha w(h_t) - \alpha + 1)} & \text{if } \alpha \frac{w_t^h}{p_t} \geq \bar{c} \end{cases} \tag{60}$$

4. *The aggregate spending in child rearing is*

$$\sum \theta n_t^{i,u} = \begin{cases} \phi^6(w(h_t), \lambda_t) & \text{if } \alpha \frac{w_t^i}{p_t} < \bar{c} \\ \phi^7(w(h_t), \lambda_t) & \text{if } \alpha \frac{w_t^u}{p_t} < \bar{c} < \alpha \frac{w_t^h}{p_t} \\ (1-\alpha-\beta) N_t & \text{if } \alpha \frac{w_t^h}{p_t} \geq \bar{c} \end{cases} \tag{61}$$

Proof. It follows from Equations 1, 3, and 13 - 15, considering that $w_t^u = A_t^m w^u(h_t)$ and $w_t^h = A_t^m w^h(h_t)$. □

Lemma 9.10. *The following conditions characterize the sectorial balance of the economy*

$$\frac{Y_t^m}{Y_t^a} = \begin{cases} \frac{\beta(A_t^a - \bar{c})a_t^m}{(1-\alpha)\bar{c}A_t^a} & \text{if } t < t^M \;\wedge\; \alpha\frac{w_t^u}{p_t} < \bar{c} < \frac{w_t^u}{p_t} \\ \frac{\beta a_t^m}{\alpha A_t^a} & \text{if } t < t^M \;\wedge\; \alpha\frac{w_t^u}{p_t} \geq \bar{c} \\ \frac{\beta A_t^m}{(1-\alpha)A_t^a}\frac{H_t(A_t^m w^h(h_t) - \bar{c}w^u(h_t)) + L_t w^u(h_t)(1-\bar{c})}{\bar{c}N_t} & \text{if } t > t^M \;\wedge\; \alpha\frac{w_t^u}{p_t} < \alpha\frac{w_t^h}{p_t} < \bar{c} \\ \frac{\beta A_t^m w^u(h_t)[H_t w^h(h_t) + L_t w^u(h_t)(1-\bar{c})((1-\alpha)A_t^a)^{-1}]}{\alpha H_t A_t^a w^h(h_t) + L_t\bar{c}} & \text{if } t > t^M \;\wedge\; \alpha\frac{w_t^u}{p_t} < \bar{c} < \alpha\frac{w_t^h}{p_t} \\ \frac{\beta A_t^m w^u(h_t)}{\alpha A_t^a} & \text{if } t > t^M \;\wedge\; \alpha\frac{w_t^u}{p_t} \geq \bar{c} \end{cases} \tag{62}$$

Proof. It follows from Equations 13-15, considering that $w_t^u = A_t^m w^u(h_t)$ and $w_t^h = A_t^m w^h(h_t)$. □

Lemma 9.11. *Consider identical technological and social conditions in A and B. If A experienced an early industrial take-off in period $\tilde{t}-1$, then*

$$l(h_{\tilde{t}+1})^A > l(h_{\tilde{t}+1})^B \;\wedge\; l(h_t)^A \geq l(h_t)^B \quad \forall t > \tilde{t}+1 \tag{63}$$

Proof. It follows from Proposition 3 and the properties of $l(h_t)$. □

9.2 Data Construction

9.2.1 Industrial Take-off

To approximate the moment when modern industrial activity widespreads in a country I use the concept of the "5% club" by Bénétrix et al. (2015). A country arrives to the "5% club" the first year in which the industrial sector has a cumulative ten-year growth rate superior to 5% per annum. Bénétrix et al. (2015) record this variable for the "periphery". I use the data they present. I assign the values of ancient political units to their current equivalents in the following way. For data on Ukraine, I impute the take-off year of Poland; for Czech Republic, the one of Czechoslovakia; for Slovenia, Croatia, and Serbia, the one of Yugoslavia; for Georgia, Belarus, Moldova, Tajikistan, Kyrgyz Republic, the one of Russia; for Armenia, the one of Turkey.

For the countries that Bénétrix et al. (2015) do not offer information, I use the data in Bairoch (1982). For the countries in Bairoch (1982), I identify their industrial take-off as the year when they reach the same industrial per capita output of the U.K. in 1750–year usually recognized as the beginning of the British Industrial Revolution. The data presented by Bairoch (1982) has a three-decades periodicity. I assume linear trends in order to have more granular data.

9.2.2 Linguistic Distance

I use the estimates by Desmet et al. (2009) and Desmet et al. (2012), who employ the Etnologue's language-trees data and the methods of (Fearon, 2003). Some notation is necessary to understand the indexes.

First, consider a country with a population of N individuals, distributed into K distinct groups, indexed by $j = 1, ..., K$. Each individual is assigned to one and only one group. The population of group j is denoted by N_j. $s_j = N_j/N$ is the share of group j in the total population of the country.

Consider the linguistic distance between groups j and k, τ_{jk}, as a measure of how far away from a common origin are the languages spoken by groups j and k. For example, Spanish is closer to Italian than Chinese, because Spanish shares with Italian a close common ancestor–i.e. Latin. The distance is a standardized metric–i.e. $\tau_{jk} \in [0, 1]$. The linguistic distance between every pair of languages is assigned into a matrix T. T is symmetric and has a 0 diagonal–i.e. $\tau_{jj} = 0 \quad \forall j \in K$ and $\tau_j k = \tau_k j \quad \forall j \wedge k \in K$.

The GI index is defined as as weighted averages of the differences between the different groups of a country.

$$GI(T) = \sum_{j=1}^{K} \sum_{k=1}^{K} s_j s_k \tau_{jk} \tag{64}$$

The ELF index is calculated in a similar way. The most important difference with the GI index is that the ELF does not consider a continuous distance but a discrete difference between groups. Thus, instead of a T matrix, we will have a T^d matrix, in which the entrance will be such that $\tau_{jk} = 1 \quad \forall j \neq k$ and 0, otherwise.

$$ELF(T^d) = 1 - \sum_{j=1}^{K} s_j^2 \tag{65}$$

An important aspect of the ELF index considered here is that it refers to the highest level of the language tree.

9.2.3 Fertility

United States: For the 19th century I use the estimates for the white population made by Haines and Steckel (2000). These estimates are based on Census information; thus, the information available is for every decade. For the 20th century, I use the estimates of Bureau

(2000), which uses data from the National Center for Health Statistics. They offer quinquennial frequency data.

United Kingdom: For the period 1600-1860 I use the quinquennial data for England constructed by Wrigley (1997). They build them using parishes records. From 1860-1960 I use the data in Mitchell (1975), who report annual birth rates estimates for England and Wales. He made his estimates based on the Statistical Abstract of the United Kingdom. Finally, I use the data presented by World Bank (2018) from 1960 onwards, which comes from official sources. This last source refers to the whole United Kingdom.

9.2.4 Miscegenation

I use the microdata from the Census available in (IPUMS, 2018). I estimate the number of single-family households in which the head of the household and her partner reported different races. There is no information available for race in the 1890 Census. I use the 1% density samples for 1850-1970. For 1980-2000 I use the 5% density samples. I expand the sample in order to have estimates for the complete unit of analysis.

I use the general variable of race, which include seven categories of race: White, Black, American Indian or Alaska Native, Chinese, Japanese, Other Asian or Pacific Islander, and Other race. Prior to 1960, the census enumerator was responsible for categorizing persons and was not specifically instructed to ask the individual his or her race. In 1970 and later years, an individual's race was reported by someone in the household or group quarters. In the 1990 U.S. census, the 2000 U.S. respondents were specifically asked what race the person "considers himself/herself" to be, although such self-description was more or less operative since 1960.

I define an interracial marriage as a situation in which the head of the household and her partner had different races.

For the elite of Antioquia, I use the data available in (Mejía, 2015). His data comes from a large-scale historiographical collection. This collection implied crossing sources of different nature that incorporated economic, demographic, historic and biographical data. Based on these data he constructs a network that evolves every decade. The data use here comes from the network metrics he presents.