

LSE Undergraduate Admissions Assessment

Mathematics – Section C

You can work on paper and scan your solutions or type out your answers onto the computer. Use of mathematical typing software is not required. You should **not use a calculator** or any other calculation device.

Paper D Total = 100 marks

1. In this question **a** is a non-zero positive integer and answers should be left in **terms of a** when needed.

$$f(x) = x^2 - 3ax + 2a^2$$

$$g(x) = x^3 + ax^2 - a^2x - a^3$$

a)

i) Write $f(x)$ as the product of 2 linear factors.

ii) Write $g(x)$ as the product of 3 linear factors.

5 marks

b) Hence or otherwise find

i) $xf(x) + g(x)$ as a product of a linear factor and a quadratic factor

ii) $\frac{f(x)}{g(x)}$ in the form $\frac{x-c}{(x+d)(x+e)}$ where c, d and e are integers and may be written in terms of a

6 marks

Total 11 marks

2. In this question A,B,C,D are real numbers and answers can be left in **terms of A,B,C,D** when needed.

For each part i) rearrange the formula to make x the subject ii) state the set of value(s) of A,B,C or D that would ensure that x is a real number

a) $y = \frac{Ax+2}{Ax-1}, x, y \neq 1, x, y \in \mathbb{R}$

4 marks

b) $\ln y = \ln x + \ln B - 2 \ln(B - 1), x, y \in \mathbb{R}, x, y > 0$

4 marks

c) $C = (x - 1)(x + 2), x \in \mathbb{R}$

4 marks

d) $D = \frac{4^x}{2^{Dx}}, x \in \mathbb{R}$

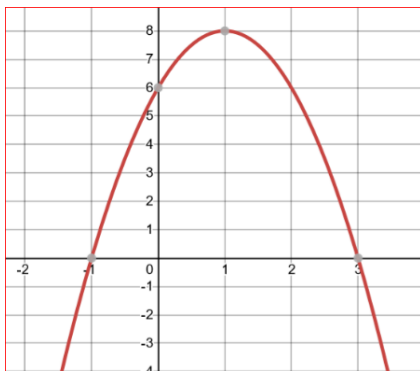
5 marks

Total 17 marks

3

Each of the graphs below is plotted accurately and shows all of the intercepts with the x and y axes and all turning points. All the intercepts are at integer values. Any asymptotes are shown with dotted lines. Find the equation of each graph.

a)



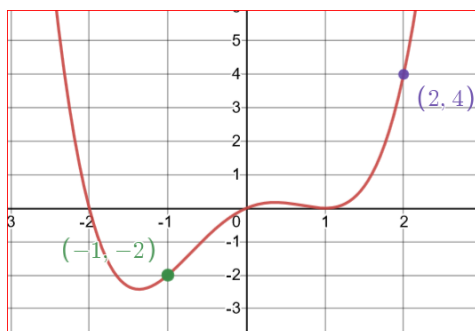
3 marks

b)



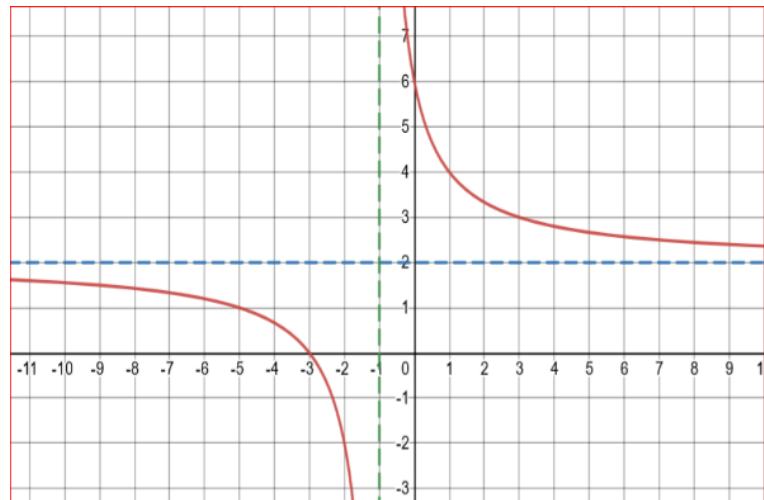
4 marks

c)



5 marks

d) The asymptotes are
 $x = -1$ and $y = 2$



4 marks

Total 16 marks

4. The table shows values of $f(x)$, $f'(x)$ and $f''(x)$ where $x = 1$ and $x = 3$

	$x = 1$	$x = 3$
$f(x)$	3	5
$f'(x)$	4	10
$f''(x)$	6	12

a) Find the values of

i) $f(x^2)$ when $x = 1$

ii) $x^2 f(x)$ when $x = 3$

iii) $f(f(x))$ when $x = 1$

3 marks

b) Find $h'(3)$ when i) $h(x) = x^2 f(x)$ ii) $h(x) = f(\frac{1}{3}x^2)$ iii) $h(x) = \frac{x^2}{f(x)}$

9 marks

c) Given that $f(x) > 0$ for all values of x and that $f(x)$ is continuous and differentiable find the values of the following integrals (giving your answers exactly).

i) $\int_1^3 f'(x) dx$

ii) $\int_1^3 \frac{f'(x)}{f(x)} dx$

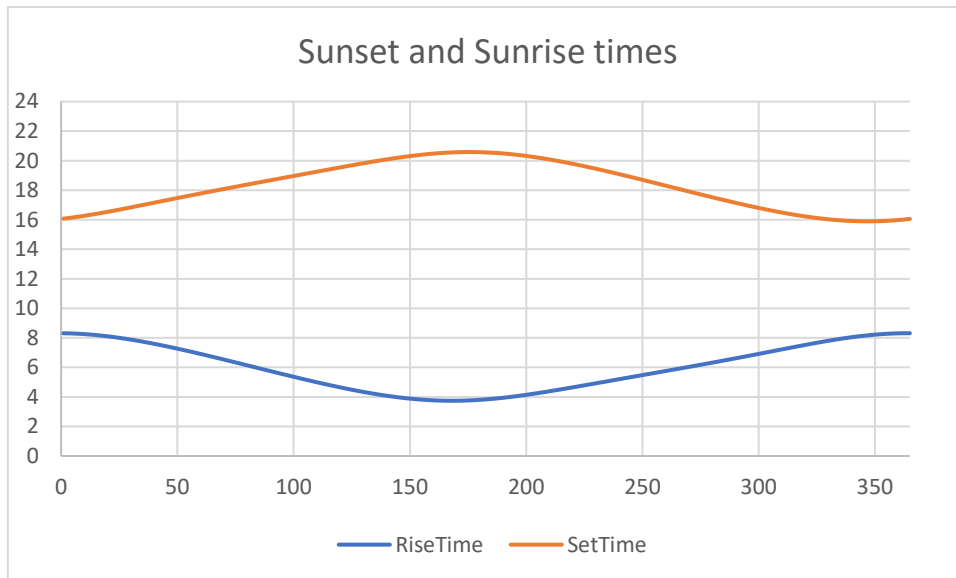
iii) $\int_1^3 f'(x)(f(x))^2 dx$

iv) $\int_1^3 x f''(x) dx$

12 marks

Total 24 marks

5. A student, Jez wants to model sunrise and sunset data. Jez collects the data for their home in the UK over a complete year and produces the following graph.



a) The horizontal and vertical axes are not labelled. Suggest suitable labels for each axis with appropriate units.

2 marks

b) Jez decides that a suitable model for the Sunset time is

$$f(x) = 2.3455\cos(0.017214(x - 175)) + 18.2395$$

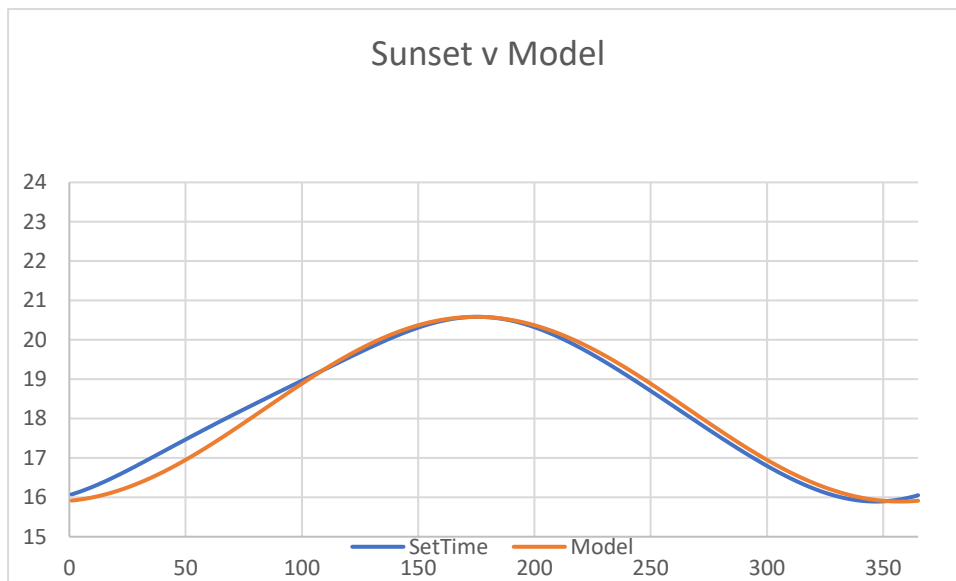
i) Give two reasons why using the cosine function as a basis for the model would be a good choice.

ii) What other function as a basis would have been a good choice?

iii) Write down the calculation Jez would have used to get the number 0.017214. **(You are not expected to do this calculation).**

4 marks

Jez compares the model with the data to get the following graph:



c) Comment on whether the model is reasonable.

2 marks

d) For Sunrise Jez devises a similar model using the earliest Sunrise time at 3.7 hours on day 168 and the latest sunrise time at 8.3 hours on day 364. Write down a suitable model $g(x)$ that Jez could use.

5 marks

e) i) Jez calculates $f(x) - g(x)$. What is Jez finding?

ii) Jez finds 2 values of x where $f'(x) - g'(x) = 0$. What do these two values of x represent?

3 marks

Total 16 marks

6. A restaurant manager commissions a water feature designed as a scaled-up cocktail glass. He is going to fill the conical part with water until it cascades down the sides.

The top of the cocktail glass is a cone with diameter 6cm and height 4cm . The total height of the cocktail glass is 9cm .

The manager wants the water feature to be 1.8m high in total.



- a) i) Write down the diameter and height of the conical part of the water feature in cm
- ii) Write down the volume of the conical part of the water feature in cm^3 giving your answer in terms of π

4 marks

b) For dramatic effect the manager wants the conical part of water feature to fill up in half an hour. What is the rate at which the cone is to be filled with water? Give your answer in terms of π in cm^3 per minute.

2 marks

- c) i) Find a formula for the volume of water (V) in the cone when the height of water is $h\text{ cm}$
- ii) Find the rate, in terms of h , at which the height of the water is increasing whilst the cone is filling. Give your answer in cm per minute .
- iii) Find the rate at which the height of the water is increasing just before it overflows. Give your answer in cm per minute .

10 marks

Total 16 marks